## CAMRAD II

COMPREHENSIVE ANALYTICAL MODEL OF ROTORCRAFT AERODYNAMICS AND DYNAMICS

Volume VII: Rotorcraft Training


Wayne Johnson
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# CAMRAD II <br> COMPREHENSIVE ANALYTICAL MODEL OF ROTORCRAFT AERODYNAMICS AND DYNAMICS 

Volume VII: Rotorcraft Training
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Wayne Johnson

Johnson Aeronautics

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## Chapter 1

## INTRODUCTION

CAMRAD II IS AN AEROMECHANICAL ANALYSIS OF HELICOPTERS AND ROTORCRAFT

INCORPORATING ADVANCED TECHNOLOGY
multibody dynamics
nonlinear finite elements
rotorcraft aerodynamics and wakes

## CAMRAD II IS A COMPREHENSIVE ANALYSIS

for design, testing, evaluation of rotors and rotorcraft
at all stages, including research, conceptual design, detailed design, and development
calculates performance, loads, vibration, response, stability
with a consistent, balanced, yet high level of technology in a single computer program
applicable to wide range of problems, wide class of rotorcraft

SUCH CAPABILITY IS ESSENTIAL FOR HELICOPTER PROBLEMS, WHICH ARE INHERENTLY COMPLEX AND MULTIDISCIPLINARY

CAMRAD II USES A BUILDING BLOCK APPROACH TO ACHIEVE FLEXIBILITY IN MODEL OF DYNAMIC AND AERODYNAMIC CONFIGURATION

CAN MODEL TRUE GEOMETRY OF ROTORCRAFT
MULTIPLE LOAD PATHS
swashplate and control system; lag dampers;
tension/torsion straps; bearingless rotors
VIBRATION CONTROL DEVICES
such as pendulum absorbers or active control
ARBITRARY HINGE ORDER
DROOPED AND SWEPT TIPS
ARBITRARY ELASTIC AXIS AND QUARTER CHORD DISSIMILAR BLADES

CAMRAD II PROVIDES A POWERFUL ANALYSIS CAPABILITY
ADVANCED ROTOR AERODYNAMICS
RIGOROUS KINEMATICS AND DYNAMICS
consistent structural loads and dynamic response; general interfaces between aerodynamic and structural dynamic components
GENERAL TRANSIENTS
CAMRAD II IS A PRACTICAL ENGINEERING TOOL CORE FOR FLEXIBILITY, ROTORCRAFT SHELL FOR EASE OF USE
shell builds typical rotorcraft and rotor models;
core input always gives capability to change model
RANGE OF COMPONENTS AND MODELLING OPTIONS
can balance efficiency and accuracy for a particular problem

CAMRAD II OFFERS A DESIGN FOR GROWTH appropriate platform for future developments, for continuing access to new technology

## 1-1 Background

## CAMRAD II INCORPORATES COMBINATION OF ADVANCED TECHNOLOGY

MULTIBODY DYNAMICS
rigid body components, frames, and joints; nonlinear kinematics; joint operations
but necessary to deal with elastic motion
STRUCTURAL DYNAMICS
substructure coupling and static residuals;
modal analysis and truncation;
and elimination of constraints
but necessary to deal with nonlinear systems

## NONLINEAR FINITE ELEMENTS

nonlinear elements, numerical integration, and beam components
but necessary to deal with large motion (so do not use nodal motion as degrees of freedom)
rotorcraft problems involve more than just structural dynamics (so no system Lagrangian; and use explicit constraint forces)

ROTORCRAFT AEROMECHANICS
aircraft dynamics, rotating systems, and aerodynamics; and define tasks and results required of analysis

## GENERAL REQUIREMENTS FOR MODERN COMPREHENSIVE ANALYSIS

> MUST CALCULATE PERFORMANCE, LOADS, VIBRATION, RESPONSE, STABILITY
> so comprehensive analysis needs technology for rotor wake, drag and stall, nonlinear dynamics, entire aircraft and must perform trim, transient, and flutter tasks

## ANALYSIS MUST HANDLE COMPLEX CONFIGURATIONS

unusual load paths and interactions, and many subsystems structural, aerodynamic, and kinematic nonlinearities arbitrary large motion, including rigid body motions and rotation of components relative to each other and not just equations and interfaces of structural dynamics

## ANALYSIS MUST BE FLEXIBLE

 separate specification of configuration, aeromechanical models, and solution procedures
## ANALYSIS MUST BE EASY TO USE

 especially for normal configurations
## ANALYSIS MUST BE PRACTICAL

efficient, accurate, and reliable

## 1-2 Approach

CAMRAD II PERFORMS NONLINEAR DYNAMIC/STATIC ANALYSIS OF AEROMECHANICAL SYSTEM

CAMRAD II TASKS
SOLVE DIFFERENTIAL, INTEGRAL, STATIC, AND IMPLICIT EQUATIONS FOR MOTION OF THE SYSTEM

## AND EVALUATE REQUIRED OUTPUT QUANTITIES FROM RESPONSE

TRIM, TRANSIENT, FLUTTER TASKS (figure 1)

TRIM TASK
EQUILIBRIUM SOLUTION FOR STEADY STATE OPERATING CONDITION
free flight: level flight, steady climb or descent, steady turns
constrained: rotor in wind tunnel; typically trim thrust and flapping to target values
identify control positions and aircraft orientation required to achieve specified operating condition

STEADY OR PERIODIC RESPONSE OF SYSTEM
UNIFORM INFLOW, NONUNIFORM INFLOW WITH
PRESCRIBED WAKE GEOMETRY, OR NONUNIFORM INFLOW WITH FREE WAKE GEOMETRY
solve equations (differential, integral, static, implicit) for motion of system evaluate required output quantities from response
 representation

Figure 1-1 CAMRAD II tasks.

## TRANSIENT TASK

NUMERICALLY INTEGRATE IN TIME (FROM TRIM SOLUTION) FOR PRESCRIBED EXCITATION

## FLUTTER TASK

DIFFERENTIAL EQUATIONS, LINEARIZED ABOUT TRIM (PROBABLY NUMERICAL PERTURBATION)
full dynamics, or quasistatic reduction of selected variables (including stability derivative representation)

RESPONSE OF TIME-INVARIANT (PERHAPS AVERAGED) OR PERIODIC EQUATIONS

## FLEXIBILITY OF CONFIGURATION MODEL

MODERN COMPREHENSIVE ANALYSIS MUST BE ABLE TO ANALYZE ARBITRARY CONFIGURATION - WHATEVER DESIGNERS CAN INVENT

## OLD APPROACH (CAMRAD/JA):

FIXED GEOMETRY AND DYNAMIC MODELS, FIXED AERODYNAMIC MODELS
new rotor or helicopter configuration requires new development of equations
structural dynamic and aerodynamic models mixed; can not change one without considering other

REQUIREMENT:
CONFIGURATION DEFINED AND CHANGED BY INPUT TO ANALYSIS

DO NOT NEED NEW CODE AS LONG AS REQUIRED PHYSICS AVAILABLE
separate physical and mathematical models from definition of configuration
separate structural dynamic and aerodynamic models

CAMRAD II:
USES BUILDING-BLOCK APPROACH TO ACHIEVE FLEXIBILITY
flexibility and generality of system configuration
obtained by assembling standard components with standard interfaces
and solving system using standard procedures

CAMRAD II SYSTEM DESCRIPTION
FOR CONFIGURATION GENERALITY, CAMRAD II SPLITS SYSTEM INTO PIECES, WITH CONNECTIONS BETWEEN

PIECES AVAILABLE TO CONSTRUCT SYSTEM:

| environmental | physical | logical |
| :--- | :--- | :--- |
| case | component | loop |
| wind | frame | part |
| operating condition | interface | transform |
| period | output | modes |
|  | input | response |
|  |  | weights |

environmental pieces: describe system operation
physical pieces: produce system equations logical pieces: solve system equations

STANDARD DESCRIPTIONS OF COMPONENTS, INTERFACES, AND SOLUTION PROCEDURES REQUIRED
structural dynamic systems are important subset of problems
aerodynamic models also important CONVENTIONS AND METHODOLOGY MUST ENCOMPASS ALL TYPES OF COMPONENTS, INTERFACES, AND SOLUTION PROCEDURES

THESE SYSTEM PIECES CONSTITUTE THE CORE ANALYSIS, PROVIDING A FLEXIBLE, BUILDING-BLOCK ORIENTED MODELLING CAPABILITY

## CAMRAD II COMPONENTS

# COMPONENTS PERFORM ALL COMPUTATIONS ASSOCIATED WITH PHYSICS OF A MODEL OF SYSTEM 

 development of improved model requires constructing new component, which will fit into existing analysis framework
## COMPONENTS AVAILABLE TO CONSTRUCT SYSTEM:

rigid body
linear normal modes
finite element beam
rod/cable

transmission
reference frame
filter
reference plane
differential equation
programmable
transfer function
Fourier series
prescribed control
gust
rigid airframe aerodynamics
airframe flow field
lifting line wing
rigid wing
wing inflow
rotor inflow
rotor dynamic wake
wing wake
wing wake geometry rotor wake geometry
wing performance rotor performance rotorcraft performance
helicopter tail boom computational fluid dynamics
plugin
structural dynamic, aerodynamic, differential equation, and performance models

## REQUIRE STANDARD FORM FOR COMPONENTS

must accommodate all kinds of components
must also handle and use special characteristics of structural dynamic components

STRUCTURAL DYNAMIC COMPONENTS HAVE COMMON CHARACTERISTICS
common implementation for structural dynamic components - differ in model of elastic motion

## OTHER KINDS OF COMPONENTS AS WELL, INCLUDING AERODYNAMICS

can not assume have symmetric, second order, differential equations
can not assume constraint equations are obtained from displacements
can not base component model on system Lagrangian

## FLEXIBILITY OF SOLUTION PROCEDURE <br> DEFINITION OF SOLUTION PROCEDURE MUST BE JUST AS FLEXIBLE AS DEFINITION OF CONFIGURATION

OLD APPROACH (CAMRAD/JA):
ONLY ONE SOLUTION METHOD; LITTLE SYSTEMATIC DEVELOPMENT OF SOLUTION PROCEDURES
solution procedure and physical models mixed can not change one without considering other

REQUIREMENT:
SOLUTION PROCEDURE DEFINED AND CHANGED BY INPUT TO ANALYSIS

DO NOT NEED NEW CODE AS LONG AS REQUIRED PROCEDURES AVAILABLE
separate solution procedure from aeromechanical model and configuration

CAMRAD II:
USES BUILDING-BLOCK APPROACH TO ACHIEVE FLEXIBILITY

## EXPANDABILITY

BUILDING-BLOCK APPROACH ESSENTIAL FOR EXPANDABILITY

MUST SEPARATE SPECIFICATION OF CONFIGURATION, AEROMECHANICAL MODEL, AND SOLUTION PROCEDURE

OTHERWISE SMALLEST CHANGE INVOLVES ENTIRE ANALYSIS
growth becomes increasingly harder as each new feature added

DEVELOPMENTS SINCE INITIAL RELEASE HAVE DEMONSTRATED EXPANDABILITY OF CAMRAD II

EVEN MAJOR EXTENSIONS TYPICALLY REQUIRE ONLY ONE OR TWO WEEKS
including documentation and testing
NOT INCLUDING DEVELOPMENT OF THEORY

# MATHEMATICAL MODEL <br> MINIMIZE APPROXIMATIONS AT TOP LEVEL OF ANALYSIS <br> ESSENTIAL FOR EXPANDABILITY - TOP LEVEL APPROXIMATIONS AND ASSUMPTIONS ARE VERY DIFFICULT TO CHANGE 

CAMRAD II:
finite-dimension description of system
components and interfaces introduce spatial discretization
solution procedures introduce time discretization structural dynamic interfaces are holonomic and independent

## NO FURTHER APPROXIMATIONS AT TOP LEVEL

coupling and solution procedures can handle arbitrary and exact models, with nonlinear and time-varying equations

## APPROXIMATIONS ARE IN COMPONENTS

required for practical solution of most problems new technology and more accurate models introduced by constructing new components, without changing framework of analysis

MATHEMATICAL MODEL

## BUILDING BLOCK APPROACH LEADS TO MORE GENERAL, MORE RIGOROUS MODELS

separate physical and logical pieces
separate structural dynamic and aerodynamic models
each piece capable of general analysis

## OLD APPROACH (CAMRAD/JA)

solution procedure: mixes part/loop solution for rotor and airframe motion; does not identify all iteration variables
wing model: assumes geometry is that of specific structural model; first order velocity and displacement

## CAMRAD II

solution procedure: sound mathematical basis; automatically finds iteration variables
wing model: general line geometry; droop and sweep no different than flap, lag, and bending, and all can be large; can connect to any structural model, with exact kinematics of velocity, displacement, force interface

MATHEMATICAL MODEL

## EXACT KINEMATICS OF RIGID BODY MOTION AND STRUCTURAL DYNAMIC INTERFACES

large displacement, exact kinematics of large rotations ASSEMBLY OF SYSTEM NOT IMPLEMENTED BY IDENTIFYING DEGREES OF FREEDOM OR VARIABLES OF TWO COMPONENTS AS EQUAL
must solve nonlinear, time-varying constraint equations for some variables (degrees of freedom or forces)

REQUIRED TO MODEL ARBITRARY CONFIGURATIONS, ARBITRARY MOTION

## ESSENTIAL FOR CAPABILITY TO MODEL ROTOR HUB CONFIGURATIONS

otherwise rotating-to-nonrotating frame connection is special (large rotation)

OLD APPROACH (CAMRAD/JA): one rotating-tononrotating frame connection, at hub node; pitch bearing with spring/actuator
control system load path model wrong without swashplate node

CAMRAD II: arbitrary, general hub and swashplate model

## TRANSPORTABILITY

CAMRAD II MUST BE EASILY INSTALLED ON MANY DIFFERENT COMPUTERS
installation on more than 10 different machines so far

FOR TRANSPORTABILITY, WRITTEN IN STANDARD FORTRAN

BUT FORTRAN DOES NOT HAVE DATA STRUCTURES NEEDED TO IMPLEMENT FLEXIBLE ANALYSIS
need records and lists; arbitrary numbers and arbitrary sizes

CAMRAD II WRITTEN USING SOFTWARE TOOL
TOOL DESIGNED FOR DEVELOPMENT, MODIFICATION, AND MAINTENANCE OF LARGE FORTRAN PROGRAM
emphasizes definition, access, and documentation of data structures
dictionary of data structures; prologue defines subroutine communication through data structures; variables uses almost like standard Fortran variables; translator produces compilable source

## EASE OF USE

COMPREHENSIVE ANALYSIS MUST BE EASY TO USE
especially for normal or typical configurations not same as saying helicopters should be easy to analyze

## FLEXIBLE ANALYSIS (CORE SYSTEM) REQUIRES LARGE AMOUNT OF DETAILED INPUT INFORMATION

helicopter is complex system - 1000's of system pieces

CAMRAD II HAS ROTORCRAFT SHELL TO CONSTRUCT CORE INPUT FOR TYPICAL CONFIGURATIONS AND TYPICAL PROBLEMS
builds multi-rotor aircraft, in free flight or wind tunnel
N -bladed rotor with general hub, perhaps with swashplate control
constructs solution procedure as well
figure 2 is simplified description of model figure 3 shows example of rotor constructed


Figure 1-2 Rotorcraft model (simplified).


Figure 1-3 Example of rotor constructed.

## SHELL ISOLATES USER FROM DETAILS OF SYSTEM DEFINITION

ACCOMPLISH 2-3 ORDERS OF MAGNITUDE REDUCTION IN AMOUNT OF INPUT INFORMATION

SHELL DOES NOT HAVE FLEXIBILITY OF CORE INPUT, AND MAY NOT BE ABLE TO MODEL EXACTLY EVERY CONFIGURATION

SHELL CONSTRUCTS MOST OF SYSTEM
minimizing need to deal directly with core input what shell constructs will provide guidance for use of core input

CAN USE CORE INPUT TO CHANGE MODEL CONSTRUCTED BY SHELL

## 1-3 Sources of Information

CAMRAD II DOCUMENTATION

| VOLUME I: | THEORY |
| :--- | :--- |
| VOLUME II: | COMPONENTS THEORY |
| VOLUME III: | ROTORCRAFT THEORY |
| VOLUME IV: | INPUT |
| VOLUME V: | COMPONENTS INPUT |
| VOLUME VI: | ROTORCRAFT INPUT |
| VOLUME VII: | ROTORCRAFT TRAINING |
| VOLUME VIII: | PROGRAM |
| VOLUME IX: | PROGRAM TRAINING |

SAMPLE JOBS
EXTRAS
NEWSLETTER
SOFTWARE SUPPORT

WEB SITE: www.johnson-aeronautics.com or www.camrad.com documentation, technical papers, sample jobs, and extras can be downloaded by customers

## 1-4 General Warnings and Limitations

## NOT YET POSSIBLE TO ACCURATELY CALCULATE OR PREDICT

 AEROMECHANICAL SYSTEM BEHAVIOR IN ALL CIRCUMSTANCES always possible to get bad answer from bad input or misuse of analysisalso possible that best results of analysis may not be accurate computer program is a tool, not substituting for but rather to be used with judgement, experience, and much testing of the actual system

## ANALYSIS CONSTRUCTS SYSTEM BY DIVIDING IT INTO COMPONENTS, WITH EACH COMPONENT A MODEL OF THE ACTUAL DEVICE

rarely are complete equations being solved for exact response empiricism and approximations are required in order to make the analysis practical
engineering judgement required to use and interpret analysis, based on experience with code, particularly projects using code to correlate with measured results

Chapter 2

## RUNNING CAMRAD II PROGRAMS

GENERAL REFERENCE FOR CHAPTER: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapter "Running CAMRAD II Programs"

## 2-1 Main Programs

CAMRAD II MAIN PROGRAMS
ROTORCRAFT ANALYSIS (CAMRADII)
INPUT AND TABLE FILE PREPARATION (INPUT)
PLOT FILE EXAMINATION (OUTPUT)
programs communicate by means of files
figure 1 shows relation between programs for typical application


Figure 2-1 Relation between CAMRAD II programs.

TYPICAL STEPS TO RUN CAMRAD II
CREATE AIRFOIL TABLE FILE
program INPUT
creates CAMRAD (unformatted) airfoil table file for entire blade
from STANDARD (free form) or C81 (fixed form) airfoil table file(s)

CREATE SHELL INPUT FILE program INPUT
creates shell input file (unformatted)
from namelist file(s) of baseline parameters (shell input blocks)

RUN ROTORCRAFT ANALYSIS
program CAMRADII
reads airfoil table file and shell input file
one or more cases per job
namelist input for each case (shell input blocks)
create job output and plot file
EXAMINE PLOT FILE
program OUTPUT
creates files for user's graphics routine
from plot file of rotorcraft analysis

## FILE NAME CONVENTIONS REQUIRED TO ORGANIZE FILES TYPICAL FILE NAME EXTENSIONS:

.STD STANDARD airfoil table file
. C81 C81 airfoil table file
.TAB CAMRAD airfoil table file
.LIST namelist input file
. DAT shell input file
. COM command file
. OUT output file
. PLOT plot file

## GENERAL USE OF Input AND output PROGRAMS

INPUT FILE PREPARATION
PREPARE UNFORMATTED FILES FROM NAMELIST
DATA; PRINT INPUT DATA batch or interactive

EXTRACT DATA TO PLOT OR DRAW INPUT;
EXAMINE INITIALIZED SYSTEM DATA interactive (or script file)

TABLE FILE PREPARATION
PREPARE FORMATTED OR UNFORMATTED FILES; PRINT TABLE DATA for each table class/type batch or interactive

EXTRACT DATA TO PLOT TABLE interactive (or script file)

## PLOT FILE EXAMINATION

READ AND EXTRACT DATA FROM PLOT FILE; PRINT LINEAR DIFFERENTIAL EQUATIONS TO LOG FILE interactive (or script file)

## 2-2 Input

## 2-2.1 Shell and Core Input

## CORE INPUT PROVIDES A FLEXIBLE, COMPONENT-ORIENTED ANALYSIS <br> core input defines tables required <br> SHELL INPUT FACILITATES APPLICATIONS TO SPECIFIC PROBLEMS

## SHELL INPUT CONSISTS OF BOTH SHELL AND CORE NAMELIST

figure 2 shows typical CAMRADII input
ANALYSIS USES SHELL INPUT TO PRODUCE CORE DATA, WHICH USER CAN THEN CHANGE USING CORE INPUT
core namelist input stored as changes to core data produced by shell, so these changes can be implemented for subsequent cases in job (without repeating core namelist for each case)

INPUT CONSISTS OF SET OF SECTIONS
identified by class, type, and name
each section corresponds to a system piece
A SHELL OR CORE SECTION IS READ FROM NAMELIST
SHELL OR CORE INPUT CAN BE STORED AS UNFORMATTED FILE
this file can be read before namelists, to initialize all sections

```
setenv BLADEAIRFOIL1 advtecheq.tab
setenv SHELLINPUT windtunnel.dat
setenv PLOTFILE sample.plot
/camrad/camradii > sample.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1, &END
    !=========================================================
    &NLDEF class='TRIM',&END
    &NLVAL WVEL=.4,COLL=10.,PITCH=-5.,LEVEL=2,NWPRNT=1, &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL MASEN=1,MBSEN=1,MHSEN=1,MPSEN=1,&END
    ! ===========================================================
    &NLDEF class='FLUTTER',&END
    &NLVAL DOFM=5*0,&END
    !=========================================================
    &NLDEF class='ROTOR',type='WAKE', name='ROTOR 1',&END
    &NLVAL OPFW=2, &END
    ! =========================================================
    &NLDEF class='AIRFRAME',type='STRUCTURE' ,&END
    &NLVAL OPAERO=1,&END
    &NLDEF class='AIRFRAME',type='AERODYNAMICS', &END
    &NLVAL VISRC=1,&END
```



```
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

Figure 2-2 Typical CAMRADII input.

INPUT SECTIONS AND TABLES ARE UNIQUELY IDENTIFIED BY CLASS, TYPE, AND NAME

ANALYSIS DEFINES ALLOWABLE CLASSES, AND TYPES FOR EACH CLASS

USER DEFINES THE NAMES
within a class, names must be unique, even if the type is different
store and search are based on class and name type defines form of data within a class

EACH CLASS/TYPE CORRESPONDS TO SET OF COMMONS POSSIBLY THERE IS NO TYPE FOR A CLASS

POSSIBLY THERE IS ONLY ONE BLOCK FOR A CLASS/TYPE name is ignored

## 2-2.2 Table Input

TABLES CORRESPOND TO FILES
TABLES PROVIDE GENERAL FILE INPUT CAPABILITY plot file used for general file output

TABLES PROVIDE MECHANISM FOR INPUT OF LARGE ARRAYS OF DATA
table data may be function of continuous variables (interpolated from the data vector)
or table data may be function of discrete variables (moved to record for use)

CORE INPUT DATA DEFINES WHICH TABLES ARE REQUIRED BY ANALYSIS

CORE SECTION CAN DEFINE MORE THAN ONE TABLE REQUIRED

SAME TABLE (SAME CLASS/TYPE/NAME) CAN BE REQUIRED BY MORE THAN ONE CORE SECTION
unit number and logical name of first requirement encountered are used to read table

USER CAN DELETE TABLES NOT USED FOR CASE, OR ALL TABLES AT END OF CASE
to minimize storage in table data vector

A TABLE SECTION IS READ FROM A FILE
FORMATTED OR UNFORMATTED
ONE FILE PER TABLE

TABLE SPECIFICATION CONSISTS OF CLASS, TYPE, AND NAME PLUS FILE UNIT NUMBER AND FILE NAME OR LOGICAL NAME

FILE NAME OR LOGICAL NAME CAN BE SAME AS "NAME" IN CLASS/TYPE/NAME SPECIFICATION
this is the default

FOR MOST INSTALLATIONS, CAN IGNORE UNIT NUMBERS

THERE IS A SPECIFIC FILE FORMAT FOR EACH TABLE CLASS/TYPE

## 2-2.3 System Definition and Names

NAMES (32-CHARACTER STRINGS)
FOR EACH CLASS THERE IS A COLLECTION OF SYSTEM PIECES, IDENTIFIED BY UNIQUE NAMES

USER IDENTIFIES THINGS IN THE INPUT BY NAME THERE ARE SHORT LABELS (8-CHARACTERS) CORRESPONDING TO EACH NAME, FOR USE BY OUTPUT ROUTINES
for core input, a blank name or label produces the default (if a default exists)

## EACH SYSTEM PIECE HAS A NAME

SAME AS NAME OF INPUT DATA SECTION FOR THE PIECE WITHIN A CLASS, NAMES MUST BE UNIQUE, EVEN IF TYPE IS DIFFERENT
class and name used to identify system piece, hence to store and search the data
type serves to define optional structure of data within a class

## SUBNAMES MAY BE NEEDED TO IDENTIFY A QUANTITY

for example, specifying a degree of freedom requires component name, vector name, and element name

TOP LEVEL NAME (FROM INPUT SECTION) MUST BE DEFINED BY USER

OTHER NAMES CAN BE DEFINED BY USER OR COMPUTER (IF DEFAULT EXISTS)
user can generally override any default name
there are defaults for all labels

WHEN USER MUST IDENTIFY SOMETHING BY NAME, OPTIONALLY IT CAN BE IDENTIFIED BY SEQUENCE NUMBER (AS A CHARACTER STRING) INSTEAD

USEFUL FOR SUBNAMES (VECTORS AND ELEMENTS), WHERE SEQUENCE NUMBER IS OBVIOUS
such as rigid body motion, force, or moment
when analysis searches for specified name and does not find it, then tries to translate name as valid sequence number

## OPTIONALLY, THERE MAY BE NO ELEMENT NAMES FOR A QUANTITY

USEFUL WHEN VECTOR IS VERY LONG
BUT AN INPUT VECTOR OR A COMPONENT DEGREE-OFFREEDOM VECTOR MUST HAVE ELEMENT NAMES otherwise analysis can not label or select the element

## 2-2.4 Units of Input Parameters

## DIMENSIONAL INPUT PARAMETERS

ENGLISH OR METRIC (SI) UNITS, USING CONSISTENT LENGTH-MASS-TIME SYSTEM
foot-slug-second or meter-kilogram-second
selected by parameter OPUNIT in class = CASE ANGLES ARE INPUT IN DEGREES

EXCEPTIONS TO CONVENTION IN ROTORCRAFT SHELL:
AIRCRAFT GROSS WEIGHT IS IN LB OR KG WIND SPEED AND AIRCRAFT SPEED CAN BE IN KNOTS

## 2-3 Shell and Core Namelist

## SHELL AND CORE NAMELIST

OCCUR IN JOB COMMAND STREAM OF PROGRAM CAMRADII OCCUR IN A FILE FOR PROGRAM input

INPUT FOR ANALYSIS PROGRAM CAMRADII BEGINS WITH NAMELIST NLJOB (WHICH ALWAYS HAS DEFAULTS)

THEN THERE IS NAMELIST INPUT FOR EACH CASE OF JOB
FIGURE 3 SHOWS FORM OF NAMELIST INPUT FOR A CASE (OR IN A FILE FOR PROGRAM INPUT)
consists of set (perhaps none) of definition and value (NLDEF-NLVAL) namelist pairs
arbitrary order
more than one namelist pair can be used for any input section (class/type/name)
case input must always conclude with a definition namelist (NLDEF) containing "action = end" (with no NLVAL namelist)
core input (perhaps just the concluding NLDEF) is required even when the shell is used

THE END OF THE INPUT FOR ONE CASE IS IDENTIFIED BY THE PAIR OF "action = end" LINES:

```
$NLDEF action='END OF SHELL',$END
$NLDEF action='END OF CORE',$END
```


## shell and core input:

```
$NLDEF class='xxxx',type='xxxx',name='xxxx',action='xxxx',$END
```



```
....
$NLDEF class='xxxx',type='xxxx',name='xxxx',action='xxxx',$END
```



```
$NLDEF action='END OF SHELL',$END
$NLDEF class='xxxx',type='xxxx',name='xxxx',action='xxxx',$END
```



```
....
....
$NLDEF class='xxxx',type='xxxx',name='xxxx',action='xxxx',$END
```



```
$NLDEF action='END OF CORE',$END
```

Figure 2-3 Program namelist input.

## DEFINITION NAMELIST (NLDEF)

ONLY FOUR PARAMETERS: CLASS, TYPE, NAME, ACTION all character strings, length 32

CLASS, TYPE, AND NAME IDENTIFY SHELL OR CORE INPUT BLOCK
possibly there is no type
possibly the name is ignored (only one block of this class/type)
lower case and some abbreviations and alternatives are accepted in CLASS and TYPE

VALUE NAMELIST (NLVAL)
CONTENTS DEPEND ON SECTION CLASS AND TYPE

EACH nLVAL NAMELIST CONTAINS CHARACTER VARIABLE C (or COMmENT, IF C IS ALREADY AN INPUT VARIABLE)
can be used to insert comments among the input data, if installation does not allow comments identified by exclamation point

PARAMETER ACtion IN nldef NAMELIST
VALUES: zero, initialize, change, delete, or end
DEFAULT IS ACTION = ' change'
only first character in ACTION is used both upper case and lower case are accepted

| ACTION | function |
| :--- | :--- |
| zero | set all parameters in block to zero before read <br> namelist |
| initialize | set all parameters in block to initial values <br> before read namelist |
| change | get current values of parameters before read <br> namelist; same as initialize for new block |
| delete | delete section |
| end | end input category |

USUALLY CAN IGNORE PARAMETER ACTION (USE DEFAULT VALUE)
except to specify end of shell input and end of core input

THERE ARE BUILT-IN PARAMETER VALUES FOR EACH CORE AND SHELL BLOCK

## IMPLEMENTED BY INITIALIZATION AND SCENARIOS

INITIALIZATION IS CONTROLLED BY PARAMETER ACTION IN nldef NAMELIST
gives option to set all parameters to default values, before reading NLVAL namelist SCENARIOS ARE CONTROLLED BY PARAMETER OPSCEN IN SOME NLVAL NAMELISTS
scenario is set of values for selected parameters, implemented immediately after reading NLVAL namelist figure 4 shows typical use of initialization and scenarios


```
!==== initialization
&NLDEF class='ROTOR',type='INFLOW',name='ROTOR 1',&END
&NLVAL &END
```



```
!==== forward flight wake scenario
&NLDEF class='ROTOR',type='WAKE', name='ROTOR 1',&END
&NLVAL OPSCEN=1,&END
```



```
!==== change wake extent
&NLDEF class='ROTOR',type='WAKE', name='ROTOR 1',&END
&NLVAL OPSCEN=0,RFW=4.,&END
```



Figure 2-4 Typical use of initialization and scenarios.

## INPUT SCENARIOS

IMPLEMENTED IMMEDIATELY AFTER EACH READ OF NAMELIST NLVAL

## PROCEDURE:

move existing section from general data storage to commons (or initialize the commons)
use namelist to read selected parameters into commons implement scenarios
move section from commons to general data storage (in compressed form)

CONTROLLED BY PARAMETER OPSCEN IN EACH BLOCK (IF AVAILABLE)

## SET OPSCEN $=0$ FOR NO SCENARIO

a scenario provides set of values for other parameters in the namelist, superseding any other input value
so in order to change a parameter that is set by a scenario:
must use separate, subsequent NLDEF-NLVAL pair, containing OPSCEN $=0$ and new parameter values

IF SCENARIO TURNED ON AFTER PARAMETERS HAVE ALREADY BEEN READ FOR A BLOCK
best to use ACTION='init' to reset all parameters to default values

## EACH SHELL INPUT BLOCK HAS AN ELEMENTARY SCENARIO CORRESPONDING TO SIMPLIFIED CONFIGURATION

these scenarios are used with initial values defined for shell input blocks

## 2-4 Summary of Input Process

## FIGURE 5 SUMMARIZES THE INPUT PROCESS

SHELL INPUT CONSISTS OF BLOCKS CORRESPONDING TO INPUT CATEGORIES
usually baseline parameters are obtained from a file then namelist input is used to change the shell for this job and case
shell then constructs core input for typical configurations CORE INPUT CONSISTS OF BLOCKS CORRESPONDING TO SYSTEM PIECES
namelist input is used to change core constructed by shell core input defines tables required

TABLE INPUT CONSISTS OF BLOCKS CORRESPONDING TO TABLE FILES

## EACH INPUT BLOCK IS IDENTIFIED BY CLASS, TYPE, AND NAME

 plus file name and unit number for tablesFIGURE 6 LISTS CORE, TABLE, AND SHELL INPUT BLOCKS IN SHELL INPUT, ALL BLOCKS NEEDED MUST BE DEFINED BY USER
even if only default parameter values are required
there is no automatic definition of shell blocks


Figure 2-5 Summary of shell and core input process.

| CLASS | TYPE | NAME |
| :--- | :--- | :--- |
| CASE | none | ignore |
| COMPONENT | type |  |
| FRAME | none |  |
| INTERFACE | STRUCTURAL DYNAMIC |  |
| INTERFACE | INPUT/OUTPUT |  |
| OUTPUT | none | ignore |
| INPUT | none |  |
| WIND | none |  |
| OPERATING CONDITION | none |  |
| PERIOD | none |  |
| TRIM LOOP | NO SOLUTION |  |
| TRIM LOOP | SUCCESSIVE SUBSTITUTION |  |
| TRIM LOOP | NEWTON RAPHSON |  |
| TRIM LOOP | REGULATOR |  |
| TRIM PART | NO SOLUTION |  |
| TRIM PART | IMPLICIT |  |
| TRIM PART | STATIC |  |
| TRIM PART | HARMONIC |  |
| TRIM PART | TIME FINITE ELEMENT |  |
| TRANSIENT | nOne |  |
| TRANSIENT LOOP | NO SOLUTION |  |
| TRANSIENT LOOP | SUCCESSIVE SUBSTITUTION |  |
| TRANSIENT PART | NO SOLUTION |  |
| TRANSIENT PART | TRIM SOLUTION |  |
| TRANSIENT PART | IMPLICIT |  |
| TRANSIENT PART | INTEGRATION |  |
| FLUTTER | none |  |
| FLUTTER LOOP | none |  |
| FLUTTER PART | NO SOLUTION |  |
| FLUTTER PART | INTERFACE |  |
| TRANSFORM | DIFFERENTIAL EQUATIONS |  |
| MODES | none |  |
| RESPONSE | ROne |  |
| WEIGHTS | VARID |  |
|  |  |  |

If there is only one block, the name is ignored. Lower case is accepted for class and type. Unique initial letters of class or type are accepted. Specific abbreviations and alternatives:

```
class = TRIM LOOP:
    type = NO SOLUTION = NULL
    type = NEWTON RAPHSON = NR
class = TRIM PART, TRANSIENT LOOP, TRANSIENT PART:
    type = NO SOLUTION = NULL
class = FLUTTER PART:
    type = NO SOLUTION = NULL
    type = DIFFERENTIAL EQUATIONS = DE = EQUATIONS
```

Figure 2-6a Core input blocks.

| CLASS | TYPE |
| :--- | :--- |
| COMPONENT | RIGID BODY |
| COMPONENT | LINEAR NORMAL MODES |
| COMPONENT | FINITE ELEMENT BEAM |
| COMPONENT | ROD/CABLE |
| COMPONENT | TRANSMISSION |
| COMPONENT | REFERENCE FRAME |
| COMPONENT | FILTER |
| COMPONENT | REFERENCE PLANE |
| COMPONENT | DIFFERENTIAL EQUATION |
| COMPONENT | PROGRAMMABLE |
| COMPONENT | TRANSFER FUNCTION |
| COMPONENT | FOURIER SERIES |
| COMPONENT | PRESCRIBED CONTROL |
| COMPONENT | GUST |
| COMPONENT | RIGID AIRFRAME AERODYNAMICS |
| COMPONENT | AIRFRAME FLOW FIELD |
| COMPONENT | LIFTING LINE WING |
| COMPONENT | WING INFLOW |
| COMPONENT | ROTOR INFLOW |
| COMPONENT | ROTOR DYNAMIC WAKE |
| COMPONENT | WING WAKE |
| COMPONENT | WING WAKE GEOMETRY |
| COMPONENT | ROTOR WAKE GEOMETRY |
| COMPONENT | WING PERFORMANCE |
| COMPONENT | ROTOR PERFORMANCE |
| COMPONENT | ROTORCRAFT PERFORMANCE |
| COMPONENT | RIGID WING |
| COMPONENT | HELICOPTER TAIL BOOM |
| COMPONENT | COMPUTATIONAL FLUID DYNAMICS |
| COMPONENT | PLUGIN |

Lower case is accepted for class and type. Unique initial letters of class or type are accepted. Specific abbreviations and alternatives:
class = COMPONENT:

$$
\begin{aligned}
& \text { type }=\text { LINEAR NORMAL MODES }=\text { NORMAL MODES }=\text { MODES } \\
& \text { type }=\text { FINITE ELEMENT BEAM }=\text { BEAM } \\
& \text { type }=\text { ROD/CABLE }=\text { ROD }=\text { CABLE } \\
& \text { type }=\text { FILTER }=\text { LOW PASS FILTER }=\text { RESOLVER }=\text { DIFFERENTIATOR } \\
& \text { type }=\text { DIFFERENTIAL EQUATION }=\text { DE }=\text { STATIC EQUATION }=\text { ADDER } \\
& \text { type }=\text { AIRFRAME FLOW FIELD }=\text { AIRFRAME INTERFERENCE VELOCITY } \\
& \text { type }=\text { ROTOR INFLOW }=\text { UNIFORM INFLOW } \\
& \text { } \begin{array}{l}
\text { type }=\text { WING WAKE }=\text { NONUNIFORM INFLOW } \\
\text { type }=\text { CMPUTATIONAL FLUID DYNAMICS }=\text { CFD }
\end{array}
\end{aligned}
$$

Figure 2-6b Core component input blocks.

| CLASS | TYPE | NAME |
| :--- | :--- | :--- |
| TWO DIMENSIONAL | STANDARD |  |
| THREE DIMENSIONAL | STANDARD |  |
| FOUR DIMENSIONAL | STANDARD |  |
| AIRFOIL | STANDARD |  |
| AIRFOIL | C81 |  |
| AIRFOIL | CAMRAD |  |
| MATRIX | STANDARD |  |
| UNSTRUCTURED | REAL |  |
| UNSTRUCTURED | INTEGER |  |

Lower case is accepted for class and type. Unique initial letters of class or type are accepted. Specific abbreviations and alternatives:

$$
\begin{aligned}
\text { class } & =\text { TWO DIMENSIONAL }=2 \mathrm{D} \\
\text { class } & =\text { THREE DIMENSIONAL }=3 \mathrm{D} \\
\text { class } & =\text { FOUR DIMENSIONAL }=4 \mathrm{D} \\
\text { class } & =\text { AIRFOIL: } \\
& \text { type }=\text { C81 }=\text { FIXED FORMAT } \\
& \text { type }=\text { CAMRAD }=\text { WING } \\
\text { class } & =\text { MATRIX }=\text { ARRAY }
\end{aligned}
$$

Figure 2-6c Table blocks.

| CLASS | TYPE | NAME |
| :--- | :--- | :--- |
| CHANGE CORE | none | ignore |
| CASE | none | ignore |
| TRIM | none | ignore |
| TRIM ROTOR | none | "Rotor n" |
| TRANSIENT | none | ignore |
| TRANSIENT ROTOR | none | "Rotor n" |
| FLUTTER | none | ignore |
| FLUTTER ROTOR | none | "Rotor n" |
| AIRFRAME | STRUCTURE | ignore |
| AIRFRAME | AERODYNAMICS | ignore |
| AIRFRAME | CONTROL | ignore |
| AIRFRAME | DRIVE TRAIN | ignore |
| ROTOR | STRUCTURE | "Rotor n" |
| ROTOR | FLEXBEAM | "Rotor n" |
| ROTOR | AERODYNAMICS | "Rotor n" |
| ROTOR | INFLOW | "Rotor n" |
| ROTOR | WAKE | "Rotor n" |
| TABLES | none | ignore |

The "Rotor n" name is defined by the parameter ROTOR in the class $=$ AIRFRAME, type = STRUCTURE input (default = 'ROTOR $1^{\prime}$, 'ROTOR 2', ..., 'ROTORkk').

If there is only one block, the name is ignored. Lower case is accepted for class and type. Unique initial letters of class or type are accepted. Specific abbreviations and alternatives:

$$
\begin{aligned}
\text { class } & =\text { ROTOR: } \\
& \text { type }=\text { INFLOW }=\text { UNIFORM INFLOW } \\
\text { type } & =\text { WAKE }=\text { NONUNIFORM INFLOW }
\end{aligned}
$$

Figure 2-6d Shell input blocks.

## 2-5 Programs

CAMRAD II MAIN PROGRAMS
ROTORCRAFT ANALYSIS (CAMRADII)
INPUT AND TABLE FILE PREPARATION (INPUT)
PLOT FILE EXAMINATION (OUTPUT)

INPUT DATA FOR CAMRADII PROGRAM
INPUT FILE, JOB NAMELISTS, TABLE FILES
INPUT FILE OPTIONAL, BUT RECOMMENDED separates baseline and job input provides configuration control of baseline input parameters ROTORCRAFT SHELL USUALLY REQUIRES AIRFOIL TABLE FILE(S)
other tables may be needed as well

TYPICAL STEPS TO RUN CAMRAD II
CREATE AIRFOIL TABLE FILE
CREATE SHELL INPUT FILE
RUN ROTORCRAFT ANALYSIS
EXAMINE PLOT FILE

## 2-6 Input and Table File Preparation: Program INPUT

PROGRAM INPUT
INPUT FILE PREPARATION
PRODUCES UNFORMATTED SHELL INPUT FILE, FROM NAMELIST FILE(S)
can also extract data to plot or draw input, and examine initialized system data

TABLE FILE PREPARATION
PRODUCES UNFORMATTED OR FORMATTED FILES, FOR EACH TABLE CLASS AND TYPE

SPECIFICALLY, CREATES AIRFOIL TABLE FILE FOR ROTORCRAFT SHELL
unformatted CAMRAD airfoil file for entire blade from STANDARD (free form) or C81 (fixed form) airfoil file(s)
can also extract data to plot table

## BATCH OR INTERACTIVE OPERATION

FIRST LINE OF JOB INPUT STREAM MUST BE "BATCH" OR "INTERACTIVE"
default is interactive
FOR INTERACTIVE MODE, PRINTED OUTPUT CAN BE DIRECTED TO LOG FILE (OUTPUTLOG)

THEN PROGRAM READS NAMELIST NLJOB, CONTAINING PARAMETERS DEFINING THE JOB

NAMELIST NLJob
AFTER SPECIFYING BATCH OPERATION, PROGRAM READS NAMELIST nlJob, CONTAINING PARAMETERS DEFINING THE JOB
all parameters have defaults, which are set (and may be changed) in main program

TABLE FILE PREPARATION REQUIRES ADDITIONAL NAMELISTS

## 2-6.1 Shell Input File Preparation

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapter "Shell Input File Preparation"
figure 7 outlines process
figure 8 shows form and example of batch job
SAMPLE JOBS PROVIDE EXAMPLES

FILES THAT MAY BE REQUIRED

| file | name or logical name | control parameter |
| :--- | :--- | :--- |
| input file | INPUTFILE | NFILEI |
| namelist file | INPUTLISTn | NLISTI |
| input file | OUTPUTFILE | NFILEO |
| namelist file | OUTPUTLIST | NLISTO |



Figure 2-7 Outline of INPUT process - input file preparation.

## BATCH JOB

define files required by job
run INPUT
BATCH
namelist NLJOB (contains parameters defining job)

## EXAMPLE (unix)

setenv INPUTLIST1 helicopter.list
setenv OUTPUTFILE helicopter.dat
/camrad/input > helicopter. out << 'endofinput'
BATCH
\&NLJOB NLISTI=1,NLISTC=0, \&END
'endofinput'

Figure 2-8 Form and example of INPUT job - input file preparation.

TYPICAL JOB TO PREPARE SHELL INPUT FILE
CREATE UNFORMATTED SHELL INPUT FILE, FROM NAMELIST FILE(S) OF BASELINE PARAMETERS; AND PRINT INPUT PARAMETERS

FILES REQUIRED BY JOB:
INPUTLIST1 to INPUTLISTn, and OUTPUTFILE
file for printed output

NAMELIST nluob PARAMETERS:
NLISTI $=n$, number of namelist files
NLISTC = first file containing core changes to shell input with no core changes, use NLISTC $=0$ (default) with core changes in only one file, make it the last file and use NLISTC = NLISTI (so the shell needs to create the core input only once)
default values generally used for other parameters

## INTERACTIVE MODE OPERATION

after reading input data, INPUT program can extract the data and write it to new files, which can then be read by user's graphics routines

## EXTRACT DATA IN ORDER TO PLOT OR DRAW SHELL INPUT

 blade parameters and geometry (structure and aerodynamic information that depends on blade radius)
## PARAMETERS ARE INPUT VARIABLES, IDENTIFIED BY NAME AS DEFINED IN INPUT MANUAL

structural parameters: at input radial stations or at Gaussian integration points aerodynamic parameters: at input radial stations, at aerodynamic panel edges, or at panel mid points

Gaussian points and panel mid points are where input data are actually used

## GEOMETRY CONSISTS OF CHORDWISE AND NORMAL OFFSETS, USED TO DRAW PLANFORM

offsets relative blade span axis, reference line, or element beam axis; normally use offset relative blade span axis (total offset)
normally twist suppressed, otherwise twist makes blade appear to be tapered
structural geometry includes elastic axis at nodes, can be compared with input elastic axis data
aerodynamic geometry includes quarter chord, leading edge, and trailing edge
script zplanform.com available to run INPUT program to extract all information needed to draw planform

## EXTRACT DATA IN ORDER TO PLOT OR DRAW CORE INPUT, OR EXAMINE INITIALIZATION

before creating core input, may override shell parameters
flutter task may be required to create all wings (normal trim task solves for motion of just one blade, so aerodynamic components created only for that blade unless flutter or transient task is executed or initialized)

## PLOT INPUT DATA

plot input as function of radius for wings and for beams, similar to plot function for shell input

## EXAMINE INITIALIZED DATA

for specified system pieces; this information can also be produced by analysis program (NPRNTD)

## DRAW GEOMETRY

three-dimensional geometry extracted, typically for "all structural dynamic components" and "lifting line wings" default for structural dynamic components: draw position as line from a reference line (such as the beam axis), for all structural dynamic interfaces
can also draw orientation, as axis system at interface for most structural dynamic components, if there are more than two locations then user must identify two locations as end points of reference line
for wings, default option is recommended

## 2-6.2 Table File Preparation

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapter "Table File Preparation"
figure 9 outlines process
figure 10 shows form and example of batch job
SAMPLE JOBS PROVIDE EXAMPLES

## FILES THAT MAY BE REQUIRED

| file | name or logical name | control parameter |
| :--- | :--- | :--- |
| source file | INPUTDECK (1) | OPSRC $=1$ |
| table file | INPUTTABLE | OPSRC $=3$ |
| table file | OUTPUTTABLE | NFILEO |

(1) INPUTDECKn or INPUTDECKnFLAPm for CAMRAD airfoil table

## InPut PROGRAM CAN PREPARE CLASS = AIRFOIL FROM SOURCE FILES OR INTERNALLY

for other table classes, table file preparation does not transform the table data

ADDITIONAL NAMELISTS, AFTER NLJOB
NAMELIST NLTABL
FOR TABLE CLASS = AIRFOIL
contains table parameters
NAMELIST NLEQN
FOR TABLE CLASS = AIRFOIL, WHEN TABLE CALCULATED INTERNALLY
contains table calculation variables


Figure 2-9 Outline of INPUT process - table file preparation.

## BATCH JOB

define files required by job
run INPUT
BATCH
namelist NLJOB (contains parameters defining job)
namelist NLTABL (contains table parameters)
namelist NLzz (table calculation parameters)

```
EXAMPLE (unix)
    setenv INPUTDECK1 airfoil.c81
setenv OUTPUTFILE airfoil.tab
/camrad/input > airfoil.out << 'endofinput'
    BATCH
    &NLJOB OPFILE=7,OPSRC=1,&END
    &NLTABL OPFORM=2,RNTRP=1,TITLE='AIRFOIL',&END
    'endofinput'
```

Figure 2-10 Form and example of INPUT job - table file preparation.

## TYPICAL JOB TO PREPARE AIRFOIL TABLE FILE, FROM AIRFOIL DECKS

create CAMRAD airfoil table file (unformatted file for entire blade), from STANDARD (free form) or C81 (fixed form) airfoil table file(s)
and print table parameters (not table data)

FILES REQUIRED BY JOB:
INPUTDECK1 (root) to INPUTDECKn (tip), and OUTPUTTABLE file for printed output

NAMELIST NLJob PARAMETERS:
OPFILE $=7$, to produce CAMRAD airfoil table OPSRC $=1$, from source files (airfoil decks)
airfoil decks should include Reynolds number
default values generally used for other parameters

## NAMELIST nLtabl PARAMETERS:

OPFORM = 1, from airfoil deck in STANDARD form OPFORM $=2$, from airfoil deck in C81 form

RNTRP $=0$, spanwise interpolation of airfoil decks RNTRP $=1$, spanwise search of airfoil decks
only required if NRB > 1; spanwise interpolation must be used with care

TITLE $=$ title for wing data
NRB $=n$, number of span stations
$R=$ span station values (root to tip)
only required if NRB $>1$; NRB is number of airfoil decks; for spanwise search $R$ is inboard edge of airfoil deck range

OPFLAP = trailing edge flap
ND(NRB) = number of flap angles at span station
default values generally used for other parameters

## INTERACTIVE MODE OPERATION

after producing the table, INPUT program can extract the data and write it to new files, which can then be read by user's graphics routines

## TWO-DIMENSIONAL, THREE-DIMENSIONAL, FOUR-DIMENSIONAL TABLES

data extracted as function of one independent variable, for selected values of second independent variable and one value of remaining variables

## AIRFOIL TABLES

EXTRACT DATA IN ORDER TO PLOT TABLE, PERHAPS WITH STALL DELAY FACTOR
$c_{\ell}(\alpha), c_{d}(\alpha), c_{m}(\alpha)$ for selected $M$
$c_{\ell}(M), c_{d}(M), c_{m}(M)$ for selected $\alpha$
$c_{d}\left(c_{\ell}\right), c_{m}\left(c_{\ell}\right)$ for one $M$

## IDENTIFY DYNAMIC STALL PARAMETERS OR AIRFOIL CHARACTERISTICS FROM AIRFOIL TABLE DATA

dynamic stall parameters define trailing edge separation point function, for Leishman-Beddoes or ONERA BH model
airfoil characteristics: zero lift angle $\alpha_{z}$; drag and moment at zero lift, $c_{d z}=c_{d}\left(\alpha_{z}\right)$ and $c_{m z}=c_{m}\left(\alpha_{z}\right)$; lift-curve slope and moment slope, $c_{\ell \alpha}$ and $c_{m \alpha}$; aerodynamic center shift $c_{m \alpha} / c_{\ell \alpha}$; drag recovery factor $\eta$

IDENTIFY SEPARATION POINT PARAMETERS AND AIRFOIL CHARACTERISTICS
at selected Mach number
and produce plot of lift coefficient and separation point function as function of angle of attack, from table data and from identified parameters

IDENTIFY CRITICAL LIFT COEFFICIENT (at all Mach numbers)

## PRODUCE TWO-DIMENSIONAL TABLE FILE

dynamic stall parameters as function of Mach number and span station, to be pasted into dynamic stall file

## PLOT AIRFOIL CHARACTERISTICS

one characteristic or parameter, as function of Mach number

CHANGE IDENTIFICATION OPTIONS

## 2-7 Analysis: Program CAMRADII

## PROGRAM CamRadil ANALYZES THE AEROMECHANICAL SYSTEM

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapter "CAMRAD II Analysis"
figure 11 outlines process
figure 12 shows form and example of batch job
SAMPLE JOBS PROVIDE EXAMPLES

## FILES THAT MAY BE REQUIRED

| file | name or logical name | control param |
| :--- | :--- | :--- |
| input data file | SHELLINPUT | OPSHLL |
| airfoil table files | BLADEAIRFOILn (3) | OPAERO |
| other table files | (2) | shell input |
| plot data file | PLOTFILE | PLFILE |
| trim initialization input | TRIMSOLUTIONINm (1) | OPITJR |
| trim initialization output | TRIMSOLUTIONOUTm (1) | OPITJW |
| transient restart input | TRANRESTARTIN | OPRNJR |
| transient restart output | TRANRESTARTOUT | OPRNJW |

(1) $m$ is case number ( 1 if only one file is read)
(2) table names are defined in shell input class $=$ TABLES
(3) $n$ is the rotor number


Figure 2-11 Outline of CAMRADII process.

## BATCH JOB

define input files required by job shell input file SHELLINPUT (contains baseline parameters) airfoil file(s) other table files
define output files required by job plot data file PLOTFILE
run CAMRADII
namelist NLJOB (contains parameters defining job)
shell and core namelist for each case (NLDEF and NLVAL)

```
EXAMPLE (unix)
    setenv SHELLINPUT helicopter.dat
    setenv BLADEAIRFOIL1 airfoil.tab
    setenv PLOTFILE sample.plot
    /camrad/camradii > sample.out << 'endofinput'
        &NLJOB PLFILE=1,&END
        &NLDEF class='TRIM',&END
        &NLVAL VKTS=100.,&END
        &NLDEF action='end of shell',&END
        &NLDEF action='end of core',&END
    'endofinput'
```

Figure 2-12 Form and example of CAMRADII job.

## MULTIPLE CASES

## SHELL INPUT FILE TYPICALLY READ ONLY FOR FIRST CASE <br> SO FOLLOWING CASES INHERIT ALL PREVIOUS INPUT

including core namelist input that changes core data constructed by shell

## SECOND AND SUBSEQUENT CASES CAN BE INITIALIZED FROM TRIM SOLUTION OF PREVIOUS CASE <br> USING THE TRIM LOOP VARIABLES (OPINIT = 1) <br> derivative matrices, control, wind, operating condition, and period parameters involved in any loop of trim task <br> variables VNAME identified by Newton-Raphson trim loop

OR THE TRIM PART TOTAL SOLUTION (OPINIT = 6)
solution of rotor, airframe, and drive train parts, plus all interfaces

OR BOTH (OPINIT $=7$ )

## CASE CAN ALSO BE INITIALIZED BY READING FILE OF THIS

 TRIM SOLUTION, WRITTEN BY A PREVIOUS JOBfigure 13 illustrates initialization of trim solution
if the system configuration changes from case to case, then names of system pieces must be appropriately changed as well; otherwise this initialization procedure will not be able to correctly match response records of two cases


Figure 2-13 Initialization of trim solution from previous case or job.

TYPICAL JOB TO ANALYZE ROTORCRAFT
READ SHELL INPUT FILE AND AIRFOIL TABLE FILES;
READ NAMELIST INPUT FOR EACH CASE;
ANALYZE ROTORCRAFT;
PRINT JOB OUTPUT AND CREATE PLOT FILE

FILES REQUIRED BY JOB
SHELLINPUT, BLADEAIRFOIL1, BLADEAIRFOIL2, and PLOTFILE airfoil tables are not required if rotor blade structure specifies no aerodynamics, OPAERO $=0$
number of airfoil tables is specified in shell input class $=$ TABLES

AFTABL $=0$, same table for all rotors (default)
AFTABL $=1$, separate tables (number of
tables $=$ number of rotors)
file for printed output

PROGRAM FIRST READS NAMELIST NLJOB, CONTAINING PARAMETERS DEFINING THE JOB
all parameters have defaults, which are set (and may be changed) in main program

## PROGRAM THEN READS SHELL AND CORE NAMELIST FOR EACH CASE

NLDEF and NLVAL pairs

NAMELIST nLJob PARAMETERS:
NCASES = number of cases
PLFILE $=1$ to write plot file for job
OPSHLL = shell input file read
defaults: one case, no plot file written, shell input file read for first case only (OPSHLL = 1)

OPINIT $=$ initialization from previous case
trim solution can be initialized from previous case using:
the trim loop variables (OPINIT $=1$ ) (derivative matrices, controls, and other variables used by Newton-Raphson trim loop)
the trim part total solution (OPINIT $=6$ ) (response of all degrees of freedom and interfaces)
or both (OPINIT =7)
default is no initialization
default values generally used for other parameters

## 2-8 Plot File Examination: Program OUTPUT

## SELECTED OUTPUT CAN BE DIRECTED TO PLOT DATA FILE

PLOT FILE INCLUDES ONLY ENOUGH HEADER INFORMATION AND TITLES TO IDENTIFY ITS DATA
reference must be made to printed output for complete description of quantities

PLOT FILE CAN BE READ USING output PROGRAM

PROGRAM OUTPUT
READS AND EXTRACTS DATA FROM PLOT FILE
create files for user's graphics routine
PRINT (TO A LOG FILE) LINEAR DIFFERENTIAL EQUATIONS OF FLUTTER ANALYSIS
figure 14 outlines process

GRAPHICS DATA REQUIRED TO PRODUCE ANIMATION OF TRIM OR TRANSIENT SOLUTION IS SENT TO PLOT FILE, AND READ BY output PROGRAM


Figure 2-14 Outline of OUTPUT process (interactive only).

## INTERACTIVE OPERATION, NOT BATCH

often run using script files
plot file data considered by section, with following operations

## LIST THE SECTIONS

move down or up sections while displaying titles
perhaps jump down to next status line, next case, or next graphics section
default (carriage return) is to move down to next section
IDENTIFY A SECTION (from a compact list)
FIND A TITLE (search for a string in the title)
FIND A TITLE AGAIN
CHANGE LIST OPTIONS
EXAMINE THE CURRENT SECTION
extract data to file for graphics routines
quantities that can be extracted:
time history data ( $F(\psi)$ or $F(t)$ );
wing sensors ( $F(r, \psi)$ for selected $r$ or selected $\psi$ ) wake geometry data (three-dimensional geometry)

UTILITY SUBROUTINES FILEXP, FILEXA, AND FILEXD WRITE EXTRACTED DATA TO FILE

USED BY Input AND output PROGRAMS
CAN BE MODIFIED AS APPROPRIATE FOR USER'S GRAPHICS ROUTINES

UTILITY FILEXP HANDLES PLOTABLE-DATA
DATA CONSISTS OF $y_{1}(x), \ldots, y_{n}(x)$ FOR SET OF POINTS $x_{i}$
figure 15 shows format of file produced
maximum 40 characters in code
maximum 12 characters in labels
blanks removed from labels

## UTILITY FILEXD HANDLES DRAWABLE-DATA <br> DATA CONSISTS OF $(x, y, z)$ COORDINATES FOR SET OF POINTS

figure 16 shows format of file produced
additional legends can be present in data section
maximum 40 characters in code maximum 80 characters in legend

## EXPORT FORMATS

CAMRAD standard, space delimiter
CAMRAD standard, tab delimiter
CAMRAD standard, comma delimiter
Tecplot
DXF (draw only)
DEFAULT EXPORT FORMAT CAN BE CHANGED BY REVISING MAIN PROGRAMS
header lines
number of dependent variables $=\mathrm{N}$, number of points $=$ NNNN
CODE
code
LABELS
xlabel y1label ... ynlabel
DATA
xvalue y1value ... ynvalue
xvalue y1value ... ynvalue

Figure 2-15 Format of file produced by FILEXP.

```
header lines
CODE
code
LEGEND
legend
DATA
xvalue yvalue zvalue penaction
xvalue yvalue zvalue penaction
```

pen action (fourth field) can have the following values:
NEW $=$ start new line (move to point with pen up)
NEWn $=$ start new line, and change to line type $n=1-4$
blank $=$ continue old line (move to point with pen down)

Figure 2-16 Format of file produced by FILEXD.

## 2-9 Graphics and Animation

A GRAPHICAL IMAGE OF THE TRIM OR TRANSIENT SOLUTION, EITHER A STILL PICTURE OR AN ANIMATION, CAN BE PRODUCED GRAPHICS DATA REQUIRED CAN BE SENT TO PLOT FILE, AND READ BY OUTPUT PROGRAM
output PROGRAM GENERATES TWO INPUT FILES FOR MATHEMATICA (COMMAND FILE AND GEOMETRY FILE) OR ONE INPUT FILE FOR TECPLOT OR ONE INPUT FILE FOR MATLAB

MATHEMATICA OR TECPLOT OR MATLAB PRODUCES THE ANIMATION
typically OUTPUT program run using script file, to facilitate changes

INPUT program used to extract geometry of system rest position, not the solution

## OBJECTS THAT CAN BE DRAWN INCLUDE STRUCTURE GEOMETRY, STRUCTURAL SENSORS, WINGS, WING SENSORS, AND WAKE GEOMETRY

structure geometry drawn as lines, or with appended shapes
wings drawn as surfaces, or with defined airfoil shapes
shapes of structure and wings are defined when OUTPUT program is run, not part of the CAMRAD II input
structural sensors drawn as arrows representing force or moment acting at a point
wing sensors display section aerodynamic quantities along wing span

MATHEMATICA INPUT CONSISTS OF A COMMAND FILE AND A GEOMETRY FILE
user should read comments in command file (either using Mathematica, or reading it as a text file)

ANIMATION PRODUCED BY OPENING MATHEMATICA NOTEBOOK
execute commands to set current working directory to location of the two input files, read the geometry file, and read the command file
then cells are executed by selecting them in order they appear (contents of most of cells are hidden), and pressing "enter"

PALETTE CREATED CONTAINING BUTTONS THAT CAN BE USED TO CONTROL DISPLAY
change orientation, size, and viewpoint
IMAGE FRAMES ARE CREATED FOR NUMBER OF AZIMUTH OR TIME STEPS, AND FROM THEM AN ANIMATION CAN BE CREATED

## MATLAB INPUT FILE CONTAINS COMMAND AND GEOMETRY DATA

 user should read comments in file
## ANIMATION PRODUCED BY OPENING MATLAB AND

 LOADING FILE (type *.m)select its folder, and execute this file to create the images and movie (mp4)
adjust the image, revise the position and orientation values, and execute the file again

IMAGE FRAMES ARE CREATED FOR NUMBER OF AZIMUTH OR TIME STEPS, AND FROM THEM THE ANIMATION IS CREATED

## TECPLOT INPUT FILE CONTAINS GEOMETRIC DATA FOR A SINGLE TIME STEP

open Tecplot and load this data file using "3D Cartesian" plot type
activate the "Contour" zone layer to display surfaces
the "Mesh" zone layer displays lines and polygon edges
color specified directly for lines for surfaces (polygons), color defined in terms of contour values

## ANIMATION PRODUCED BY WRITING SCRIPT TO RUN output PROGRAM AND PRODUCE TECPLOT INPUT FILE FOR EACH TIME STEP

then Tecplot macro can be used to load the files and create the animation

## 2-10 Data Vectors and Memory

A DATA STRUCTURE IS A SET OF REAL, INTEGER, AND CHARACTER VARIABLES

STRUCTURE TYPES:
DATA VECTORS
RECORDS
LISTS
COMMONS

## DATA VECTORS

PROGRAM STORES MOST GLOBAL DATA IN DATA-VECTOR-
TYPE DATA STRUCTURE (A "HEAP")
DATA VECTORS OF CAMRAD II:
core input
table input
shell input
principal data vector of analysis
DATA VECTORS IMPLEMENTED AS COMMONS
separate commons used for real, integer, and character variables

SO MEMORY AVAILABLE TO ANALYSIS IS FIXED WHEN PROGRAM EXECUTABLE IS CREATED

## STORAGE IN DATA VECTOR

DATA VECTOR IS PARTITIONED INTO SECTIONS analysis creates sections as required sections stored in data vector starting from beginning FREE SPACE
free space exists in data vector from last section to end there may also be free space between sections, if sections have been moved
usage of space in data vectors is printed at end of each case, and this information can be saved in statistics file

## NEW SECTIONS

added after last section in data vector
if there is not enough free space after last section, vector is packed
if after the pack there still is not enough free space, then analysis halts
with the following error message:
"SPACE REQUIRED GREATER THAN FREE SPACE"

## WHEN ANALYSIS RUNS OUT OF FREE SPACE IN A DATA VECTOR: <br> EITHER CHANGE PARAMETERS TO CREATE SMALLER PROBLEM <br> OR INCREASE SIZE OF DATA VECTOR <br> revise definition of data vector size in main programs, following instructions given with error message <br> OR USE DYNAMIC MEMORY EXECUTABLE assign data vector sizes at run time

## 2-11 Sample Jobs

## PURPOSE

## CHECK INSTALLATION

DEMONSTRATE USE OF ANALYSIS PROVIDE STARTING POINT FOR PROJECTS

## SHELL INPUT FILE PREPARATION

NAMELIST FILES
COMMAND FILES
RESULTING OUTPUT
elementary rotor, rotor in wind tunnel, teetering rotor, helicopter, bearingless rotor, tandem helicopter, tiltrotor in wind tunnel, tiltrotor aircraft

## AIRFOIL TABLE FILE PREPARATION

AIRFOIL DECK
COMMAND FILES
RESULTING OUTPUT simulated advanced technology airfoils (from airfoil equations) NACA 0012 airfoil (from standard C81 table)

SAMPLE DYNAMIC STALL TABLE
ROTORCRAFT ANALYSIS
SAMPLE JOBS following table summarizes configuration, operating condition, aerodynamic model, dynamic model

SEE LISTINGS OF SAMPLE JOBS, FOR SPECIFIC INSTALLATIONS

| case | configuration | characteristics |
| :---: | :---: | :---: |
| 1 | elementary rotor in wind tunnel | hover, no trim dynamic inflow rigid blade eigenvalues, frequency response flutter and transient time history |
| 2 | teetering rotor in wind tunnel | hover <br> hover wake (free geometry) <br> rigid blade <br> periodic eigenvalues (Floquet theory) |
| 3 | rotor in wind tunnel | forward flight <br> low speed wake (free geometry) rigid blade |
| 4 | swept tip rotor in wind tunnel | forward flight, no trim forward flight wake (rigid geometry) airframe-to-rotor aerodynamic interference elastic blade, swashplate, elastic airframe dynamic stall; eigenvalues |
| 5 | swept tip rotor in wind tunnel | hover, no trim <br> uniform inflow elastic blade, swashplate, elastic airframe eigenvalues |
| 6 | main-rotor and tail-rotor helicopter | hover <br> uniform inflow <br> rigid blade, drive train <br> flight dynamics <br> flutter and transient time history |
| 7 | main-rotor and tail-rotor helicopter | forward flight <br> high speed wake (rigid geometry) <br> elastic blade, swashplate <br> vibration |
| 8 | bearingless rotor in wind tunnel | hover, no trim uniform inflow elastic blade, swashplate eigenvalues |


| case | configuration | characteristics |
| :--- | :--- | :--- |
| 9 | tandem helicopter | forward flight |
|  |  | forward flight wake (rigid geometry) |
|  |  | rotor-to-rotor aerodynamic interference |
|  | rigid blade |  |
|  |  | axial flow |
|  |  | uniform inflow |
|  |  | elastic blade, swashplate, gimballed hub |
|  |  | elastic airframe, drive train |
|  |  | eigenvalues |
|  |  | forward flight |
|  |  | uniform inflow |
|  |  | airframe-to-rotor aerodynamic interference |
|  |  | rigid blade, swashplate, gimballed hub |
|  |  | elastic airframe, drive train |
|  |  | vibration, eigenvalues |
|  |  |  |

## OTHER RESOURCES

FILES AVAILABLE TO HELP PREPARE INPUT (CAMRAD II EXTRAS FOLDER)

## TYPICAL INPUT

conversion of CAMRAD/JA input (ztemplate.list)
hover analysis (zhover.list)
simplified models (zsimple.list)
windtunnel case (zwindtunnel.list)
blade frequency calculation (zfrequency.list)
script to draw planform (zplanform.com)

## EXAMPLES

external aeroacoustic analysis, using post-trim
demonstration of graphics generation
generic programs to extract data from output file
nonrotating lifting-line wing
frozen wake geometry for second case or job generic wind turbine
individual blade control
interblade damper
dual load path lag damper
demonstration of trailing-edge flap
elastic beams for flap structure
tip-path plane sensors
autopilot for transient task
prescribed airframe motion for transient task revised trim variables or matrix

DOCUMENTS DEMONSTRATING USE OF CORE INPUT
construction of elastic cantilever wing for tiltrotor bearingless rotor analysis
closed-loop higher-harmonic control
pendulum absorber
sample jobs and extras folder can be downloaded by customers from CAMRAD II web site

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#$! simulated advanced technology airfoils for sample jobs
#$!
setenv OUTPUTTABLE $airfoil/advtecheq.tab
$camrad/input > $airfoil/advtecheq.out << 'endofinput'
    BATCH
    &NLJOB OPFILE=7,OPSRC=2,&END
    &NLTABL RNTRP=1,
        TITLE='SIMULATED ADVANCED TECHNOLOGY AIRFOILS (CONSTRUCTED DATA)',
        NRB=2,R=0.,.85,
    &END
    &NLEQN
    ! 11 percent thick airfoil for inboard sections
        CLMAX=1.55,ALPHAD=15.5,MCRIT=.800,MDIV =.675,DEL0=.0070,CMAC=-.005,
        RETBL1=5.E6,
    &END
    &NLEQN
    ! }8\mathrm{ percent thick airfoil for tip
        CLMAX=1.20,ALPHAD=12.0,MCRIT=.835,MDIV =. 710, DEL0=.0065,CMAC=-.005,
        RETBL1=5.E6,
    &END
    endofinput'
#$! NACA 0012 airfoil, standard C81 table
#$!
setenv INPUTDECK1 $airfoil/naca0012.c81
setenv OUTPUTTABLE $airfoil/naca0012.tab
$camrad/input > $airfoil/naca0012.out << 'endofinput
    BATCH
    &NLJOB OPFILE=7,OPSRC=1,&END
    &NLTABL OPFORM=2,RNTRP=1,
        TITLE='NACA 0012 AIRFOIL (STANDARD C81 TABLE)',
    &END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#$! sample jobs: elementary rotor
#$!
setenv INPUTLIST1 $input/elementary.list
setenv OUTPUTFILE $input/elementary.bin
$camrad/input > $input/elementary.out << 'endofinput'
    BATCH
    &NLJOB NLISTI=1,NLISTC=0,&END
'endofinput'
```

```
#$! sample jobs: rotor in wind tunnel
#$!
setenv INPUTLIST1 $input/windtunnel.list
setenv OUTPUTFILE $input/windtunnel.bin
$camrad/input > $input/windtunnel.out << 'endofinput'
    BATCH
    &NLJOB NLISTI=1,NLISTC=1,&END
'endofinput'
```

\#\$! sample jobs: teetering rotor
\#\$!
setenv INPUTLIST1 \$input/teeter.list
setenv OUTPUTFILE \$input/teeter.bin
\$camrad/input > \$input/teeter.out << 'endofinput'
BATCH
\&NLJOB NLISTI=1,NLISTC=0, \&END
'endofinput'
\#\$! sample jobs: single main rotor and tail rotor helicopter
\#\$ !
setenv INPUTLIST1 \$input/helicopter.list
setenv OUTPUTFILE \$input/helicopter.bin
\$camrad/input > \$input/helicopter.out << 'endofinput'
BATCH
\&NLJOB NLISTI=1,NLISTC=0, \&END
'endofinput'
\#\$! sample jobs: bearingless rotor
\#\$ !
setenv INPUTLIST1 \$input/bearingless.list
setenv OUTPUTFILE \$input/bearingless.bin
\$camrad/input > \$input/bearingless.out << 'endofinput'
BATCH
\&NLJOB NLISTI=1,NLISTC=0, \&END
'endofinput'

```
#$! sample jobs: tandem helicopter
#$!
setenv INPUTLIST1 $input/tandema.list
setenv INPUTLIST2 $input/tandem1.list
setenv INPUTLIST3 $input/tandem2.list
setenv OUTPUTFILE $input/tandem.bin
$camrad/input > $input/tandem.out << 'endofinput'
    BATCH
    &NLJOB NLISTI=3,NLISTC=0,&END
'endofinput'
#$! sample jobs: tiltrotor in wind tunnel
#$!
setenv INPUTLIST1 $input/tiltrotorw.list
setenv INPUTLIST2 $input/tiltrotorl.list
setenv OUTPUTFILE $input/tiltrotorwt.bin
$camrad/input > $input/tiltrotorwt.out << 'endofinput'
    BATCH
    &NLJOB NLISTI=2,NLISTC=0,&END
'endofinput'
#$! sample jobs: tiltrotor aircraft
#$ !
setenv INPUTLIST1 $input/tiltrotora.list
setenv INPUTLIST2 $input/tiltrotor1.list
setenv INPUTLIST3 $input/tiltrotor2.list
setenv OUTPUTFILE $input/tiltrotor.bin
$camrad/input > $input/tiltrotor.out << 'endofinput'
    BATCH
    &NLJOB NLISTI=3,NLISTC=0,&END
'endofinput'
```

```
! Elementary Rotor
```



```
&NLDEF class='CASE',&END
&NLVAL
    TITLE='SAMPLE JOBS: ELEMENTARY ROTOR MODEL', ! description
&END
```



```
&NLDEF class='TRIM',&END
&NLVAL OPSCEN=1,
    WINDIN=2,VTIPIN=1,VTIP=650., ! operating condition
    LEVEL=1, ! wake loop
    OPTRIM=0,COLL=10., ! no trim
    MHARMR=1, ! harmonics
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL OPSCEN=1,&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL OPSCEN=0,MHSEN=0,MPSEN=0,&END ! output
!==================================================================================
&NLDEF class='TRANSIENT',&END
&NLVAL OPSCEN=1,&END
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 1',&END
&NLVAL OPSCEN=1,&END
```



```
&NLDEF class='FLUTTER',&END
&NLVAL OPSCEN=1,
    DOFA=6*0, ! degrees of freedom
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL OPSCEN=1,
    OPWAKE=3,DOFL=3*1, ! dynamic inflow
&END
```



```
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL OPSCEN=1,
    TITLE='SAMPLE JOBS: WIND TUNNEL SUPPORT', ! description
    CONFIG=0,OPFREE=0,OPAERO=0, ! wind tunnel
    MASSR=9.584, ! inertia
&END
&NLDEF class='AIRFRAME',type='CONTROL',&END
&NLVAL OPSCEN=1,&END
!===============================================================================
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
&NLVAL OPSCEN=1,
    TITLE='SAMPLE JOBS: ELEMENTARY ROTOR MODEL', ! description
    RADIUS=20.,NBLADE=4,ROTATE=1,SIGMA=.075,VTIPN=650.,
    EFLAP=0.,EPITCH=0.,TWISTL=-8.,
    KFLAP=108823.325,MASS=.1198, ! nu=1.15,gamma=8.
    NRPOS=0, ! sensors
&END
!===============================================================================
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL OPSCEN=1,
    CHORD=1.1781,
    BTIP=1.,USMODL=0,OPCOMP=0, ! ideal aerodynamics
&END
!=================================================================================
&NLDEF class='ROTOR',type='INFLOW', name='ROTOR 1',&END
&NLVAL OPSCEN=1,
    KHLMDA=1.,KFLMDA=1.,OPFFLI=0, ! ideal aerodynamics
&END
!===============================================================================
&NLDEF class='TABLES',&END
&NLVAL &END
```



```
! turn scenarios off
&NLDEF class='TRIM',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='TRANSIENT',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='FLUTTER',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='AIRFRAME',type='CONTROL',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
&NLDEF class='ROTOR',type='INFLOW',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
```



```
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
```

```
! Rotor in Wind Tunnel
```



```
&NLDEF class='CASE',&END
&NLVAL
    TITLE='SAMPLE JOBS: ROTOR IN WIND TUNNEL', ! description
    OPUNIT=1,OPDENS=3,DENSE=.00237689,TEMP=59., ! environment
&END
!=================================================================================
&NLDEF class='TRIM',&END
&NLVAL
    VELIN=2,WINDIN=2,VTIPIN=1,VTIP=650., ! operating condition
    LEVEL=1, ! wake loop
    COLL=10.,CTTRIM=.08,MTRIM=3, ! wind tunnel trim
    MNAME='CT/S ','BETAS ','BETAC ',
    MHARMR=10,MHARMA=10, ! harmonics
    DOFA=6*0,DOFM=5*0, ! degrees of freedom
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL OPMODE=0,DOFB=12*1,&END ! degrees of freedom
```



```
&NLDEF class='FLUTTER',&END
&NLVAL DOFA=6*0,DOFM=5*1,DOFD=8*0,&END ! degrees of freedom
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL
    OPWAKE=4,OPVATR=2,OPVRTA=2, ! trim inflow
    OPMODE=1,DOFM=2*1,38*0,DOFL=3*1,NMPRNT=1, ! degrees of freedom
&END
```



```
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL
    TITLE='SAMPLE JOBS: WIND TUNNEL SUPPORT' ! description
    CONFIG=0,OPFREE=0,OPAERO=0, ! wind tunnel
    MASSR=9.6, ! inertia
    HSP=2.,OPSPM=0, ! control system
    NLOC=6, ! locations
    FSLOC(5)=5.,-5.,WLLOC(5)=-7.,-7., ! 5=center,6=nose
    NMODE=5, ! test module
    QMASS=5*100.,QFREQ=2.1,2.6,20.,22., 23.,QDAMP=2*.02, 3*.2,
    LSHAPE(1,1,1) = 1., 0., 0.,
    LSHAPE (1,1,2) = 0., 1., 0.,
    LSHAPE(1,1,3) = 1., 0., 0.,
    LSHAPE (1,1,4) = 0., 1., 0.,
    LSHAPE (1,1,5) = 0., 0., 1.,
    ASHAPE(1,1,1) = 0., -1.5, 0.,
    ASHAPE (1,1,2) = 1.5, 0., 0.,
    ASHAPE(1,1,3) = 0., 0., 0.,
    ASHAPE (1,1,4) = 0., 0., 0.,
    ASHAPE (1,1,5) = 0., 0., 0.,
&END
&NLDEF class='AIRFRAME',type='AERODYNAMICS' ,&END
&NLVAL
    VISRC=0, ! test module
    NBODY=1,LOCBC=5,LOCBN=6,
    LENGTH=20.,THICK=.5,SHAPE=3,
&END
&NLDEF class='AIRFRAME',type='CONTROL',&END
&NLVAL &END
&NLDEF class='AIRFRAME',type='DRIVE TRAIN',&END
&NLVAL
    CONFIG=0,OPGOV=0,
    EGEAR=7.,KRS=10000.,IRS=.1,
    KES=10000.,IENG=50.,DENG=1.,
&END
```



```
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
&NLVAL ! description
    TITLE='SAMPLE JOBS: HELICOPTER MAIN ROTOR, SWEPT/TAPERED TIP',
    RADIUS=20.,NBLADE=4,ROTATE=1,SIGMA=.075,
    VTIPN=650.,
    HINGE=6,EFLAP=.04,EPITCH=.065, ! blade root
    KFLAP=1000.,KLAG=1000.,DLAG=2000.,
    CONTRL=2,PITCH=1,KPITCH=0.,LOCKP=1, ! control system
        XSP=-.023,YSP=.045,ZSP=-.1,
        XPH=-.023,YPH=.045, ZPH=0.,
        EPH=.07,KPL=80000.,LOCKPL=1,LOCKSP=0,
    OPBEAM=2,DRELST=.05,KNODE=3,RNODE=.94,.35,.64, ! elastic blade
    NSEN=3,QUANT=3*4,RLOAD=.4,.6,.8, ! loads
    TWIN=1,TWISTL=-10., ! linear twist
    ETPP=.94, ! tpp sensor
    NPROP=3,RPROP=0.,.94,1., ! section properties
    ZEA=3*0., ZQC=3*0., ZC=3*0.,ZI=3*0.,
    XEA=3*0.,XQC=3*0.,XC=3*0.,XI=3*0.,
    XEA=2*0.,.03464, ! 30 deg sweep
    XQC=2*0.,.03464,
    KP=3*.011,KT=3*.011,EA=3*20000000.,
    EIFLAP=3*68000., EILAG=3*800000.,GJ=3*90000.,
    MASS=3*.12,ITHETA=3*.0056,IPOLAR=3*.0056,
&END
!================================================================================
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL
    NPANEL=20, ! aero panels
    REDGE=. 12,. 20,. 28,.35,.42,.48,.54,.59,.64,.69,.73,
        . 77,.81,.84,.87,.90,.92,.94,.96,.98,1.0,
    NPROP=4,RPROP=0.,.94,.9401,1., ! aero properties
    CHORD=3*1.1781,.70686,ASWEEP=2*0.,2*30., ! 60% taper
    NSEN=6,OPREF=6*4,
    QUANT= 5,6,25,35,82,82,
    IDENT= 1,1, 0, 0, 0, 0,
    AXIS= 3,3, 0, 0, 1, 3,
    OPSCL= 2,2, 1, 1, 2, 2,
    NAPLOT=1,1, 4, 1, 0, 0, !
&END
!================================================================================
&NLDEF class='ROTOR',type='INFLOW', name='ROTOR 1',&END
&NLVAL &END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL
    OPSCEN=1, ! forward flight wake
    TWIST=-10.,RICWG=.15, ! hover wake
&END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
```



```
&NLDEF class='TABLES',&END
&NLVAL &END
!==================================================================================
&NLDEF action='end of shell',&END
```



```
! hydraulic lag damper (core input)
&NLDEF class='COMPONENT',type='BEAM',name='ROTOR 1 BLADE 1 ELEMENT 1',&END
&NLVAL
    CTYPE (2)=3,CLIN (2)=0.,
    NCHYDA (2)=0, CHYDA (1,2)=1000.,
    NCHYDB (2)=2,CHYDB (1,2)=0.,0.,100000.,
&END
&NLDEF class='COMPONENT',type='BEAM',name='ROTOR 1 BLADE 2 ELEMENT 1',&END
&NLVAL
    CTYPE (2)=3,CLIN (2 )=0.,
```

```
    NCHYDA ( 2) = 0, CHYDA (1, 2)=1000.,
    NCHYDB(2)=2,CHYDB (1, 2)=0.,0.,100000.,
&END
&NLDEF class='COMPONENT',type='BEAM',name='ROTOR 1 BLADE 3 ELEMENT 1',&END
&NLVAL
    CTYPE (2)=3,CLIN (2)=0.,
    NCHYDA ( 2)=0,CHYDA (1, 2)=1000.,
    NCHYDB (2)=2,CHYDB (1, 2)=0.,0., 100000.,
&END
&NLDEF class='COMPONENT',type='BEAM',name='ROTOR 1 BLADE 4 ELEMENT 1',&END
&NLVAL
    CTYPE (2)=3,CLIN (2)=0.,
    NCHYDA(2)=0,CHYDA(1, 2)=1000.,
    NCHYDB(2)=2,}\operatorname{CHYDB}(1,2)=0.,0.,100000.
&END
!================================================================================
&NLDEF action='end of core',&END
```

! Teetering Rotor in Wind Tunnel

\&NLDEF class='CASE', \&END
\& NLVAL
TITLE='SAMPLE JOBS: TEETERING ROTOR IN WIND TUNNEL',! description
OPUNIT=1,OPDENS=3,DENSE=.00237689,TEMP=59., ! environment
\&END

\&NLDEF class='TRIM', \&END
\&NLVAL

```
    VELIN=2,WINDIN=2,VTIPIN=1,VTIP=650., ! operating condition
    LEVEL=2*1, ! wake loop
    COLL=10.,CTTRIM=.08,MTRIM=1, ! wind tunnel trim
    MNAME='CT/S ',VNAME='COLL ',
    RELAXT=.5, ! trim loop
    OPPART=3, ! teetering rotor
    MHARMR=0,MHARMA=0,MPSIAV=1, ! harmonics
    DOFA=6*0,DOFM=2*0, ! degrees of freedom
```

\&END
\&NLDEF class='TRIM ROTOR', name='ROTOR 1',\&END
\& NLVAL
NAPRNT $=1$, NAFILE $=1$, MATIME $=1$, ! output
OPMODE $=0, \mathrm{DOFG}=1, \mathrm{DOFB}=1,0,1,9 * 0$, ! degrees of freedom
\&END

\&NLDEF class='FLUTTER', \&END
\& NLVAL
MPSIAV=1, ! no average
$\mathrm{DOFA}=6 * 0, \mathrm{DOFM}=2 * 1, \mathrm{DOFD}=8 * 0, \quad$ ! degrees of freedom
\&END
\&NLDEF class='FLUTTER ROTOR', name='ROTOR 1',\&END
\&NLVAL
OPWAKE=4, OPVATR=2, OPVRTA=2, ! trim inflow
OPMODE $=0, \mathrm{DOFG}=1, \mathrm{DOFB}=1,0,1,9 * 0, \mathrm{DOFL}=3 * 0, \quad$ ! degrees of freedom
\&END

\&NLDEF class='AIRFRAME',type='STRUCTURE',\&END
\&NLVAL
TITLE='SAMPLE JOBS: WIND TUNNEL SUPPORT' ! description
CONFIG $=0$, OPFREE $=0$, OPAERO $=0$, ! wind tunnel
MASSR=8.4, ! inertia
NMODE=2, ! test module
$\mathrm{QMASS}=2 * 1000 ., \mathrm{QFREQ}=2 ., 4 ., \mathrm{QDAMP}=2 * .1$,
$\operatorname{LSHAPE}(1,1,1)=1 ., 0 ., 0 .$,
$\operatorname{LSHAPE}(1,1,2)=0 ., 1 ., 0 .$,
$\operatorname{ASHAPE}(1,1,1)=0 .,-1.5,0 .$,
$\operatorname{ASHAPE}(1,1,2)=1.5,0 ., 0 .$,
\&END
\&NLDEF class='AIRFRAME',type='AERODYNAMICS', \&END
\&NLVAL \&END
\&NLDEF class='AIRFRAME',type='CONTROL', \&END
\&NLVAL \&END
\&NLDEF class='AIRFRAME',type='DRIVE TRAIN',\&END
\&NLVAL \&END

\&NLDEF class='ROTOR', type='STRUCTURE', name='ROTOR 1', \&END
\&NLVAL
TITLE='SAMPLE JOBS: TEETERING MAIN ROTOR', ! description
RADIUS $=20$. , NBLADE=2, ROTATE=1,SIGMA=. 05 ,
VTIPN=650.,
GIMBAL=1, CONTRL $=0$, HINGE $=0$, PITCH $=2$, ! blade root
CONE=2., ZUS =-. 01, $\mathrm{EPITCH}=.05, \mathrm{LOCKP}=0$,
OPBEAM=0,KNODE=0, ! rigid blade
TWIN=1,TWISTL=-10., ! linear twist
NPROP=2,RPROP=0.,1., $\quad$ ! section properties

```
    ZEA=2*0., ZQC=2*0.,ZC=2*0.,ZI=2*0.,
    XEA=2*0., XQC=2*0.,XC=2*0.,XI=2*0.,
    KP=2*.015,KT=2*.015,EA=2*60000000.,
    EIFLAP=2*500000., EILAG=2*5000000.,GJ=2*93000.,
    MASS=2*.21,ITHETA=2*.018,IPOLAR=2*.018,
&END
!======================================================================================
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL
    NPANEL=20,
    REDGE=. 12,. 20,. 28,.35,.42,.48,.54,.59,.64,.69,.73,
        .77,.81,.84,.87,.90,.92,.94,.96,.98,1.0,
    NPROP=2,RPROP=0.,1., ! aero properties
    CHORD=2*1.5708,
    NSEN=5,OPREF=5*4, ! aerodynamic sensors
    QUANT= 5,25,35,82,82,
    IDENT= 1, 0, 0, 0, 0,
    AXIS= 3, 0, 0, 1, 3,
    OPSCL= 2, 1, 1, 2, 2,
    NAPLOT=1, 4, 1, 0, 0,
&END
!==================================================================================
&NLDEF class='ROTOR',type='INFLOW',name='ROTOR 1',&END
&NLVAL &END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL
    OPSCEN=1, ! forward flight wake
    TWIST=-10.,RICWG=.20, ! hover wake
&END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
```



```
&NLDEF class='TABLES',&END
&NLVAL &END
!==================================================================================
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
```

```
! Single Main Rotor and Tail Rotor Helicopter
```



```
&NLDEF class='CASE',&END
&NLVAL ! description
    TITLE='SAMPLE JOBS: SINGLE MAIN ROTOR AND TAIL ROTOR HELICOPTER',
    OPUNIT=1,OPDENS=3,DENSE=.00237689,TEMP=59., ! environment
&END
```



```
&NLDEF class='TRIM',&END
&NLVAL
    VELIN=1,WINDIN=1,VTIPIN=1,VTIP=650., ! operating condition
    LEVEL=2*1, ! wake loop
    COLL=10.,MTRIM=6, ! free flight trim
    MNAME='FORCE X ','FORCE Y ','FORCE Z ',
            'MOMENT X','MOMENT Y','MOMENT Z',
    VNAME='COLL ','LATCYC ','LNGCYC ',
    MHARMR=10,1,MHARMA=10,1,MHARMD=10,1, ! harmonics
    DOFA=6*0,DOFD=8*0, ! degrees of freedom
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL OPMODE=0,DOFB=1,0,1,9*0,&END ! degrees of freedom
&NLDEF class='TRIM ROTOR',name='ROTOR 2',&END
&NLVAL OPMODE=0,DOFB=1,0,1,9*0,&END ! degrees of freedom
!=================================================================================
&NLDEF class='TRANSIENT',&END
&NLVAL DOFA=6*1,DOFD=8*0,&END ! degrees of freedom
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 1',&END
&NLVAL
    OPWAKE=4,OPVATR=2,OPVRTA=2, 
&END
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 2',&END
&NLVAL
    OPWAKE=4,OPVATR=2,OPVRTA=2, 
&END
&NLDEF class='FLUTTER', &END
&NLVAL DOFA=6*1,&END ! degrees of freedom
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL
    OPWAKE=4,OPVATR=2,OPVRTA=2, 
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 2',&END
&NLVAL
    OPWAKE=4,OPVATR=2,OPVRTA=2, ! trim inflow
    OPMODE=0,DOFB=1,0,1,9*0,DOFL=3*0, ! degrees of freedom
&END
```



```
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL
    TITLE='SAMPLE JOBS: HELICOPTER AIRFRAME', ! description
    CONFIG=1,RGEAR(2)=4., ! single mr/tr
    WEIGHT=8000.,IXX=2600.,IYY=12000.,IZZ=10000., ! inertia
    MASSR=9.6,.48,
    HSP=2., ! control
    FSCG=0.,WLCG=0., ! geometry
    FSRTR=0.,24.,WLRTR=6.,0., BLRTR=0.,0.,
    ASHAFT=-4.,0.,
    FSWB=0.,WLWB=0.,FSHT=20.,WLHT=0.,FSVT=24.,WLVT=0.,
    NLOC=8, ! locations
    FSLOC(5)=0.,-4., 2*-9., ! 5-6 for sensor
    WLLOC(5)=0.,0., 2*3., ! 7-8 for inflow
```

```
    BLLOC(5)=0.,2., 0.,5.,
    NSEN=3,LOCSEN=5,5,6,QUANT=4,5,17,OPSCL=2*1,0, ! airframe sensors
&END
&NLDEF class='AIRFRAME',type='AERODYNAMICS',&END
&NLVAL
    LFTAW=50.,DRGOW=12.,DRGVW=0., AMAXW=20.,MOMAW=400.,
    SIDEB=-150.,YAWB=-1500.,
    LFTAH=50.,AMAXH=20.,IHTL=5.,LFTAV=40.,AMAXV=20.,
&END
&NLDEF class='AIRFRAME',type='CONTROL',&END
&NLVAL &END
&NLDEF class='AIRFRAME',type='DRIVE TRAIN',&END
&NLVAL
    CONFIG=1,OPGOV=0,IGEAR=6.,EGEAR=12.,GAINE=100.,
    KRS=50000.,1000.,IRS=.1,.1,
    KIS=10000.,KES=5000 , ,IENG=.02,
&END
!===============================================================================
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
&NLVAL
    TITLE='SAMPLE JOBS: HELICOPTER MAIN ROTOR', ! description
    RADIUS=20.,NBLADE=4,ROTATE=1,SIGMA=.075,
    VTIPN=650.,
    HINGE=3, ! blade root
    EFLAP=.04,ELAG=.06,EPITCH=.065,DLAG=2000.,
    CONTRL=2,PITCH=1,KPITCH=0.,LOCKP=1, ! control system
        XSP=-.03,YSP=.035,ZSP=-.1,
        XPH=-.03,YPH=.035, ZPH=0.,
        EPH=.07,KPL=80000.,LOCKPL=1,LOCKSP=0,
    OPBEAM=2,DRELST=.1,KNODE=2,RNODE=.35,.64, ! elastic blade
    NSEN=3,QUANT=3*4,RLOAD=.4,.6,.8, ! loads
    TWIN=1,TWISTL=-10., ! linear twist
    NPROP=2,RPROP=0.,1., ! section properties
    ZEA=2*0., ZQC=2* 0., ZC=2*0.,ZI=2*0.,
    XEA=2*0., XQC=2*0.,XC=2*0.,XI=2*0.,
    KP=2*.011,KT=2*.011,EA=2*20000000.,
    EIFLAP=2*68000.,EILAG=2*800000.,GJ=2*90000.,
    MASS=2*. 12,ITHETA=2*.0056,IPOLAR=2*.0056,
&END
```



```
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL
    NPANEL=20,
    REDGE=.12,.20,. 28,.35,.42,.48,.54,.59,.64,.69,.73,
        . 77,.81,.84,.87,.90,.92,.94,.96,.98,1.0,
    NPROP=2,RPROP=0.,1., ! aero properties
    CHORD=2*1.1781,
    NSEN=5,OPREF=5*4, ! aerodynamic sensors
    QUANT= 5,25,35,82,82, ! lambda
    IDENT= 1, 0, 0, 0, 0, ! alpha,theta
    AXIS=3,0, 0, 1, 3, ! Fx,Fz
    OPSCL= 2, 1, 1, 2, 2,
i
    NAPLOT=1, 4, 1, 0, 0, !
&END
```



```
&NLDEF class='ROTOR',type='INFLOW',name='ROTOR 1',&END
&NLVAL KINTFW=1.5,KINTFH=1.8,&END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL
    OPSCEN=1, ! forward flight wake
    TWIST=-10.,RICWG=.15, ! hover wake
    NPOFF=2,LOCOFF=7,8, ! inflow off rotor
&END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL OPSCEN=0,&END
```

```
!======================================================================================
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 2',&END
&NLVAL
    TITLE='SAMPLE JOBS: HELICOPTER TAIL ROTOR', ! description
    RADIUS=5.,NBLADE=4,ROTATE=1,SIGMA=.15,
    VTIPN=650.,
    CONTRL=0,HINGE=3,PITCH=2, ! blade root
    EFLAP=0.,ELAG=.1,EPITCH=.15,
    AFLAP=30.,LOCKP=0,DLAG=10.,
    OPBEAM=0,KNODE=0, ! rigid blade
    TWIN=1,TWISTL=0.,
    NPROP=2,RPROP=0.,1.,
    ZEA=2*0.,ZQC=2*0.,ZC=2*0.,ZI=2*0.,
    XEA=2*0., XQC=2*0., XC=2*0., XI=2*0.,
    KP=2*.022,KT=2*.022,EA=2*2000000.,
    EIFLAP}=2*850.,EILAG=2*25000.,GJ=2*1500.,
    MASS=2*.024,ITHETA=2*.00028,IPOLAR=2*.00028,
&END
!=======================================================================================
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 2',&END
&NLVAL
    NPANEL=8,REDGE=.2,.3,.4,.5,.6,.7,.8,.9,1.,
    NPROP=2,RPROP=0.,1., ! aero properties
    CHORD=2*.58905,BTIP=.95,
&END
!=======================================================================================
&NLDEF class='ROTOR',type='INFLOW',name='ROTOR 2',&END
&NLVAL KINTHV=.7,KINTFV=.7,&END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 2',&END
&NLVAL &END
l========================================================================================
&NLDEF class='TABLES',&END
&NLVAL AFTABL=1,&END ! separate tables
!======================================================================================
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
```

```
! Bearingless Rotor
```



```
&NLDEF class='CASE',&END
&NLVAL
    TITLE='SAMPLE JOBS: BEARINGLESS ROTOR', ! description
    OPUNIT=1,OPDENS=3,DENSE=.00237689,TEMP=59., ! environment
&END
!================================================================================
&NLDEF class='TRIM',&END
&NLVAL
    VELIN=1,WINDIN=1,WKTS=0.,VTIPIN=1,VTIP=650., ! operating condition
    LEVEL=2*1, ! wake loop
    OPTRIM=0,COLL=10., ! no trim
    MHARMR=0,MHARMA=0,DOFA=6*0,MPSIAV=1,MPSI=4, ! hover
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL OPMODE=0,DOFB=12*1,&END ! no blade modes
!===============================================================================
&NLDEF class='FLUTTER',&END
&NLVAL
    MPSIAV=1, ! no average
    DOFA=6*0, ! degrees of freedom
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL
    OPWAKE=4,OPVATR=2,OPVRTA=2, ! trim inflow
    OPMODE=1,DOFM=4*1,36*0,DOFL=3*0, ! degrees of freedom
    GDAMPM=40*.01,NMPRNT=1,
&END
```



```
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL
    TITLE='SAMPLE JOBS: WIND TUNNEL SUPPORT', ! description
    CONFIG=0,OPFREE=0,OPAERO=0, ! wind tunnel
    MASSR=12.26, ! inertia
    HSP=2., ! control
&END
&NLDEF class='AIRFRAME',type='AERODYNAMICS',&END
&NLVAL &END
&NLDEF class='AIRFRAME',type='CONTROL',&END
&NLVAL &END
&NLDEF class='AIRFRAME',type='DRIVE TRAIN',&END
&NLVAL &END
!==================================================================================
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
&NLVAL
    TITLE='SAMPLE JOBS: BEARINGLESS ROTOR', ! description
    RADIUS=20. ,NBLADE=5,ROTATE=1,SIGMA=.075,
    VTIPN=650.,
    CONFIG=2,CONTRL=2,LOCKSP=0,HINGE=0,PITCH=0, ! blade root
        XSP=.04,YSP=.05,ZSP=-.1, ! te pitch link
        XPH=.04,YPH=.05,ZPH=0.,
        EPH=.05,LOCKPL=1,KPL=100000.,
    CONE=2.,
    OPBEAM=2,KNODE=3,RNODE=.28,.48,.73, ! elastic blade
    DRELST=.04,KRIGID=1,RRGDB=0.,RRGDE=.28,
    TWIN=1,TWISTL=-10., ! linear twist
    NPROP=4,RPROP=0.,.2499,.2501,1., ! section properties
    ZEA=4 * 0., ZQC=4*0., ZC=4*0.,ZI=4*0.,
    XEA=4*0., XQC=4*0., XC=4*0.,XI=4*0.,
    KP=2*.020,2*.011,KT=2*.020, 2*.011, EA= 4* 40000000.,
    EIFLAP=2*600000.,2*60000.,EILAG=4*1650000.,GJ=2*900000.,2*90000.,
    MASS=4*. 12,ITHETA=2*.02, 2*.0056,IPOLAR=2*.001, 2*.0056,
&END
```



```
&NLDEF class='ROTOR',type='FLEXBEAM',name='ROTOR 1',&END
&NLVAL
    EBLADE=.02,EFB=.25,SNUB=1, ! blade root
        XSNUB=0.,YSNUB=.025,ZSNUB=.005873, ! snubber
        XROOT=0.,EROOT=.025, ZROOT=.005000,
        KXSNUB=0.,KYSNUB=0.,KZSNUB=100000.,
        DXSNUB=10.,DYSNUB=0.,DZSNUB=0.,
    OPBEAM=2,KNODE=2,RNODE=.03,.23, ! elastic flexbeam
    DRELST=.04,KRIGID=2,RRGDB=0.,.23,RRGDE=.03,.25,
    NPROP=2,RPROP=0.,1.,
    ZEA=2*0.,ZC=2*0.,ZI=2*0.,
    XEA=2*0., XC=2*0.,XI=2*0.,
    KP=2*.011,KT=2*.011,EA=2*40000000.,
    EIFLAP=2*36000.,EILAG=2*440000.,GJ=2*5000.,
    MASS=2*.02,ITHETA=2*.001,IPOLAR=2*.001,
&END
!====================================================================================
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL
    NPANEL=20,
    REDGE=.12,.20,.28,.35,.42,.48,.54,.59,.64,.69,.73,
        .77,.81,.84,.87,.90,.92,.94,.96,.98,1.0,
    NPROP=2,RPROP=0.,1., ! aero properties
    CHORD=2*.94248,
    NSEN=5,OPREF=5*4, ! aerodynamic sensors
    QUANT= 5,25,35,82,82,
    IDENT= 1, 0, 0, 0, 0,
    AXIS= 3, 0, 0, 1, 3,
    OPSCL= 2, 1, 1, 2, 2,
    NAPLOT=1, 4, 1, 0, 0,
        lambda
    ! alpha,theta
        Fx,Fz
    !OT=1, 4, 1, 0, 0, !
&END
```



```
&NLDEF class='ROTOR',type='INFLOW',name='ROTOR 1',&END
&NLVAL &END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL &END
!======================================================================================
&NLDEF class='TABLES',&END
&NLVAL &END
!=======================================================================================
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
```

! Tandem Helicopter

\&NLDEF class='CASE', \&END
\& NLVAL
$\begin{array}{ll}\text { TITLE='SAMPLE JOBS: TANDEM HELICOPTER', } & \text { ! description } \\ \text { OPUNIT=1,DENSE=.00237689,TEMP=59., } & \text { ! environment }\end{array}$
\& END

\&NLDEF class='TRIM', \&END
\&NLVAL

```
    VELIN=1,WINDIN=1,VTIPIN=1,VTIP=650., ! operating condition
```

    LEVEL \(=2 * 2\), ! wake loop
    COLL=10., MTRIM=6, ! free flight trim
    MNAME='FORCE X ','FORCE Y ','FORCE Z ',
            'MOMENT X ', 'MOMENT \(\mathrm{Y}^{\prime},{ }^{\prime}\) MOMENT Z ',
    \(\begin{array}{cccc}\text { VNAME }=\text { 'COLL } & \text { ','LATCYC } & \text { ',' LNGCYC } & \text { ', } \\ \text { 'PEDAL } & \text { ','PITCH } & \text { ','ROLL } & \text { ', }\end{array}\)
    MHARMR \(=2 * 1\), MHARMA \(=2 * 1\), ! harmonics
    DOFA \(=6 * 0\), \(\quad!\) degrees of freedom
    CNTRLF=0., ! auxiliary propulsion
    \& END
\&NLDEF class='TRIM ROTOR', name='ROTOR 1',\&END
$\& N L V A L O P M O D E=0, D O F B=1,0,1,9 * 0, \& E N D \quad$ ! degrees of freedom
\&NLDEF class='TRIM ROTOR', name='ROTOR 2',\&END
$\& N L V A L$ OPMODE $=0$, DOFB $=1,0,1,9 * 0, \& E N D \quad!$ degrees of freedom

\&NLDEF class='FLUTTER', \&END
\&NLVAL DOFA $=6 * 1, \& E N D \quad$ ! degrees of freedom
\&NLDEF class='FLUTTER ROTOR', name='ROTOR 1',\&END
$\&$ NLVAL OPMODE $=0, \mathrm{DOFB}=3 * 1,9 * 0, \mathrm{DOFL}=1,2 * 0, \& E N D \quad$ ! degrees of freedom
\&NLDEF class='FLUTTER ROTOR', name='ROTOR 2', \&END
$\& N L V A L O P M O D E=0, \mathrm{DOFB}=3 * 1,9 * 0, \mathrm{DOFL}=1,2 * 0, \& E N D \quad!$ degrees of freedom

\&NLDEF class='AIRFRAME', type='STRUCTURE', \&END
\&NLVAL
TITLE='SAMPLE JOBS: TANDEM HELICOPTER AIRFRAME', ! description
CONFIG=2, ! tandem
WEIGHT=16000.,IXX=6000.,IYY=60000.,IZZ=55000., ! inertia
FSRTR=-15.,14.,WLRTR=6.,10.,ASHAFT=-5.,-2., ! geometry
$\mathrm{NFRC}=1$, LOCFRC $=5$, $\mathrm{AXFRC}=1$, $\mathrm{NLOC}=5$,
auxiliary propulsion
\&END
\&NLDEF class='AIRFRAME',type='AERODYNAMICS' , \&END
\& NLVAL
LFTAW=300., IWBM=-5., MOMAW=1200.,
DRG0W=40., DRGVW=40., DRGIW=. $01, \mathrm{AMAXW}=20 .$,
SIDEB $=-300 .$, ROLLB $=-1200 .$, YAWB $=-700$,
\&END
\&NLDEF class='AIRFRAME',type='CONTROL', \&END
\&NLVAL KF $0=1.1, \mathrm{KR} 0=.9, \& \mathrm{END}$
\&NLDEF class='AIRFRAME',type='DRIVE TRAIN',\&END
\&NLVAL \&END

\&NLDEF class='TABLES', \&END
\&NLVAL \&END

\&NLDEF action='end of shell', \&END
\&NLDEF action='end of core', \&END
! Tandem Helicopter Front Rotor

\&NLDEF class='AIRFRAME', type='STRUCTURE', \&END
\&NLVAL MASSR (1)=9.6, \&END

```
&NLDEF class='ROTOR',type='STRUCTURE', name='ROTOR 1',&END
&NLVAL
    TITLE='SAMPLE JOBS: TANDEM HELICOPTER FRONT ROTOR', ! description
    RADIUS=20.,NBLADE=4,ROTATE=-1,SIGMA=.075,
    VTIPN=650.,
    CONTRL=0,HINGE=3,PITCH=2, ! blade root
    EFLAP=.02,ELAG=.06,EPITCH=.05,
    DLAG=1000.,LOCKP=0,
    OPBEAM=0,KNODE=0, ! rigid blade
    TWIN=1,TWISTL=-10., ! linear twist
    NPROP=2,RPROP=0.,1., ! section properties
    ZEA=2*0.,ZQC=2*0.,ZC=2*0.,ZI=2*0.,
    XEA=2*0.,XQC=2*0.,XC=2*0.,XI=2*0.,
    KP=2*.011,KT=2*.011,
    EIFLAP=2*68000., EILAG=2*800000.,GJ=2*90000.,EA=2*20000000.,
    MASS=2*.12,ITHETA=2*.0056,IPOLAR=2*.0056,
&END
```


\&NLDEF class='ROTOR',type='AERODYNAMICS', name='ROTOR 1', \&END
\&NLVAL
NPANEL=20, ! aero panels
REDGE $=.12$, $20, .28, .35, .42, .48, .54, .59, .64, .69, .73$,
$.77, .81, .84, .87, .90, .92, .94, .96, .98,1.0$,
NPROP $=2$, RPROP $=0 ., 1 ., \quad$ ! aero properties
CHORD $=2$ * 1.1781 ,
NSEN $=6, \mathrm{OPREF}=6 * 4$, ! aerodynamic sensors
QUANT $=5,6,25,35,82,82$, ! lambda,lambda-int
IDENT $=1,1,0,0,0,0$, ! alpha,theta
AXIS $=3,3,0,0,1,3, \quad$ ! Fx,Fz
OPSCL $=2,2,1,1,2,2$,
$!$
NAPLOT $=1,1,4,1,0,0$, !
\& END

\&NLDEF class='ROTOR',type='INFLOW', name='ROTOR 1', \&END
\&NLVAL
OPFFLI=2,FMLMDA=2*0., ! uniform inflow
$\operatorname{KINTFR}(2)=1.8, \operatorname{KINTHR}(2)=.06, \quad$ interference inflow
\&END
\&NLDEF class='ROTOR',type='WAKE', name='ROTOR 1',\&END
$\& N L V A L$ OPSCEN=1,\&END ! forward flight wake
\&NLDEF class='ROTOR',type='WAKE', name='ROTOR 1',\&END
\&NLVAL OPSCEN=0,
CORE=.5, COREWG=.5, ! vortex core
$\mathrm{OPFWG}=4, \mathrm{OPDISP}=2 * 0, \mathrm{RFW}=3 ., \mathrm{MFWG}=3, \quad$ ! wake geometry
\&END

\&NLDEF action='end of shell', \&END
\&NLDEF action='end of core',\&END
! Tandem Helicopter Rear Rotor

\&NLDEF class='AIRFRAME', type='STRUCTURE', \&END
\&NLVAL MASSR (2)=9.6, \&END

```
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 2',&END
&NLVAL
    TITLE='SAMPLE JOBS: TANDEM HELICOPTER REAR ROTOR', ! description
    RADIUS=20.,NBLADE=4,ROTATE=1,SIGMA=.075,
    VTIPN=650.,
    CONTRL=0,HINGE=3,PITCH=2, ! blade root
    EFLAP=.02,ELAG=.06,EPITCH=.05,
    DLAG=1000.,LOCKP=0,
    OPBEAM=0,KNODE=0, ! rigid blade
    TWIN=1,TWISTL=-10., ! linear twist
    NPROP=2,RPROP=0.,1., ! section properties
    ZEA=2*0.,ZQC=2*0.,ZC=2*0.,ZI=2*0.,
    XEA=2*0., XQC=2*0.,XC=2*0.,XI=2*0.,
    KP=2*.011,KT=2*.011,
    EIFLAP=2*68000., EILAG=2*800000.,GJ=2*90000.,EA=2*20000000.,
    MASS=2*.12,ITHETA=2*.0056,IPOLAR=2*.0056,
&END
!=================================================================================
```

\&NLDEF class='ROTOR',type='AERODYNAMICS', name='ROTOR 2', \&END
\&NLVAL
NPANEL=20, $\quad$ aero panels
REDGE $=.12, .20, .28, .35, .42, .48, .54, .59, .64, .69, .73$,
$.77, .81, .84, .87, .90, .92, .94, .96, .98,1.0$,
NPROP $=2$, RPROP $=0 ., 1 ., \quad$ aero properties
CHORD $=2$ * 1.1781 ,
NSEN $=6, \mathrm{OPREF}=6 * 4$, ! aerodynamic sensors
QUANT $=5,6,25,35,82,82$, ! lambda,lambda-int
IDENT $=1,1,0,0,0,0$, ! alpha,theta
AXIS $=3,3,0,0,1,3, \quad$ ! Fx,Fz
OPSCL $=2,2,1,1,2,2$,
$!$
NAPLOT $=1,1,4,1,0,0$, !
\& END

\&NLDEF class='ROTOR',type='INFLOW', name='ROTOR 2', \&END
\&NLVAL
OPFFLI=2,FMLMDA=2*0., ! uniform inflow
$\operatorname{KINTFR}(1)=0 ., \operatorname{KINTHR}(1)=.06, \quad$ ! interference
\&END
\&NLDEF class='ROTOR',type='WAKE', name='ROTOR 2', \&END
\&NLVAL OPSCEN=1,\&END ! forward flight wake
\&NLDEF class='ROTOR',type='WAKE', name='ROTOR 2',\&END
\&NLVAL OPSCEN=0,
CORE $=.5$, COREWG $=.5$, ! vortex core
$\mathrm{OPFWG}=4, \mathrm{OPDISP}=2 * 0, \mathrm{RFW}=2 \cdot, \mathrm{MFWG}=2, \quad$ ! wake geometry
\&END

\&NLDEF action='end of shell', \&END
\&NLDEF action='end of core',\&END

```
! Tiltrotor in Wind Tunnel
```



```
&NLDEF class='CASE',&END
&NLVAL
    TITLE='SAMPLE JOBS: TILTROTOR IN WIND TUNNEL', ! description
    OPUNIT=1,OPDENS=3,DENSE=.00237689,TEMP=59., ! environment
&END
```



```
&NLDEF class='TRIM',&END
&NLVAL
    VELIN=1,WINDIN=1,VTIPIN=1,VTIP=600., ! operating condition
    LEVEL=1, ! wake loop
    COLL=10.,CTTRIM=.08,MTRIM=1, ! wind tunnel trim
    MNAME='CT/S ',VNAME= COLL ',
    OPPART=3, ! gimballed rotor
    MHARMR=0,MHARMA=0,MHARMD=0,MPSIAV=1, ! harmonics
    DOFA=6*0,DOFM=3*0,DOFD=8*0, ! degrees of freedom
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL OPMODE=0,DOFG=1,DOFB=12*1,&END ! degrees of freedom
```



```
&NLDEF class='FLUTTER',&END
&NLVAL
    MPSIAV=1, ! no average
    DOFA=6*0,DOFM=3*1,DOFD=1,7*0,DOFORD=5*1, ! degrees of freedom
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL
    OPWAKE=4,OPVATR=2,OPVRTA=2, ! trim inflow
    OPMODE=1,DOFG=1,DOFM=4*1,36*0,DOFL=2,2*0, ! degrees of freedom
    GDAMPM=40*.06,NMPRNT=1,
&END
```



```
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL
    TITLE='SAMPLE JOBS: CANTILEVER WING (ATILT=0)', ! description
    CONFIG=0,OPFREE=0, ! wind tunnel
    OPAERO=1, ! wing aerodynamics
    OPTRAN=1, ! drive train
    ASHAFT=-90., ! geometry
    NMODE=3, ! elastic modes
    MENAME='BEAM','CHORD','TORSION',
    MELABL='BEAM','CHORD','TORSION',
    QMASS=80.,80.,1.2,QFREQ=3.,5.,11.,
    QDAMP=3*.04,QAEROD=100.,1.,3.,
    LSHAPE(1,1,1)= 0., 0.,-1.,
    LSHAPE(1,1,2)= -1.,.45, 0.,
    LSHAPE (1,1,3)= 0., 0.,-.2,
    ASHAPE (1,1,1)= -. 1,.04, 0.,
    ASHAPE (1,1,2)= 0., 0., .1,
    ASHAPE(1,1,3)=.02,.06, 0.,
&END
&NLDEF class='AIRFRAME',type='AERODYNAMICS',&END
&NLVAL &END
&NLDEF class='AIRFRAME',type='CONTROL',&END
&NLVAL &END
&NLDEF class='AIRFRAME',type='DRIVE TRAIN',&END
&NLVAL
    CONFIG=0,OPGOV=0,
    EGEAR=1.,KRS=1000000.,KES=10000.,
&END
```



```
&NLDEF class='TABLES',&END
&NLVAL &END
```


\&NLDEF action='end of shell',\&END \&NLDEF action='end of core',\&END

```
! Tilting Proprotor Aircraft
```



```
&NLDEF class='CASE',&END
&NLVAL
    TITLE='SAMPLE JOBS: TILTING PROPROTOR AIRCRAFT', ! description
    OPUNIT=1,DENSE=.00237689,TEMP=59., ! environment
&END
```



```
&NLDEF class='TRIM',&END
&NLVAL
    VELIN=1,WINDIN=1,VTIPIN=1,VTIP=600., ! operating condition
    LEVEL=2*1, ! wake loop
    GOV=10.,MTRIM=3, ! free flight trim
    MNAME='FORCE X ','FORCE Z ','MOMENT Y',
    VNAME='GOV ','LNGCYC ','PITCH ',
    OPPART=2*3, ! gimballed rotors
    MHARMR=2*10,MHARMA=2*10,MHARMD=2*10, ! harmonics
    DOFA=6*1,DOFM=10*1,DOFD=8*1, ! degrees of freedom
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL
    CNTRLR=25., 2*0.,
    OPMODE=0,DOFG=1,DOFB=12*1, ! degrees of freedom
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 2',&END
&NLVAL
    CNTRLR=25., 2*0.,
    OPMODE=0,DOFG=1,DOFB=12*1, ! degrees of freedom
&END
```



```
&NLDEF class='FLUTTER',&END
&NLVAL
    DOFA=6*1,DOFM=8*1,2*0,DOFD=8*1, ! degrees of freedom
    OPEQN=4*0,2*1,3*0, ! sym/antisym eqns
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL
    OPWAKE=3,OPVATR=2,OPVRTA=2, ! dynamic inflow
    OPMODE=1,DOFG=1,DOFM=4*1,36*0,DOFL=2,2*0, ! degrees of freedom
    GDAMPM=40*.06,
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 2',&END
&NLVAL
    OPWAKE=3,OPVATR=2,OPVRTA=2,
    OPMODE=1,DOFG=1,DOFM=4*1,36*0,DOFL=2,2*0, ! degrees of freedom
    GDAMPM=40*.06,
&END
```



```
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL ! description
    TITLE='SAMPLE JOBS: TILTING PROPROTOR AIRCRAFT AIRFRAME',
    CONFIG=4,ATILT=0.,OPTRAN=1, ! tiltrotor
    WEIGHT=15000., ! inertia
    IXX=50000.,IYY=15000.,IZZ=60000,
    FSCG=-1.,WLCG=-2., ! geometry
    FSRTR=0., BLRTR=20.,WLRTR=5.,ASHAFT=0.,ACANT=1.,
    FSPIV=0.,BLPIV=20.,WLPIV=0.,ASPIV=0.,ADPIV=-1.,
    FSWB=0.,WLWB=0.,FSHT=25.,WLHT=0.,FSVT=25.,WLVT=0.,
    NLOC=17,
    ! locations
    FSLOC(5)=-7.,WLLOC (5)=-4., ! 5=sensor
    FSLOC(6)= 0.,2., 0., 25.,25.,25., ! 6-11=wings
    BLLOC(6)=-20.,0.,20., -7., 0., 7.,
    WLLOC(6)= 0.,0., 0., 0., 0., 0.,
    FSLOC(12)=-3.,-15.5, 0.,-8., 0.,-8., ! 12-17=bodies
    BLLOC(12)=2*0., 2*-20., 2*20.,
```

```
WLLOC(12)=2*-2., 2*0., 2*0.,
NSEN=1,LOCSEN=5,QUANT=3,OPSCL=1, ! sensor
NMODE=10, ! elastic modes
QMASS=10*12.,QDAMP=3*.03,.2,3*.03,3*.2,
QFREQ=3.,6.,8.,18.,6.,8.,7.,20.,15.,22.,
QSYM=4*1,4*-1,2*1,
QAEROD=10.,2.,20.,12.,24.,80.,14.,16.,0.,0.,
QAEROC= 15., -10., 2*0.,
            0., 8., 2*0.,
            -4., 35., 2*0.,
                -32., -15., 2*0.,
                2*0.,-16., -12.,
                2*0., 5., -40.,
                2*0., 0., 15.,
                2*0.,-25., 11.,
            0., 0., 2*0.,
            0., 0., 2*0.,
! location1=rotor1(right)
LSHAPE(1,1,1)= .0, .0, -.2,
LSHAPE(1,1,2)= -.2, .2, .2,
LSHAPE}(1,1,3)= -.1, .1, -.3,
LSHAPE(1,1,4)= .0, .4, .0,
LSHAPE}(1,1,5)= .0, .1, -.2
LSHAPE}(1,1,6)= -.1, .1, -.1
LSHAPE (1, 1,7)= .0, .0, -.3,
LSHAPE}(1,1,8)= .0, .3, .0
LSHAPE(1,1,9)= .0, .1, .0,
LSHAPE}(1,1,10)= .0, .2, .1
ASHAPE}(1,1,1)= -.03, .0, .0
ASHAPE (1,1,2)= .0, -.06, .05,
ASHAPE}(1,1,3)= .0, .1, .04
ASHAPE(1,1,4)= -.06, .0, .2,
ASHAPE}(1,1,5)= -.04, .04, .0,
ASHAPE(1,1,6)= .02, .07, .07,
ASHAPE (1,1,7)= .0, .1, .0,
ASHAPE}(1,1,8)=-.1, .0, .2
ASHAPE(1,1,9)= .0, .0, .03,
ASHAPE}(1,1,10)= .0, -.1, .1
! location2=rotor2(left)
LSHAPE(1,2,1)= .0, -.0, -.2,
LSHAPE}(1,2,2)= -.2, -.2, .2
LSHAPE(1,2,3)= -.1, -.1, -.3,
LSHAPE (1,2,4)= .0, -.4, .0,
LSHAPE (1,2,5)= .0, -.1, -.2,
LSHAPE(1,2,6)= -.1, -.1, -..1,
LSHAPE}(1,2,7)= .0, -.0, -.3
LSHAPE (1,2,8)= .0, -.3, .0,
LSHAPE}(1,2,9)= .0, -.1, .0
LSHAPE (1,2,10)= .0, -.2, .1,
ASHAPE}(1,2,1)= .03, .0, -.0
ASHAPE}(1,2,2)= -.0, -.06,-.05
ASHAPE}(1,2,3)= -.0, .1, -.04
ASHAPE}(1,2,4)= .06, .0, -.2,
ASHAPE (1,2,5)= .04, .04,-.0,
ASHAPE}(1,2,6)= -.02, .07,-.07
ASHAPE (1,2,7)= -.0, .1, -.0,
ASHAPE (1,2,8)= .1, .0, -.2,
ASHAPE}(1,2,9)= .0, .0, -.03
ASHAPE(1,2,10)= .0, -.1, -.1,
! location5=sensor
LSHAPE(1,5,1)= .0, .0, .1,
LSHAPE (1,5,2)= .1, .0, .03,
LSHAPE (1,5,3)= .04, .0, .08,
LSHAPE(1,5,4)= -.04, .0, -.06,
LSHAPE}(1,5,5)= .0, -. 16, .0
```

```
    LSHAPE (1,5,6)= .0, -. 16, .0,
    LSHAPE (1,5,7)= .0, . 1, .0,
    LSHAPE (1,5,8)= .0, .04,.0,
    LSHAPE (1,5,9)= -.03,.0, 1.,
    LSHAPE (1,5,10)= -.02, .0, -.02,
&END
&NLDEF class='AIRFRAME',type='AERODYNAMICS',&END
&NLVAL
    LFTAW=1000.,IWBL=4.,IWBD=4.,IWBM=-2., LFTDW=200., LFTFW=300.,
    DRG0W=12.,DRGVW=12.,DRGIW=.0005,DRGDW=12.,DRGFW=5 .,
    AMAXW=20.,MOMAW=1400.,MOMDW=-200.,MOMFW=-300.,
    SIDEB=-100.,ROLLB=200.,ROLLP=-90000.,ROLLR=9000., ROLLDA=-3500.,
    YAWB=-1500.,YAWP=-2000.,YAWR=-2000.,YAWDA=70.,
    LFTAH=250., LFTEH=150.,AMAXH=15.,LFTAV=180.,LFTRV=70.,AMAXV=20.,
    EHTAIL=.0005,LHTAIL=25.,HVTAIL=3.,
!
    VISRC=2*0,
    NWING=2,LOCWL=6,9,LOCWM=7,10,LOCWR=8,11,
    CIRC=2*0.,FCIRCW=1.,0.,FCIRCH=0.,1.,FCIRCV=0.,0.,AXS=5., 2.,
    XCIRC=.19,.25,XTHICK=.27,.375,SPAN=40.,14.,CHORD=6.,4.,
    NBODY=3,LOCBC=12,14,16,LOCBN=13,15,17,
    LENGTH=25.,2*16.,THICK=.25,2*.18,SHAPE=3*1,
&END
&NLDEF class='AIRFRAME',type='CONTROL',&END
&NLVAL
    K0=0.,KC=0.,KS=0.,KP=0.,
    KT=4.,KA=6 .,KE=2.,KR=5.,
&END
&NLDEF class='AIRFRAME',type='DRIVE TRAIN',&END
&NLVAL
    CONFIG=3,OPGOV=1,
    IGEAR=16.,EGEAR=35.,GAINE=12.,
    KRS=2*750000.,IRS=2*2.,KIS=5000.,IIS=2 .,
    KES=12000.,IENG=.1,DENG=.2,
    KIGOV=.02,KPGOV=0 . ,KRGOV=2*1.,KEGOV=0 .,
    WGOV=6.2832,ZGOV=.7 ,
&END
```



```
&NLDEF class='TABLES',&END
&NLVAL &END
```



```
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
```

```
! Tilting Proprotor Aircraft Right Rotor
!=================================================================================
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL
    MASSR(1)=10.8, 
&END
```



```
&NLDEF class='ROTOR',type='STRUCTURE', name='ROTOR 1',&END
&NLVAL ! description
    TITLE='SAMPLE JOBS: TILTING PROPROTOR AIRCRAFT RIGHT ROTOR',
    RADIUS=15.,NBLADE=3,ROTATE=1,SIGMA=.10,
    VTIPN=600.,
    GIMBAL=1,HINGE=0, ! blade root
    CONE=1.5,EPITCH=.09,KGMBL=15000.,
    CONTRL=2,PITCH=1,KPITCH=0.,LOCKP=1, ! control system
    XSP=.06,YSP=.016,ZSP=-.1,
    XPH=.06,YPH=.016, ZPH=.03,
    EPH=.11,KPL=25000.,LOCKPL=1,LOCKSP=0,
    OPBEAM=2,DRELST=.04,KNODE=2,RNODE=.35,.64, ! elastic blade
    NSEN=2,QUANT=2*4,RLOAD=.05,.35,
    NPROP=3,RPROP=0.,.5,1., ! section properties
    ZEA=3*0.,ZQC=3*0.,ZC=3*0.,ZI=3*0.,
    XEA=3*0., XQC=3*0.,XC=3*0.,XI=3*0.,
    TWISTA=34.5,7.,-7.,THETAC=34.5,7.,-7.,THETAI=34.5,7.,-7.,
    KP=3*.019,KT=3*.019,EA=3*250000000.,
    EIFLAP=3*1500000., EILAG=3*20000000.,GJ=3*200000.,
    MASS=3*.24,ITHETA=3*.02,IPOLAR=3*.02,
&END
```



```
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL
    NPANEL=20,
    REDGE=. 12,. 20,. 28,.35,.42,.48,.54,.59,.64,.69,.73,
        . 77,.81,.84,.87,.90,.92,.94,.96,.98,1.0,
    NPROP=11,RPROP=0.,.1,.2,.3,.4,.5,.6,.7,.8,.9,1., ! aero properties
    CHORD=11*1.5708,
    KSDL=3*.81,.50,.33,.23,.16,.10,.07,.04,.01,
    KSDD=3*.54,.34,.20,.12,.06,.02,.00,.00,.00,
    NSEN=9,OPREF=9*4,
    QUANT= 5, 25,31, 32, 33, 35,71, 82,82,
    IDENT= 1, 0, 0, 0, 0, 0, 0, 0, 0,
    AXIS= 3, 0, 0, 0, 0, 0, 0, 1, 3,
    OPSCL= 2, 1, 2, 2, 2, 1, 2, 2, 2,
    NAPLOT=1, 4, 1, 1, 1, 1, 1, 0, 0,
&END
!==================================================================================
&NLDEF class='ROTOR',type='INFLOW', name='ROTOR 1',&END
&NLVAL
    KHLMDA=1.085,KFLMDA=2 . ,FMLMDA=2 * 0.,
    KINTFR(2)=-.085,KINTHW=1.5,KINTFH=1.8,
&END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL OPSCEN=2,TWIST=-28.,RICWG=.3,&END ! hover wake
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
&NLVAL OPSCEN=0,RNW=.25,WKMODL=8*2,&END
!==================================================================================
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
```

```
! Tilting Proprotor Aircraft Left Rotor
!==================================================================================
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL
```



```
&END
```



```
&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 2',&END
&NLVAL ! description
    TITLE='SAMPLE JOBS: TILTING PROPROTOR AIRCRAFT LEFT ROTOR',
    RADIUS=15.,NBLADE=3,ROTATE=-1,SIGMA=. 10,
    VTIPN=600.,
    GIMBAL=1,HINGE=0, ! blade root
    CONE=1.5, EPITCH=.09,KGMBL=15000.,
    CONTRL=2,PITCH=1,KPITCH=0.,LOCKP=1, ! control system
    XSP=.06,YSP=.016,ZSP=-.1,
    XPH=.06,YPH=.016, ZPH=.03,
    EPH=.11,KPL=25000.,LOCKPL=1,LOCKSP=0,
    OPBEAM=2,DRELST=.04,KNODE=2,RNODE=.35,.64, ! elastic blade
    NSEN=2,QUANT=2*4,RLOAD=.05,.35,
    NPROP=3,RPROP=0.,.5,1., ! section properties
    ZEA=3*0.,ZQC=3*0.,ZC=3*0.,ZI=3*0.,
    XEA=3*0., XQC=3*0.,XC=3*0.,XI=3*0.,
    TWISTA=34.5,7.,-7.,THETAC=34.5,7.,-7.,THETAI=34.5,7.,-7.,
    KP=3*.019,KT=3*.019,EA=3*250000000.,
    EIFLAP=3*1500000., EILAG=3*20000000.,GJ=3*200000.,
    MASS=3*.24,ITHETA=3*.02,IPOLAR=3*.02,
&END
```



```
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 2',&END
&NLVAL
    NPANEL=20,
    REDGE=. 12,. 20,. 28,.35,.42,.48,.54,.59,.64,.69,.73,
        . 77,.81,.84,.87,.90,.92,.94,.96,.98,1.0,
    NPROP=11,RPROP=0.,.1,.2,.3,.4,.5,.6,.7,.8,.9,1., ! aero properties
    CHORD=11*1.5708,
    KSDL=3*. 81,.50,.33,.23,.16,.10,.07,.04,.01,
    KSDD=3*.54,.34,.20,.12,.06,.02,.00,.00,.00,
    NSEN=9,OPREF=9*4,
    QUANT= 5, 25,31, 32, 33,35,71,82,82,
    IDENT= 1, 0, 0, 0, 0, 0, 0, 0, 0,
    AXIS= 3, 0, 0, 0, 0, 0, 0, 1, 3,
    OPSCL= 2, 1, 2, 2, 2, 1, 2, 2, 2,
    NAPLOT=1, 4, 1, 1, 1, 1, 1, 0, 0,
&END
!=================================================================================
&NLDEF class='ROTOR',type='INFLOW', name='ROTOR 2',&END
&NLVAL
    KHLMDA=1.085,KFLMDA=2 . ,FMLMDA=2 * 0.,
    KINTFR(1)=-.085,KINTHW=1.5,KINTFH=1.8,
&END
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 2',&END
&NLVAL OPSCEN=2,TWIST=-28.,RICWG=.3,&END ! hover wake
&NLDEF class='ROTOR',type='WAKE',name='ROTOR 2',&END
&NLVAL OPSCEN=0,RNW=.25,WKMODL=8*2,&END
!==================================================================================
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! ELEMENTARY ROTOR IN WIND TUNNEL
#$! hover, no trim; rotor stability and response
#$! flutter time history; transient time history
#$! flap motion, rigid blade and rigid wing; dynamic inflow
#$! elementary scenarios
#$!
setenv BLADEAIRFOIL1 $airfoil/naca0012.tab
setenv SHELLINPUT $input/elementary.bin
setenv PLOTFILE $jobs/sample1.plot
$camrad/camradii > $jobs/samplel.out << 'endofinput'
    &NLJOB NCASES=3,PLFILE=1,OPSHLL=2,&END
    !================================================================================
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='RESPONSE',&END ! flutter; description
    &NLDEF class='TRIM',&END
    &NLVAL
        WVEL=0.,COLL=5., ! operating condition
        MHARMR=0, ! hover
    &END
```



```
    &NLDEF class='FLUTTER',&END
    &NLVAL
        MPSIAV=1, ! no average
        TASK=1,0,1,0,MFREQ=1, ! frequency response
    &END
    &NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
    &NLVAL
        CONR=3*1, ! rotor controls
        MPSEN=1, ! output
        OPWAKE=4,DOFL=3*0, ! trim inflow
        OPWAKE=1,DOFL=3*0,
        OPWAKE=3,DOFL=3*2, ! quasistatic inflow
        OPWAKE=3,DOFL=3*1, ! dynamic inflow
    &END
```



```
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    ! reduced output
    &NLDEF class='FLUTTER',&END
    &NLVAL OPSTEP=0,KPRNT(3)=0,OPSCL(3)=5,&END ! output
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    !=================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
    !##############################################################################
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='TIME HISTORY',&END ! flutter; description
    &NLDEF class='TRIM',&END
    &NLVAL
        WVEL=0.,COLL=5., ! operating condition
        MHARMR=0, ! hover
    &END
```



```
    &NLDEF class='FLUTTER',&END
    &NLVAL
\begin{tabular}{ll} 
MPSIAV \(=1\), & ! no average \\
\(\operatorname{GUST}=2 * 0,1\) & ! vertical gust \\
TASK \(=1,1,0,0, \mathrm{KPRNT}(2)=1\), & ! time history \\
OPHIST \(=7, \mathrm{TPER}=.2, \mathrm{TIMEB}=.1, \mathrm{TSTEP}=.02, \mathrm{TMAX}=.4\), TOUT \(=.02, \mathrm{MHIST}=0\), \\
NLPRNT \(=0\), NMPRNT \(=0\), & ! matrices not printed
\end{tabular}
```

```
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL
    CONR=3*0,
    MHSEN=1,MPSEN=1,MASEN=1, ! output
    DOFMBC=1,5*0,SENMBC=1,5*0, ! collective motion
    OPWAKE=4,DOFL=3*0, ! trim inflow
    OPWAKE=1,DOFL=3*0, ! uniform inflow
    OPWAKE=3,DOFL=2,2*0, ! quasistatic inflow
    OPWAKE=3,DOFL=1,2*0, ! dynamic inflow
&END
!===================================================================================
&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
&NLVAL
    NSEN=2,OPREF=2*4, ! aerodynamic sensors
    QUANT =5,25, ! induced velocity
    IDENT =0, 0, ! angle of attack
    AXIS =3, 0,
    OPSCL =2, 1,
    NAPLOT=1, 1,
&END
```



```
!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
! reduced output
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=0,MASEN=0,&END ! output
!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
!=================================================================================
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
!##############################################################################
&NLDEF class='CASE',&END
&NLVAL TNTASK=1,CODE='TIME HISTORY',&END ! transient; desc
&NLDEF class='TRIM',&END
&NLVAL
    WVEL=0.,COLL=5., ! operating condition
    MHARMR=0, ! hover
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL
    MHSEN=1,NHPRNT=1,NHFILE=1,MHTIME=1, ! output
    MPSEN=1,NPPRNT=1,NPFILE=1,MPTIME=1, ! output
    MASEN=1,NAPRNT=1,NAFILE=1,MATIME=1, ! output
&END
```



```
&NLDEF class='TRANSIENT',&END
&NLVAL
    WGUST=20., ! vertical gust
    TIMEB=0.,TIMEE=.5,OPHIST=7,TPER=.2,TIMEZ=.1, ! time history
    TRACEP=2,
&END
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 1',&END
&NLVAL
    MHSEN=1,MPSEN=1,MASEN=1, ! output
    DOFMBC=6*1, ! degrees of freedom
    OPVATR=2,OPVRTA=2, ! no rotor/body int
    OPWAKE=1, ! uniform inflow
    OPWAKE=4, ! trim inflow
    OPWAKE=3, ! dynamic inflow
&END
```

```
l=================================================================================
```

l=================================================================================
\&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',\&END
\&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',\&END
\&NLVAL
\&NLVAL
NSEN=2,OPREF=2*4, ! aerodynamic sensors
NSEN=2,OPREF=2*4, ! aerodynamic sensors
QUANT =5,25, ! induced velocity

```
    QUANT =5,25, ! induced velocity
```

```
    IDENT =0, 0, ! angle of attack
    AXIS =3, 0,
    OPSCL =2, 1,
    NAPLOT=1, 1,
&END
```



```
!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
! reduced output
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=0,MASEN=0,&END ! output
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=0,MASEN=0,&END ! output
!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



```
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! TEETERING ROTOR IN WIND TUNNEL
#$! hover, axisymmetric trim; hover wake, free geometry
#$! periodic coefficient system stability
#$! teetering rotor; rigid blade and rigid wing
#$!
setenv BLADEAIRFOIL1 $airfoil/naca0012.tab
setenv SHELLINPUT $input/teeter.bin
setenv PLOTFILE $jobs/sample2.plot
$camrad/camradii > $jobs/sample2.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='WAKE AND STABILITY',&END ! flutter; description
    &NLDEF class='TRIM',&END
    &NLVAL
        WVEL=0., ! operating condition
        LEVEL=3,NWPRNT=1, ! wake loop
        COLL=8.3,CTTRIM=.08, ! wind tunnel trim
        TOLERC=.2,ITERF=4,RELAXF=.5, ! hover free wake
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL MASEN=1,MWSEN=1,&END ! output
    &NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
    &NLVAL NAPLOT=5*2,MSPAN=25,&END
                                    ! output
    !==================================================================================
    &NLDEF class='ROTOR',type='WAKE', name='ROTOR 1',action='init', &END
    &NLVAL OPSCEN=2,TWIST=-10.,RICWG=.20,&END ! hover wake
    &NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
    &NLVAL OPSCEN=0,&END
```



```
    &NLDEF class='FLUTTER',&END
    &NLVAL OPFLUT=1,OPMEAN=0,MSTEP=100,&END ! periodic system
    !===================================================================================
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    ! reduced output
    &NLDEF class='TRIM',&END
    &NLVAL NWPRNT=0,&END ! wake loop
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL MWSEN=0,&END
    l output
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    !=================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! ROTOR IN WIND TUNNEL
#$! forward flight trim; low speed wake
#$! rigid blade and rigid wing (from elastic blade model)
#$! wind tunnel operation (from free flight model)
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv SHELLINPUT $input/helicopter.bin
setenv PLOTFILE $jobs/sample3.plot
$camrad/camradii > $jobs/sample3.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL CODE='LOW SPEED WAKE',&END ! description
    &NLDEF class='TRIM',&END
    &NLVAL
        WINDIN=2,WVEL=.15, ! operating condition
        LEVEL=3,NWPRNT=1, ! wake loop
        CTTRIM=.08,XTRIM=-12.,MTRIM=4, ! wind tunnel trim
        MNAME='CT/S ','BETAS ','BETAC ','CX/S ',
        VNAME='COLL ','LATCYC ','LNGCYC ','PITCH ',
        COLL=7.06,LATCYC=-3.65,LNGCYC=3.02,PITCH=-1.16,
        RELAXT=.25, ! trim loop
        MHARMR=2*1,MHARMA=2*1, ! harmonics
        RELAXC(2)=2*.05, ! convergence
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL
        OPMODE=0,DOFB=1,0,1,9*0, ! degrees of freedom
        MASEN=1,MWSEN=1, ! output
    &END
    !================================================================================
    &NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
    &NLVAL RFW=3.,MFWG=3,NPOFF=0,&END ! wake extent
    !=================================================================================
    &NLDEF class='AIRFRAME',type='STRUCTURE',&END
    &NLVAL
        TITLE='WIND TUNNEL SUPPORT', ! description
        CONFIG=0,OPFREE=0,OPAERO=0, ! wind tunnel
        ASHAFT=0.,ACANT=0., ! geometry
    &END
    ! ==================================================================================
    &NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
    &NLVAL OPBEAM=0,KNODE=0,LOCKPL=0,&END ! rigid blade
```



```
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    ! reduced output
    &NLDEF class='TRIM',&END
    &NLVAL NWPRNT=0,&END ! wake loop
    ! %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    !=================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! ROTOR IN WIND TUNNEL
#$! forward flight trim; high speed wake; rotor stability
#$! dynamic stall; loads; airframe-to-rotor aerodynamic interference
#$! swashplate, elastic blade; swept tip
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv BLADESTALL1 $airfoil/dynamicstall.2std
setenv SHELLINPUT $input/windtunnel.bin
setenv PLOTFILE $jobs/sample4.plot
$camrad/camradii > $jobs/sample4.out << 'endofinput'
    &NLJOB PLFILE=1,&END
    !================================================================================
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='HIGH SPEED WAKE',&END ! flutter; description
    &NLDEF class='TRIM',&END
    &NLVAL
        WVEL=.4,PITCH=-5., ! operating condition
        LEVEL=2,NWPRNT=1, ! wake loop
        OPTRIM=0,COLL=10.,LATCYC=-3.,LNGCYC=5., ! no trim
        RELAXR=.5, ! convergence
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL MASEN=1,MBSEN=1,MCSEN=1,MHSEN=1,MPSEN=1,&END ! output
    !================================================================================
    &NLDEF class='FLUTTER',&END
    &NLVAL DOFM=5*0,&END ! degrees of freedom
    !==================================================================================
    &NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
    &NLVAL OPSTLL=2,&END ! dynamic stall
    &NLDEF class='ROTOR',type='WAKE',name='ROTOR 1',&END
    &NLVAL OPFW=2,OPNW=0,&END ! dual peak model
```



```
    &NLDEF class='AIRFRAME',type='STRUCTURE',&END ! aerodynamic int
    &NLVAL OPAERO=1,&END
    &NLDEF class='AIRFRAME',type='AERODYNAMICS',&END
    &NLVAL VISRC=1,&END
```



```
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    ! reduced output
    &NLDEF class='TRIM',&END
    &NLVAL NWPRNT=0,&END ! wake loop
    &NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
    &NLVAL NMFILE=0,&END ! output
    ! faster
    &NLDEF class='FLUTTER',&END
    &NLVAL MPSIAV=4,&END ! averaged equations
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    !===============================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! ROTOR IN WIND TUNNEL
#$! hover, no trim; ground resonance
#$! swashplate, elastic blade; swept tip
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv SHELLINPUT $input/windtunnel.bin
setenv PLOTFILE $jobs/sample5.plot
$camrad/camradii > $jobs/sample5.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='GROUND RESONANCE',&END ! flutter, description
    &NLDEF class='TRIM',&END
    &NLVAL
        WVEL=0.,VTIPIN=2,RPM=200., ! operating condition
        LEVEL=1, ! wake loop
        COLL=4.,OPTRIM=0, ! no trim
        MHARMR=0,MHARMA=0,MPSIAV=1,MPSI=4, ! hover
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL DOFB=3*1,9*0,&END ! degrees of freedom
    !=================================================================================
    &NLDEF class='FLUTTER',&END
    &NLVAL
        MPSIAV=1, ! no average
    &END
    &NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
    &NLVAL
        DOFMBC=0,2*1,3*0,OPMODE=0,DOFB=0,1,10*0, ! lag only (cyclic)
        DOFMBC=0,2*1,3*0,OPMODE=1,DOFM=4*1,36*0, ! elastic (cyclic)
    &END
```



```
    &NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
    &NLVAL
        OPAERO=0, ! no aerodynamics
        OPAERO=1, ! aerodynamics
        DLAG=500., ! lag damping
    &END
    !================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! SINGLE MAIN ROTOR AND TAIL ROTOR HELICOPTER
#$! hover, free flight trim; flight dynamics (stability)
#$! flutter time history; transient time history
#$! rigid blade and rigid wing (from elastic model); drive train
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv BLADEAIRFOIL2 $airfoil/naca0012.tab
setenv SHELLINPUT $input/helicopter.bin
setenv PLOTFILE $jobs/sample6.plot
$camrad/camradii > $jobs/sample6.out << 'endofinput'
    &NLJOB NCASES=3,OPINIT=7,PLFILE=1,&END
    !==================================================================================
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='FLIGHT DYNAMICS',&END ! flutter, description
    &NLDEF class='TRIM',&END
    &NLVAL
        VKTS=0., ! operating condition
        LEVEL=1, ! wake loop
        COLL=9.81,LATCYC=.51,LNGCYC=-2.42,PEDAL=-8.68, ! trim
        PITCH=1.57,ROLL=-4.07,THROTL=10.54,
        RELAXT=.25, ! trim loop
    &END
    &NLDEF class='FLUTTER',&END
    &NLVAL
        OPEQN=3*0,6*1, ! equation sets
        DOFORD=5*1, ! first order states
        !#not used# OPSTAB=1, ! flight dynamics
        !#not used# NRPRNT=2*2,NAPRNT=2,NDPRNT=2, ! stability deriv
    &END
```



```
    &NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
    &NLVAL
        OPBEAM=0,KNODE=0, ! rigid blade
        CONTRL=0,PITCH=2,LOCKP=0, ! no swashplate
    &END
    ! =================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
    !##############################################################################
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='TIME HISTORY',&END ! flutter, description
    &NLDEF class='TRIM',&END
    &NLVAL
        VKTS=0., ! operating condition
        LEVEL=1, ! wake loop
        COLL=9.81,LATCYC=.51,LNGCYC=-2.42,PEDAL=-8.68, ! trim
        PITCH=1.57,ROLL=-4.07,THROTL=10.54,
        RELAXT=.25, ! trim loop
    &END
```



```
    &NLDEF class='FLUTTER',&END
    &NLVAL
        OPEQN=3*0,1,5*0, ! equation sets
        MPSIAV=4, ! loop solution
        CONP=1,4*0, ! collective
        NOUT=1,KINDO=3,NAMEV='PILOT INPUT',NAMEE='COLLECTIVE',
        TASK=1,1,0,0,MHIST=0, ! time history
    OPHIST=3,TPER=.6,NCYCLE=1,TIMEB=. 1,TOUT=.02,TMAX=1.0,
    MSSEN=1,
    ! output
```

```
&END
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=1,MPSEN=1,&END ! output
&NLDEF class='FLUTTER ROTOR',name='ROTOR 2',&END
&NLVAL DOFMBC=2,5*0,&END
quasistatic tail rtr
!=================================================================================
&NLDEF class='TRIM',&END ! drive train
&NLVAL MTRIM=7,MNAME(7)='TORQUE',VNAME(7)='THROTL',&END
&NLDEF class='FLUTTER',&END
&NLVAL
DOFD=1,7*0,
NOUT=2,KINDO(2)=1,NAMEC(2)='DRIVE TRAIN ROTOR 1 SHAFT',
    NAMEV(2)='DRIVE TRAIN AZIMUTH',NAMEE(2)='1',
&END
&NLDEF class='AIRFRAME',type='STRUCTURE',&END
&NLVAL OPTRAN=1,&END
```



```
!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
! reduced output
&NLDEF class='FLUTTER ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=0,MPSEN=0,&END ! output
!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
!================================================================================
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
!##############################################################################
&NLDEF class='CASE',&END
&NLVAL FLTASK=0,TNTASK=1,CODE='TIME HISTORY',&END ! transient, desc
&NLDEF class='TRIM',&END
&NLVAL
    VKTS=0., ! operating condition
    LEVEL=1, ! wake loop
    COLL=9.81,LATCYC=.51,LNGCYC=-2.42,PEDAL=-8.68, ! trim
    PITCH=1.57,ROLL=-4.07,THROTL=10.54,
    RELAXT=.25, ! trim loop
    MSSEN=1, ! output
&END
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=1,MPSEN=1,&END ! output
!=================================================================================
&NLDEF class='TRANSIENT',&END
&NLVAL
    WGUST=20., ! vertical gust
    TIMEB=0.,TIMEE=1.1,
! time history
    OPHIST=3,TPER=.6,NCYCLE=1,TIMEZ=.1,TSTEPS=.01,
    MSSEN=1, ! output
    TRACEP=2,
&END
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=1,MPSEN=1,&END ! output
```



```
!%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
! reduced output
&NLDEF class='TRIM',&END
&NLVAL NSPRNT=1,NSFILE=1,MSTIME=4,&END ! output
&NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=0,MPSEN=0,&END ! output
&NLDEF class='TRANSIENT ROTOR',name='ROTOR 1',&END
&NLVAL MHSEN=0,MPSEN=0,&END ! output
! % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % 
```



```
&NLDEF action='end of shell',&END
&NLDEF action='end of core',&END
endofinput'
```

A35 (Section 2-10

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! SINGLE MAIN ROTOR AND TAIL ROTOR HELICOPTER
#$! forward flight, free flight trim; forward flight wake
#$! loads; vibration; inflow off rotor
#$! swashplate, elastic blade
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv BLADEAIRFOIL2 $airfoil/naca0012.tab
setenv SHELLINPUT $input/helicopter.bin
setenv PLOTFILE $jobs/sample7.plot
$camrad/camradii > $jobs/sample7.out << 'endofinput'
    &NLJOB PLFILE=1,&END
    !================================================================================
    &NLDEF class='CASE',&END
    &NLVAL CODE='LOADS AND VIBRATION',&END ! description
    &NLDEF class='TRIM',&END
    &NLVAL
        VKTS=100., ! operating condition
        LEVEL=2, ! wake loop
        COLL=8.6,LATCYC=-2.1,LNGCYC=3.5,PEDAL=-3.8, ! trim
        PITCH=-.5,ROLL=-2.2,
        RELAXT=.25, ! trim loop
        DOFA=6*1, ! degrees of freedom
        RELAXR=.5, ! convergence
        MSSEN=1, ! output
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL
        OPMODE=1,DOFM=6*1,34*0, ! blade modes
        MASEN=1,MBSEN=1,MCSEN=1,MHSEN=1,MVSEN=1, ! output
    &END
```



```
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    ! faster
    &NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
    &NLVAL NPANEL=10,REDGE=. 12,.28,.42,.54,.64,.73,.81,.87,.92,.96,1., &END
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    !==================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! BEARINGLESS ROTOR IN WIND TUNNEL
#$! hover, no trim; isolated blade stability
#$! bearingless hub with snubber, swashplate
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv SHELLINPUT $input/bearingless.bin
setenv PLOTFILE $jobs/sample8.plot
$camrad/camradii > $jobs/sample8.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='STABILITY',&END ! flutter; description
    &NLDEF class='TRIM',&END
    &NLVAL WKTS=0.,COLL=10.,&END ! operating condition
```



```
    &NLDEF class='FLUTTER',&END
    &NLVAL OPBLD=1,&END ! independent blade
    !=================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
    endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! TANDEM HELICOPTER
#$! forward flight, free flight trim; nonuniform inflow
#$! rotor-rotor interference
#$! rigid blade and rigid wing; auxiliary force
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv SHELLINPUT $input/tandem.bin
setenv PLOTFILE $jobs/sample9.plot
$camrad/camradii > $jobs/sample9.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL CODE='TANDEM',&END ! description
    !===================================================================================
    &NLDEF class='TRIM',&END
    &NLVAL
        VKTS=100., ! operating condition
        LEVEL=2*3,NWPRNT=0, ! wake loop
        COLL=9.4,LATCYC=-0.6,LNGCYC=2.2,PEDAL=0.3, ! uniform inflow trim
        PITCH=-2.4,ROLL=0.3,
        COLL=8.1,LATCYC=-0.1,LNGCYC=1.2,PEDAL=0.7, ! free wake trim
        PITCH=-1.9,ROLL=0.6,
        RELAXT=.25,OPPID=2, ! trim loop
        ITERC=100,
        CNTRLF=1500., ! auxiliary propulsion
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL MASEN=1,MWSEN=1,&END ! output
    &NLDEF class='TRIM ROTOR',name='ROTOR 2',&END
    &NLVAL MASEN=1,MWSEN=1,&END ! output
    !==================================================================================
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    ! reduced output
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL MWSEN=0,&END ! output
    &NLDEF class='TRIM ROTOR',name='ROTOR 2',&END
    &NLVAL MWSEN=0,&END ! output
    ! faster
    &NLDEF class='TRIM',&END
    &NLVAL OPTRIM=0,&END ! no trim
    !%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    l=================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! TILTROTOR IN WIND TUNNEL
#$! axial flow, axisymmetric trim; tiltrotor stability
#$! gimballed hub, swashplate, elastic blade; drive train
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv SHELLINPUT $input/tiltrotorwt.bin
setenv PLOTFILE $jobs/sample10.plot
$camrad/camradii > $jobs/samplel0.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='STABILITY',&END ! flutter; description
    &NLDEF class='TRIM',&END
    &NLVAL
        WKTS=300., ! operating condition
        LEVEL=1, ! wake loop
        CPTRIM=0.,COLL=53.77, ! wind tunnel trim
        MTRIM=1 ,MNAME='CP/S ',VNAME= COLL ',
        RELAXT=.25, ! trim loop
        TOLERC=.05, ! circulation loop
            RELAXR=.5, ! convergence
            MPSI=4, ! uniform inflow only
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL MBSEN=1,&END ! output
    !================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```

```
#!/bin/csh
set camrad='/home/camrad/'
set airfoil='/home/camrad/sample/airfoil'
set input='/home/camrad/sample/input'
set jobs='/home/camrad/sample/jobs'
#
#$! TILTING PROPROTOR AIRCRAFT
#$! airplane mode, symmetric trim; tiltrotor stability
#$! airframe-to-rotor aerodynamic interference
#$! vibration; higher harmonic control
#$! gimballed hub, swashplate, rigid blade; drive train
#$!
#$!
setenv BLADEAIRFOIL1 $airfoil/advtecheq.tab
setenv SHELLINPUT $input/tiltrotor.bin
setenv PLOTFILE $jobs/sample11.plot
$camrad/camradii > $jobs/sample11.out << 'endofinput'
    &NLJOB PLFILE=1,&END
```



```
    &NLDEF class='CASE',&END
    &NLVAL FLTASK=1,CODE='VIBRATION/STABILITY',&END ! flutter; description
    &NLDEF class='TRIM',&END
    &NLVAL
        VKTS=250., ! operating condition
        LEVEL=2*1, ! wake loop
        GOV=24.32,LNGCYC=.66,PITCH=.26,
        ! trim
        RELAXT=.25, ! trim loop
        RELAXR=2*.5, ! convergence
        MSSEN=1, ! output
    &END
```



```
    &NLDEF class='TRIM ROTOR',name='ROTOR 1',&END
    &NLVAL
        MHSEN=1,MASEN=1,MPSEN=1, ! output
        MHHCN=1,TCHHC=-.2,TCHHS=-.3,TSHHC=.3,TSHHS=-.1, ! higher harmonic
    &END
    &NLDEF class='TRIM ROTOR',name='ROTOR 2',&END
    &NLVAL
        MHHCN=1,TCHHC=-.2,TCHHS=-.3,TSHHC=.3,TSHHS=-.1, ! higher harmonic
    &END
```



```
    &NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 1',&END
    &NLVAL OPBEAM=0,KNODE=0,LOCKPL=0,&END ! rigid blade and wing
    &NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR 2',&END
    &NLVAL OPBEAM=0,KNODE=0,LOCKPL=0,&END ! rigid blade and wing
    !===================================================================================
    &NLDEF class='AIRFRAME',type='AERODYNAMICS',&END
    &NLVAL VISRC=2*1,&END ! aerodynamic int
```



```
    ! faster
    &NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 1',&END
    &NLVAL NPANEL=2,REDGE=.3,.7,1.,&END ! two wing panels
    &NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR 2',&END
    &NLVAL NPANEL=2,REDGE=.3,.7,1.,&END ! two wing panels
    ! %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    !=================================================================================
    &NLDEF action='end of shell',&END
    &NLDEF action='end of core',&END
'endofinput'
```


## 2-12 CAMRAD II Jobs For Training

RUN SAMPLE JOB
OBJECTIVES:
establish access to program executables establish approach to handling files and directories become familiar with basic use of programs

## COPY SAMPLES/EXTRAS TO USER'S DIRECTORY

CHOOSE A SAMPLE JOB
SAMPLE8 is fast

RUN input PROGRAM TO GENERATE AIRFOIL FILE one of jobs in AIRFOIL. UNIX (revise directories) interactively run INPUT program to examine or plot airfoil file data

RUN Input PROGRAM TO GENERATE SHELL INPUT FILE one of jobs in AINPUT. UNIX (revise directories)
interactively run INPUT program to examine, plot, or draw input file data

RUN CAMRADII PROGRAM
one of jobs in JOBS. UNIX (revise directories)
examine output file

INTERACTIVELY RUN OUTPUT PROGRAM TO PLOT OR DRAW PLOTFILE DATA
can use existing plot file from sample jobs
(SAMPLE3 has free wake geometry)

## IDENTIFY ROTORCRAFT FOR INITIAL CALCULATIONS

## OBJECTIVES:

simple enough for efficient progress during training, representative of actual near-term calculations
typically single main rotor and tail rotor configuration; rotor only (wind tunnel) and complete helicopter

## PREPARE INPUT

## GENERATE INPUT DATA

namelist files for main rotor, tail rotor, airframe, wind tunnel
input file for wind tunnel (from wind tunnel, main rotor namelists)
input file for helicopter (from airframe, main rotor, tail rotor namelists)

RESOURCES: ztemplate.list, helicopter.list, windtunnel.list (and other sample job input)

## CREATE AIRFOIL FILES

for main rotor and tail rotor
RESOURCES: sample job input

## CHECK INPUT DATA

run INPUT program to plot (2D) and draw (3D) input file and airfoil file data

RESOURCES: zplanform.com (script file for 2D plot of planform)

## RUN CALCULATIONS

## BLADE FREQUENCIES <br> frequencies in vacuum, for varying rotor speed <br> RESOURCES: zfrequency.list

## STABILITY

rotor only, hover
rotor only, forward flight
rotor and airframe (ground resonance)
rotor and airframe (free flight)
RESOURCES: sample6, sample5

## HOVER PERFORMANCE

rotor only (axisymmetric)
helicopter (not axisymmetric)
RESOURCES: zhover.list, sample2

FORWARD FLIGHT PERFORMANCE, LOADS, VIBRATION
rotor only (wind tunnel)
trim rotor lift, propulsive force, flapping; or fixed collective and shaft angle, trim flapping
helicopter
trim aircraft force and moment
trim aircraft force and moment, specify climb or descent angle
trim aircraft force and moment and power
RESOURCES: sample4, sample7

## MAXIMUM THRUST

rotor only (wind tunnel)
collective sweep with zero shaft angle, trim flapping
helicopter
turn rate sweep, trim aircraft force and moment (and maybe power)

## GUST RESPONSE OR MANEUVER

vertical gust input or longitudinal cyclic input flutter task (linear response) and transient task (nonlinear) helicopter, forward flight

RESOURCES: sample6

## Chapter 3

## TABLE FORMATS

GENERAL REFERENCES FOR CHAPTER:
CAMRAD II Documentation, Volume VI, Rotorcraft input; Chapter "Table Formats"

CAMRAD II Documentation, Volume I, Theory; Chapter "Tables"

## 3-1 Table Class and Type

## TABLES CORRESPOND TO FILES

PROVIDING GENERAL FILE INPUT CAPABILITY

## EACH TABLE CLASS/TYPE HAS SPECIFIC FILE FORMAT AND DATA ORGANIZATION

CLASS = MATRIX
table data function of discrete variables, not interpolated; initialization moves table data to record array for use

CLASS = UNSTRUCTURED
table data unstructured set of numbers, not interpolated; use of table data by analysis depends on component OTHER CLASSES
table data function of continuous variables, interpolated from data vector

COMPONENT IDENTIFIES TABLE BY CLASS, TYPE, AND NAME USER (OR ROTORCRAFT SHELL) DEFINES NAME names must be unique with class, even if type is different TABLE SPECIFICATION ALSO INCLUDES FILE UNIT NUMBER, AND FILE NAME OR LOGICAL NAME
file name or logical name can be same as name in class/type/name specification (this is the default) for most installations, can ignore unit numbers

## FIGURE 1 LISTS THE TABLE CLASSES AND TYPES IMPLEMENTED

InPut PROGRAM CAN PREPARE CLASS = AIRFOIL FROM SOURCE FILES OR INTERNALLY
for other table classes, table file preparation does not transform the table data

| CLASS | TYPE | NAME |
| :--- | :--- | :--- |
| TWO DIMENSIONAL | STANDARD |  |
| THREE DIMENSIONAL | STANDARD |  |
| FOUR DIMENSIONAL | STANDARD |  |
| AIRFOIL | STANDARD |  |
| AIRFOIL | C81 |  |
| AIRFOIL | CAMRAD |  |
| MATRIX | STANDARD |  |
| UNSTRUCTURED | REAL |  |
| UNSTRUCTURED | INTEGER |  |

Lower case is accepted for class and type. Unique initial letters of class or type are accepted. Specific abbreviations and alternatives:

$$
\begin{aligned}
\text { class } & =\text { TWO DIMENSIONAL }=2 \mathrm{D} \\
\text { class } & =\text { THREE DIMENSIONAL }=3 \mathrm{D} \\
\text { class } & =\text { FOUR DIMENSIONAL }=4 \mathrm{D} \\
\text { class } & =\text { AIRFOIL: } \\
& \text { type }=\text { C81 }=\text { FIXED FORMAT } \\
& \text { type }=\text { CAMRAD }=\text { WING } \\
\text { class } & =\text { MATRIX }=\text { ARRAY }
\end{aligned}
$$

Figure 3-1 Table blocks.

## 3-2 Table File Formats

## TABLE FILE CAN BE FORMATTED OR UNFORMATTED

## FORMATTED FILE CAN BE FIXED FORM OR FREE FORM

## FREE FORM:

each line consists of number of fields, separated by blanks or commas or tabs
maximum 132 characters in line maximum 20 fields in line maximum 40 characters in each field ignore any equals sign and part of field to its left so format "name = value" can be used comments (exclamation mark and what follows on line) are ignored, except for title and identification lines

## FIXED FORM:

data found in fields of specific width and position

## GENERAL FORMAT CONVENTIONS

EACH TABLE HAS TITLE, AND FILE HAS IDENTIFICATION, WHICH SHOULD BE UNIQUE

INDEPENDENT AND DEPENDENT VARIABLES MAY HAVE LABELS
maximum 80 characters in title
maximum 20 characters in file identification
maximum 8 characters in labels
blank lines typically interpreted as zeros and blank labels
END OF EACH TABLE HAS VALUES OF REFERENCE QUANTITIES

## 3-3 Evaluation of Dependent Variables

TABLE QUANTITIES ARE FUNCTIONS OF CONTINUOUS VARIABLES, ACCESSED BY INTERPOLATION

EXCEPTION: CLASS = MATRIX or UNSTRUCTURED

## LINEAR INTERPOLATION, WITH NO EXTRAPOLATION BEYOND RANGE OF TABLE

OPTIONAL CYCLICAL OR ANGLE INTERPOLATION FOR INDEPENDENT VARIABLE $\alpha$
$\alpha$ is angle in degrees
table range typically 0 to 360 degrees, or -180 to 180 degrees
multiple of 360 degrees added to or subtracted from $\alpha$, until value within table range
then dependent variable interpolated
with no extrapolation, in case table range does not extend full 360 degrees

## FOR THREE-DIMENSIONAL TABLE, LAST VARIABLE $\gamma$ CAN BE SEARCHED OR INTERPOLATED

for interpolation, $\gamma_{k}$ defines value for $k$-th two-dimensional array
for search, $\gamma_{k}$ defines beginning of range of $k$-th two-dimensional array

## INDEPENDENT VARIABLES CAN HAVE UNIFORM INCREMENTS

 increment determined from first two values of independent variable (but all values must still be present in table)
## 3-4 Summary of Tables

## CLASS $=$ TWO DIMENSIONAL, TYPE $=$ STANDARD

SET OF $C_{\gamma}(\alpha, \beta)$
one or more 2D coefficients, identified by label $\gamma$
separate rectangular $(\alpha, \beta)$ array for each
$\alpha$ and $\beta$ can be cyclical, and uniform
FORMATTED, FREE FORM

CLASS $=$ THREE DIMENSIONAL, TYPE $=$ STANDARD
$C(\alpha, \beta, \gamma)$
3D coefficient, arranged as separate rectangular ( $\alpha, \beta$ ) arrays for set of $\gamma$ values
$\alpha$ and $\beta$ and $\gamma$ can be cyclical, and uniform
$\gamma$ can be searched or interpolated
FORMATTED, FREE FORM

CLASS $=$ FOUR DIMENSIONAL, TYPE $=$ STANDARD
$C(\alpha, \beta, \gamma, \delta)$
4D coefficient, arranged as separate rectangular ( $\alpha, \beta$ ) arrays for set of $(\gamma, \delta)$ values
same $(\alpha, \beta)$ values for all $(\gamma, \delta)$, same $\gamma$ values for all $\delta$ $\alpha$ and $\beta$ and $\gamma$ and $\delta$ can be cyclical, and uniform

FORMATTED, FREE FORM

CLASS $=$ MATRIX, TYPE $=$ STANDARD
SET OF $C_{M}(i, j)$ one or more 2D matrices or arrays, identified by label or number $M$
$i$ and $j$ can be uniform
no interpolation
FORMATTED, FREE FORM

CLASS $=$ UNSTRUCTURED, TYPE $=$ REAL or INTEGER
SET OF $T(i)$
real or integer numbers, counted as read, stored in record no interpolation

FORMATTED, FREE FORM

CLASS $=$ AIRFOIL, TYPE $=$ STANDARD
$c_{\ell}(\alpha, M), c_{d}(\alpha, M), c_{m}(\alpha, M) ;$
$c_{\ell f}(\alpha, M), c_{d f}(\alpha, M), c_{m f}(\alpha, M)$
lift, drag, moment coefficients (any order) and perhaps flap coefficients, for one flap angle function of angle of attack $\alpha$, and Mach number $M$
separate ( $\alpha, M$ ) array for each coefficient $\alpha$ can be row or column, $\alpha$ cyclical

FORMATTED, FREE FORM

CLASS $=$ AIRFOIL, TYPE $=$ C81
$c_{\ell}(\alpha, M), c_{d}(\alpha, M), c_{m}(\alpha, M) ;$
$c_{\ell f}(\alpha, M), c_{d f}(\alpha, M), c_{m f}(\alpha, M)$
lift, drag, moment coefficients (in this order)
and perhaps flap coefficients, for one flap angle
function of angle of attack $\alpha$, and Mach number $M$
separate ( $\alpha, M$ ) array for each coefficient
$\alpha$ must be row, $\alpha$ cyclical
FORMATTED, FIXED FORM

CLASS $=$ AIRFOIL, TYPE $=$ CAMRAD
$c_{\ell}(\alpha, M, \phi, r), c_{d}(\alpha, M, \phi, r), c_{m}(\alpha, M, \phi, r) ;$
$c_{\ell f}(\alpha, M, \phi, r), c_{d f}(\alpha, M, \phi, r), c_{m f}(\alpha, M, \phi, r)$
lift, drag, moment coefficients
and perhaps flap coefficients
function of angle of attack $\alpha$, Mach number $M$,
flap angle $\phi$, and span station $r$
special ( $\alpha, M$ ) array for efficient interpolation
$\alpha$ cyclical, $r$ can be searched or interpolated
CONSTRUCTED FROM SET OF AIRFOIL TABLES (STANDARD OR C81)

UNFORMATTED

## 3-5 Class = Airfoil

## AIRFOIL TABLES CONTAIN LIFT COEFFICIENT, DRAG COEFFICIENT, AND MOMENT COEFFICIENT DATA

AS FUNCTION OF ANGLE OF ATTACK $\alpha$ AND MACH NUMBER $M$
two-dimensional airfoil data: $c_{\ell}(\alpha, M), c_{d}(\alpha, M), c_{m}(\alpha, M)$
angle of attack is in range -180 to 180 degrees
angle of attack is cyclical

## LOADS AND ANGLE OF ATTACK DEFINED AS IN TWODIMENSIONAL AIRFOIL TEST

$\alpha$ varied by pitching airfoil
lift positive upward
drag positive in direction of free stream velocity moment positive nose up
figure 2 shows the conventions
figure 3 illustrates expected behavior of airfoil characteristics
moments measured about reference axis usually this axis is at $25 \%$ chord
$\alpha$ measured to reference chord line in airfoil usually not zero-lift line
component that uses airfoil data must have consistent definition of reference chord line and reference axis
consistent definition of blade twist required


Figure 3-2 Conventions for airfoil loads and angle of attack.


Figure 3-3 Sketch of two-dimensional airfoil characteristics.

FIRST REFERENCE VALUE IS REYNOLDS NUMBER $R e_{1}$ (DEFAULT VALUE OF ZERO)
this is Reynolds number of table corresponding to Mach number $M=1$

Reynolds number at arbitrary Mach number will then be calculated as $R e=M R e_{1}$
figure 4 illustrates the approximation
this linear relation is usually an approximation to true relation between Reynolds number and Mach number for airfoil data

SYNTHESIZED AIRFOIL DATA
WHEN DATA ARE NOT AVAILABLE FOR REQUIRED AIRFOIL CHARACTERISTICS CAN BE SYNTHESIZED FROM EQUATIONS THAT REPRESENT TYPICAL AIRFOIL BEHAVIOR
equations can also be used to generate simplified airfoil characteristics

PARAMETERS OF THESE EQUATIONS ARE INPUT THROUGH NAMELIST NLEQN

REFERENCE FOR AIRFOIL EQUATIONS: CAMRAD II Documentation, Volume I, Theory; Chapter "Tables"


Figure 3-4 Relation between Reynolds number and Mach number.

## AIRFOIL CAN HAVE TRAILING EDGE FLAP

 coefficients are functions of flap angle $\phi$, as well as angle of attack and Mach number tables include coefficients of lift, drag, and moment on flap: $c_{\ell f}$, $c_{d f}, c_{m f}$$c_{\ell}, c_{d}, c_{m}$ are total loads on airfoil, including flap loads second reference value is the flap angle $\phi$

TYPE = STANDARD OR C81: table is for one flap angle $\phi$ six two-dimensional coefficients in table

TYPE $=$ CAMRAD:
wing can have trailing edge flap at any span station six four-dimensional coefficients in table

## 3-6 Class = Airfoil, Type = Standard

## TABLE CONTAINS THREE OR SIX TWO-DIMENSIONAL COEFFICIENTS

$c_{\ell}(\alpha, M), c_{d}(\alpha, M), c_{m}(\alpha, M) ;$
$c_{\ell f}(\alpha, M), c_{d f}(\alpha, M), c_{m f}(\alpha, M)$ with trailing edge flap
separate, rectangular ( $\alpha, M$ ) array for each:

$$
\begin{array}{cc}
\alpha_{i} & \text { for } i=1 \text { to } N_{\alpha} \\
M_{j} & \text { for } j=1 \text { to } N_{M}
\end{array}
$$

dependent variable data consists of values $c_{\ell}(i, j), c_{d}(i, j)$, $c_{m}(i, j) ; c_{\ell f}(i, j), c_{d f}(i, j), c_{m f}(i, j)$
coefficients can be in any order
angle of attack can be row or column
angle of attack $\alpha$ is cyclical
table for one flap angle $\phi$

## FILE FORMAT: FORMATTED, FREE FORM

figure 5 defines the file format
three or six coefficients, each function of $A$ and $B$ : set of $C(A, B)$
labels LABELA and LABELB identify independent variables A and $B$ as
angle of attack: label = ALPHA
Mach number: label $=\mathrm{MACH}$
either order
table is for airfoil with trailing edge flap if keyword "FLAP" is present
second reference value is the flap angle
labels LABELGk identify the dependent variables as lift coefficient: label = CL
drag coefficient: label = CD
moment coefficient: label = CM
flap lift coefficient: label = CLF
flap drag coefficient: label = CDF
flap moment coefficient: label = CMF
any order
limitations:

| parameter | maximum | minimum |
| :--- | :--- | :--- |
| TITLE | 80 characters |  |
| IDENT | 20 characters |  |
| LABELA, LABELB, LABELG | 8 characters |  |
| NAk, NBk | $400 \alpha, 100 \mathrm{M}$ | 2 |
| number of reference values | 2 |  |

values of $A$ and $B$ must be unique and sequential
more than one line can be used for each table row; new table row must start on new line.
nothing is read after "END OF FILE"

```
general format
    TITLE
    IDENT
LABELA LABELB FLAP
NAk NBk lllabELGk lllllol
REF1 REF2
END OF FILE
```

typical format

```
TITLE
IDENT
ALPHA MACH
\begin{tabular}{ccccc} 
NAL & NML & CL & & \\
& MMM & MMM & \(\ldots\) & MMM \\
AAA & CCC & CCC & \(\ldots\) & CCC \\
AAA & CCC & CCC & \(\ldots\) & CCC \\
\(\ldots\). & \(\ldots\) & \(\ldots\) & \(\ldots\) & \(\ldots\)
\end{tabular}
... ... \(\quad . . \quad\)... ...
AAA CCC CCC ... CCC
NAD NMD CD
    MMM MMM ... MMM
AAA CCC CCC ... CCC
AAA CCC CCC ... CCC
... ... .. ... ...
... ... ... ... ...
AAA CCC CCC ... CCC
AAA CCC CCC ... CCC
NAM NMM CM
    MMM MMM ... MMM
AAA CCC CCC ... CCC
AAA CCC CCC ... CCC
... ... ... ... ...
AAA CCC CCC \cdots ツ...
AAA CCC CCC ... CCC
RETBL1
END OF FILE
```

Figure 3-5 Table File Format: Class = Airfoil, Type = Standard.

## 3-7 Class = Airfoil, Type = C81

## TABLE CONTAINS THREE OR SIX TWO-DIMENSIONAL COEFFICIENTS

$c_{\ell}(\alpha, M), c_{d}(\alpha, M), c_{m}(\alpha, M) ;$
$c_{\ell f}(\alpha, M), c_{d f}(\alpha, M), c_{m f}(\alpha, M)$ with trailing edge flap
separate, rectangular ( $\alpha, M$ ) array for each:

$$
\begin{aligned}
\alpha_{i} & \text { for } i=1 \text { to } N_{\alpha} \\
M_{j} & \text { for } j=1 \text { to } N_{M}
\end{aligned}
$$

dependent variable data consists of values $c_{\ell}(i, j), c_{d}(i, j)$, $c_{m}(i, j) ; c_{\ell f}(i, j), c_{d f}(i, j), c_{m f}(i, j)$
coefficients must be in fixed order angle of attack must be row angle of attack $\alpha$ is cyclical table for one flap angle $\phi$

## FILE FORMAT: FORMATTED, FIXED FORM

figure 6 defines the file format three or six coefficients, each a function of $A$ and $M$ : set of C(A,M)
independent variable $A$ is angle of attack, and independent variable M is Mach number
table is for airfoil with trailing edge flap if NA and NM for flap are present in header line
second reference value is the flap angle
dependent variables are lift, drag, and moment coefficients; and flap lift, drag, and moment coefficients (fixed order)
limitations:

| parameter | maximum | minimum |
| :--- | :--- | :--- |
| HEADER | 30 characters |  |
| NML, NAL, NMLF, NALF | 99 | 2 |
| NMD, NAD, NMDF, NADF | 99 | 2 |
| NMM, NAM, NMMF, NAMF | 99 | 2 |
| number of reference values | 2 |  |

values of $A$ and $M$ must be unique and sequential header is used as title
basic read format is 10 columns, each 7 characters wide; first column only used for A values
if NM greater than 9, more than one line used for each table row; new table row must start on new line
with trailing edge flap, header line includes NMLF, NALF, NMDF, NADF, NMMF, NAMF; and CLF, CDF, CMF arrays follow CM array
without trailing edge flap, there must be two blanks after NAM in the header line
nothing is read after reference values (or one blank line)

## EXTENSIONS OF ORIGINAL C81 FILE FORMAT

reference values added to end of table
can include coefficients for a trailing edge flap
without trailing edge flap, there must be two blanks after NAM in the header line
general format

| HEADER | NML , NAL , NMD , NAD , NMM , NAM |  | read format |
| :---: | :---: | :---: | :---: |
|  |  |  | A30,6I2,6I2 |
|  | $\mathrm{M}(1, k)$ | . . . M $\mathrm{M}(\mathrm{NML}, \mathrm{k})$ | 7X,9F7.0 |
| A (1, k) | CL ( $1,1, \mathrm{k}$ ) | ... ... CL (1,NML, k) | 10F7.0/(7X,9F7.0) |
| - | - | . | - |
| . | - | - | - |
| A (NAL, k) | CL ( $\mathrm{NAL}, 1, \mathrm{k}$ ) | ... ... CL (NAL, NML , k ) | 10F7.0/(7X,9F7.0) |
|  | M ( $1, k$ ) | ... ... M(NMD, k) | 7X,9F7.0 |
| A ( $1, \mathrm{k}$ ) | CD ( $1,1, k$ ) | ... ... CD (1,NMD, k) | 10F7.0/(7X,9F7.0) |
| - | - | . | - |
| - | - | - | - |
| A ( $\mathrm{NAD}, \mathrm{k}$ ) | CD ( $\mathrm{NAD}, 1, \mathrm{k}$ ) | ... ... CD (NAD, NMD , k ) | 10F7.0/(7X,9F7.0) |
|  | $\mathrm{M}(1, k)$ | . . . . . M (NMM, k) | 7X,9F7.0 |
| A ( $1, \mathrm{k}$ ) | CM ( $1,1, \mathrm{k}$ ) | ... ... CM (1,NMM, k) | 10F7.0/(7X,9F7.0) |
| - | - | . | - |
| - | - | - |  |
| A ( $\mathrm{NAM}, \mathrm{k}$ ) | CM ( $\mathrm{NAM}, 1, \mathrm{k}$ ) | . . . . . CM ( $\mathrm{NAM}, \mathrm{NMM}, \mathrm{k}$ ) | 10F7.0/(7X,9F7.0) |
| REF1 REF |  |  | 2F14.0 |

typical format

| HEADERHEADERHEADERHEADERHEADER N N N N N N |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | MMMMMM | MMMMMM | MMMMMM |  |  |
| AAAAAA | CLIFT | CLIFT | CLIFT |  |  |
| AAAAAA | CLIFT | CLIFT | CLIFT |  |  |
| AAAAAA | CLIFT | CLIFT | CLIFT |  |  |
|  | MMMMMM | MMMMMM | MMMMMM |  |  |
| AAAAAA | CDRAG | CDRAG | CDRAG |  |  |
| AAAAAA | CDRAG | CDRAG | CDRAG |  |  |
| AAAAAA | CDRAG | CDRAG | CDRAG |  |  |
|  | MMMMMM | MMMMMM MMMMMM |  |  |  |
| AAAAAA | CMOM | CMOM | CMOM |  |  |
| AAAAAA | CMOM | CMOM | CMOM |  |  |
| AAAAAA | CMOM | CMOM | CMOM |  |  |
| RETBL1 |  |  |  |  |  |

Figure 3-6 Table File Format: Class = Airfoil, Type = C81.

# USE OF TABLE DATA (STANDARD AND C81 TYPES) <br> LINEAR INTERPOLATION, WITH NO EXTRAPOLATION beyond range of Table 

figure 7 illustrates use of table data

## ANGLE OF ATTACK $\alpha$

to avoid errors (since there is no extrapolation), range should always extend from -180 to 180 deg
data at large $|\alpha|$ likely from generic test (usually NACA 0012 data), not from test of this airfoil
careful joining two sets of data

## MACH NUMBER $M$

data from $M_{\text {min }}$ to $M_{\text {max }}$
evaluation of coefficients (with no extrapolation) for $M<$ $M_{\text {min }}$ good, as long as $M_{\text {min }}$ is below about 0.3
evaluation of coefficients (with no extrapolation) for $M>$ $M_{\text {max }}$ can produce significant errors

## NO ERROR MESSAGE PRODUCED IF TABLE DATA ACCESSED OUTSIDE AVAILABLE RANGE

when analyze rotors operating at high advancing tip Mach number and/or high thrust, be sure within range of valid airfoil data


Figure 3-7 Use of airfoil table data (Standard or C81 type).

## 3-8 Class $=$ Airfoil, Type $=$ CAMRAD

TABLE CONTAINS THREE THREE-DIMENSIONAL COEFFICIENTS
$c_{\ell}(\alpha, M, r), c_{d}(\alpha, M, r), c_{m}(\alpha, M, r)$
last independent variable is wing span station $r$
if wing has trailing edge flap at any span station:
$c_{\ell f}(\alpha, M, \phi, r), c_{d f}(\alpha, M, \phi, r), c_{m f}(\alpha, M, \phi, r)$ as well
common, rectangular ( $\alpha, M$ ) array:

$$
\begin{aligned}
\alpha_{i} & \text { for } i=1 \text { to } N_{\alpha} \\
M_{j} & \text { for } j=1 \text { to } N_{M}
\end{aligned}
$$

for set of values $\phi_{l}\left(l=1\right.$ to $\left.N_{\phi}\right)$ and $r_{k}\left(k=1\right.$ to $\left.N_{r}\right)$
special convention used for ( $\alpha, M$ ) array, designed for efficient interpolation
dependent variable data consists of values $c_{\ell}(i, j, l, k)$, $c_{d}(i, j, l, k), c_{m}(i, j, l, k) ; c_{\ell f}(i, j, l, k), c_{d f}(i, j, l, k), c_{m f}(i, j, l, k)$
angle of attack $\alpha$ is cyclical span station $r$ can be searched or interpolated for interpolation, $r_{k}$ defines value for $k$-th airfoil for search, $r_{k}$ defines beginning of range of $k$-th airfoil

## FILE FORMAT: UNFORMATTED

TABLE IS CONSTRUCTED BY PROGRAM Input FROM SET OF AIRFOIL TABLES
source table is STANDARD or C81 type
one source table for each span station and each flap angle STANDARD and C81 tables are for single airfoil section and flap angle

CAMRAD table is for entire wing (one or more airfoil sections)

DATA IN SOURCE TABLE INTERPOLATED TO VALUES OF $\alpha$ and $M$ REQUIRED FOR CAMRAD TABLE
resolution of $(\alpha, M)$ in CAMRAD table normally small enough that behavior of data in source table is accurately captured

## USE OF TABLE DATA

## LINEAR INTERPOLATION, WITH NO EXTRAPOLATION BEYOND RANGE OF TABLE

figure 8 illustrates use of table data

## FOR EVALUATION OF CAMRAD TABLE FROM SOURCE TABLE

be sure range in CAMRAD table is larger than range in STANDARD or C81 table, so no information is lost primary concern is maximum Mach number FOR EVALUATION OF COEFFICIENTS FROM CAMRAD TABLE, IN AERODYNAMIC ANALYSIS
same considerations as for STANDARD and C81 tables


Figure 3-8 Use of airfoil table data (CAMRAD type).

## SPECIAL CONVENTION USED FOR ( $\alpha, M$ ) ARRAY, DESIGNED FOR EFFICIENT INTERPOLATION

## COEFFICIENT DATA DEFINED AT SET OF ANGLE OF ATTACK POINTS

THESE POINTS CONSIST OF SEVERAL GROUPS
WITH SAME ANGLE OF ATTACK INCREMENT WITHIN EACH GROUP

SET OF ANGLE OF ATTACK POINTS COMPLETELY
SPECIFIED BY $\alpha$ VALUES AT BOUNDARIES BETWEEN GROUPS, AND INDICES OF THESE POINTS
figure 9 illustrates the convention
figure 10 shows typical boundaries (defaults)
there are $N_{a}$ angle of attack boundaries ( $N_{a}-1$ groups) index $n$ counts actual angle of attack values, with $n=n_{k}$ at boundaries ( $k=1$ to $N_{a}$ )
$n_{1}=1$ always, and $n_{N_{a}}$ is total number of angle of attack values in table
corresponding angles of attack at the boundaries are $\alpha_{k}$
no extrapolation, so angle of attack range should be -180 to 180 degrees


Figure 3-9 Convention for angle of attack points and boundaries.


Figure 3-10 Typical angle of attack and Mach number boundaries.

## PURPOSE OF THIS ORGANIZATION IS TO MINIMIZE THE SEARCH

 REQUIRED TO INTERPOLATE DATAONLY NECESSARY TO SEARCH TABLE IN TERMS OF GROUPS

INTERPOLATION WITHIN GROUP REQUIRES ONLY NUMERICAL OPERATIONS
reduction in search operations will be significant if large number of points can be divided into small number of groups

ADDITIONALLY, SEARCH CAN BE STARTED WITHIN SPECIFIED GROUP, RATHER THAN AT BEGINNING OF RANGE

SIMILAR CONVENTION USED FOR VARIATION WITH MACH NUMBER

## TABLE DATA DESCRIBED BY FOLLOWING QUANTITIES: BOUNDARIES

angle of attack: $\quad n_{k}$ and $\alpha_{k}$, for $k=1$ to $N_{a}$
Mach number: $\quad n_{k}$ and $M_{k}$, for $k=1$ to $N_{m}$
flap angle:
span station:
$\phi_{l}$, for $l=1$ to $N_{\phi}$
$r_{k}$, for $k=1$ to $N_{r}$

## COEFFICIENT VALUES

$c\left(n_{N_{a}}, n_{N_{m}}, N_{\phi}, N_{r}\right)$ for lift, drag, and moment (total and flap)

SAME BOUNDARIES ARE USED FOR ALL COEFFICIENTS

## DEFAULT BOUNDARIES

## DEFAULT ANGLE OF ATTACK INCREMENTS:

1 deg from $\alpha=-30$ to 30 ,
2 deg from $\alpha=150$ to -150 , 10 deg otherwise

DEFAULT MACH NUMBER INCREMENTS:
.100 from $M=0$. to .6 , .025 from $M=.6$ to .95

IF WING HAS TRAILING EDGE FLAP
AT EACH SPAN STATION $k, N_{\phi k}$ SETS OF COEFFICIENTS, FOR FLAP ANGLES $\phi_{l}, l=1$ to $N_{\phi k}$
interpolate over $\alpha$ and $M$, then $\phi$, then $r$

## SPAN STATION $r$ CAN BE SEARCHED OR INTERPOLATED

$N_{r}$ SETS OF COEFFICIENTS, CORRESPONDING TO SPAN STATIONS $r_{k}, k=1$ to $N_{r}$
for interpolation, $r_{k}$ defines value for $k$-th airfoil for search, $r_{k}$ defines beginning of range of $k$-th airfoil
$k$-th airfoil used over range $r=r_{k}$ to $r=r_{k+1}$
in order to use spanwise interpolation on wing with trailing edge flaps, there should be tables at the span stations of the flap edges

## WING GEOMETRY

CROSS-SECTION GEOMETRY DEFINED BY SPECIFIC AIRFOIL SECTIONS AT CERTAIN SPAN STATIONS
typically linear variation of geometry between these specified sections
figure 11 illustrates the geometry

CAMRAD TABLE FOR WING - CAN INTERPOLATE SPANWISE
TABLE SECTIONS CORRESPOND TO SPAN STATIONS WHERE WING GEOMETRY SPECIFIED

CAREFUL USING SPANWISE INTERPOLATION with nonlinear aerodynamics, linear variation of geometry does not imply linear variation of loads interpolation can produce problems with stall behavior, compressible drag rise, or zero lift angle check behavior by plotting data from CAMRAD table at intermediate span stations

CAMRAD TABLE FOR WING - CAN SEARCH SPANWISE
SPAN STATIONS IN TABLE CORRESPOND TO BOUNDARIES BETWEEN SECTIONS typically these boundaries correspond to edges of aerodynamic panels

IF TRANSITION REGION BETWEEN TWO SECTIONS IS LONG, MAY NEED TO INCLUDE SEPARATE TRANSITION SECTION
such transition sections are not usually tested, so the data for this section must be generated by the user (perhaps by linear interpolation)

## WING GEOMETRY



CAMRAD TABLE FOR WING
span station interpolated

span station searched

span station searched


Figure 3-11 Spanwise specification of airfoil tables.

## THEORY

GENERAL REFERENCE FOR CHAPTER: CAMRAD II Documentation, Volume I, Theory

## 4-1 System Pieces

FOR CONFIGURATION GENERALITY, SPLIT SYSTEM INTO PIECES, WITH CONNECTIONS BETWEEN

ENVIRONMENTAL PIECES
PHYSICAL PIECES
LOGICAL PIECES (SOLUTION PROCEDURE)

PIECES AVAILABLE TO CONSTRUCT SYSTEM:

| environmental | physical | logical |
| :--- | :--- | :--- |
| case | component | loop |
| wind | frame | part |
| operating condition | interface | transform |
| period | output | modes |
|  | input | response |
|  |  | weights |

## ENVIRONMENTAL PIECES

 provide standard description of system operation
## PHYSICAL PIECES

correspond to physical description of system produce system equations

## LOGICAL PIECES

define procedure for solving system equations
transform and modes provide standard means to introduce various transformations of system variables and equations
response provides standard means to define characteristics of all system variables
weights provides standard perturbation and convergence weights for response

## 4-2 System Equations

PHYSICAL PIECES DEFINE EQUATIONS THAT DESCRIBE SYSTEM figure 1 illustrates relations between physical pieces ASSOCIATED WITH EACH PIECE ARE CERTAIN VARIABLES:

| system piece | variables |
| :--- | :--- |
| component $n$ | degree of freedom $\xi_{n}$ <br> interface $l$ <br> input $m$ |
| constraint variables $f_{l}$  <br> input $u_{m}$  <br> output $q$ output $y_{q}$ |  |

ALL THESE VARIABLES ARE VECTORS; EACH VECTOR HAS ONE OR MORE ELEMENTS

## 4-2.1 Components

A COMPONENT CONTAINS THE PHYSICS OF THE MODEL, AND PERFORMS ALL CALCULATIONS FOR THE SYSTEM

## COMPONENT $n$ CAN HAVE DEGREES OF FREEDOM $\xi_{n i}$

dependence on degrees of freedom means that equations of motion exist, and so usually a differential equation formulation is possible


Figure 4-1 System equations.

COMPONENT CAN DEPEND ON FRAME MOTION $\beta$
FRAME MOTION CAN BE PRESCRIBED
OR FRAME MOTION CAN EQUAL RIGID BODY MOTION OF THIS OR OTHER COMPONENT (CALLED "FRAME DEGREES OF FREEDOM")

COMPONENT HAS INPUT $f_{n i}$ AND OUTPUT $x_{n j}$
COMPONENT INPUT IS OF STRUCTURAL DYNAMIC KIND OR INPUT/OUTPUT KIND
can be connected to an interface or to a system input piece

COMPONENT OUTPUT IS OF STRUCTURAL DYNAMIC KIND OR INPUT/OUTPUT KIND
can be connected to an interface or to a system output piece

FOR STRUCTURAL DYNAMIC INTERFACE, INPUT AND OUTPUT OCCUR IN PAIRS
input is vector of force and moment at the connection output is motion of axes at connection

HENCE COMPONENT PRODUCES MOTION AND OUTPUT EQUATIONS, DEPENDING ON DEGREES OF FREEDOM, FRAME MOTION, AND INPUT:

$$
\begin{aligned}
0 & =A_{n j}\left(\xi_{n i}, \beta, f_{n i}\right) \\
x_{n j} & =B_{n j}\left(\xi_{n i}, \beta, f_{n i}\right)
\end{aligned}
$$

in general, nonlinear and time varying
if component does not have degrees of freedom, motion equations do not exist

## 4-2.2 Interfaces

## AN INTERFACE CONNECTS TWO (OR MORE) COMPONENTS

INTERFACE HAS CONSTRAINT VARIABLES $f_{l}$
INTERFACE PRODUCES CONSTRAINT EQUATIONS
to be solved for constraint variables or degrees of freedom

## AND CONSTRAINT FORCES

equations to evaluate component input $f$ in terms of constraint variables

THESE EQUATIONS IN GENERAL DEPEND ON COMPONENT INPUT $x$ AND CONSTRAINT VARIABLES:

$$
\begin{aligned}
& 0=B_{l}\left(x, f_{l}\right) \\
& f=C_{l}\left(f_{l}\right)
\end{aligned}
$$

INTERFACE TYPE CAN BE STRUCTURAL DYNAMIC OR INPUT/OUTPUT

## INPUT/OUTPUT INTERFACE

OUTPUT OF COMPONENT A SENT TO INPUT OF COMPONENT B
or to input of more than one component
CONSTRAINT VARIABLES $f_{l}$ ARE COMPONENT OUTPUT
EQUATIONS:

$$
\begin{aligned}
0 & =B_{l}=x_{A}-f_{l} \\
f_{B} & =C_{l}=f_{l}
\end{aligned}
$$

component output $x_{A}$ need not be displacement, so can depend on derivatives of component A degrees of freedom

## STRUCTURAL DYNAMIC INTERFACE

TWO COMPONENTS CONNECTED (COMPONENTS A AND B) CONSTRAINT VARIABLES $f_{l}$ ARE REACTION FORCES OF CONNECTION
component input is force and moment at connection (acting on the component) component output is linear and angular motion of axes at connection

## EQUATIONS:

$$
\begin{aligned}
& 0=B_{l}=x_{A}-x_{B} \\
& f_{A}=C_{l}=+f_{l} \\
& f_{B}=C_{l}=-f_{l}
\end{aligned}
$$

EQUATIONS ARE ACTUALLY MORE COMPLICATED THAN THIS
constraint equation obtained from exact kinematics of interface
derivatives of $B_{l}$ required

## 4-2.3 Output

AN OUTPUT PROVIDES AN EXTERNAL CONNECTION TO THE SYSTEM

OUTPUT HAS VARIABLES $y_{q}$
VARIABLES MAY BE CONNECTED TO OUTPUT $x$ OF COMPONENT
this component output must be of input/output kind
OR OUTPUT VECTOR MAY BE IDENTIFIED AS SOME DEGREE OF FREEDOM, INTERFACE, INPUT, OR FRAME $\operatorname{VECTOR}\left(\xi_{n}, f_{l}, u_{m}, \operatorname{OR} \beta\right.$ )

EQUATIONS:

$$
y_{q}=B_{y q}=\left\{\begin{array}{l}
x \\
\xi_{n} \\
f_{l} \\
u_{m} \\
\beta
\end{array}\right.
$$

## 4-2.4 Input

AN INPUT PROVIDES AN EXTERNAL CONNECTION TO THE SYSTEM

INPUT HAS VARIABLES $u_{m}$
VARIABLES ARE CONNECTED TO INPUT OF ONE OR MORE COMPONENTS

EQUATIONS:

$$
f=u_{m}
$$

component input must be of input/output kind

## 4-2.5 System

## COMPLETE SYSTEM COMBINES ALL SYSTEM PIECES

SYSTEM VARIABLES INCLUDE DEGREES OF FREEDOM $\xi$, CONSTRAINT VARIABLES $f$, INPUT $u$, AND OUTPUT $y$.
frame degrees of freedom are included in $\xi$
all system variables are partitioned into vectors, the vectors associated with individual system pieces

SYSTEM HAS MOTION EQUATIONS, CONSTRAINT EQUATIONS, AND OUTPUT EQUATIONS:

$$
\begin{aligned}
& 0=A(\xi, f, u) \\
& 0=B(\xi, f, u) \\
& y=B_{y}(\xi, f, u)
\end{aligned}
$$

THESE EQUATIONS ARE ASSOCIATED WITH DEGREES OF FREEDOM, CONSTRAINT VARIABLES, AND OUTPUT QUANTITIES RESPECTIVELY
usually for structural dynamic interfaces, derivatives of constraint equation are also required

IN GENERAL, EQUATIONS ARE TIME VARYING AND NONLINEAR CAN BE DIFFERENTIAL EQUATIONS (IF SYSTEM HAS DEGREES OF FREEDOM)

OR IMPLICIT EQUATIONS (EVALUATING OUTPUT FROM INPUT)

## 4-3 Solution Procedure

## 4-3.1 Loops and Parts

TO MAINTAIN A FLEXIBLE ABILITY TO SOLVE AEROMECHANICAL PROBLEMS, A BUILDING-BLOCK APPROACH IS REQUIRED FOR LOGICAL PIECES OF SYSTEM, JUST AS FOR PHYSICAL PIECES

IN GENERAL IT WILL BE NECESSARY TO IMPLEMENT ITERATIVE SOLUTION METHODS, SINCE EQUATIONS ARE OFTEN LARGE AND NONLINEAR

APPROACH USED IS TO DEFINE SOLUTION PROCEDURE IN TERMS OF LOOPS AND PARTS
figure 2 illustrates approach

A PART SOLVES SET OF EQUATIONS FOR RESPONSE
PHYSICAL SYSTEM (COMPONENTS, INTERFACES, AND OUTPUT) IS DIVIDED INTO PARTS
so each part has subset of system motion, constraint, and output equations

USING METHOD THAT DEPENDS ON PART TYPE, PART SOLVES EQUATIONS FOR CORRESPONDING DEGREE OF FREEDOM, CONSTRAINT, AND OUTPUT VARIABLES


Figure 4-2 Solution procedure.

## A LOOP ITERATES BETWEEN PART SOLUTIONS

USING METHOD THAT DEPENDS ON LOOP TYPE, THE LOOP ITERATES UNTIL CONVERGED SYSTEM SOLUTION IS OBTAINED

LOOPS CAN BE HIERARCHICAL, EXECUTING CHILD LOOPS AS WELL AS SPECIFIED PART SOLUTIONS

LOOPS ALSO CONTROL WRITE OF OUTPUT AND OTHER INFORMATION, BY EXECUTING SPECIFIED WRITE MODULES

## FACTORS THAT CAN MAKE IT ADVANTAGEOUS OR ESSENTIAL TO SOLVE THE EQUATIONS BY PARTS (SUBSETS OF THE SYSTEM EQUATIONS)

a) iterative solution of smaller problems can be more efficient than direct solution of large problem
b) different kinds of equations (implicit, static, differential, integral) may require different solution procedures
c) different subsystems may require or allow different parameters in solution procedure
d) certain approximations can be implemented only with partitioned solution (such as using averaged equations in solution procedure; or handling system with more than one period)
e) subsystem motion or constraint equations can be simplified when the effects of variables in other parts are entirely contained in the right-hand-side force vectors

## 4-3.2 Iterative Methods

AN ITERATIVE LOOP OR PART SOLUTION METHOD IS OFTEN REQUIRED, BECAUSE OF EITHER NONLINEARITY OR SIZE OF PROBLEM

ITERATIVE METHODS GENERALLY HAVE RELAXATION FACTOR (AND OTHER PARAMETERS) TO IMPROVE CONVERGENCE

AND TOLERANCE TO MEASURE CONVERGENCE
convergence is tested in terms of an error, which is typically some norm of difference between two iterations

BASIC APPROACH IS TO FIND LARGEST VALUE OF RELAXATION FACTOR FOR WHICH ITERATION WILL CONVERGE

HAVING ACHIEVED CONVERGENCE, LARGEST VALUE OF TOLERANCE THAT PRODUCES ACCURATE SOLUTION IS FOUND
using smaller relaxation factor or smaller tolerance can be too inefficient

## ASSESSING TOLERANCE USUALLY DONE BY LOOKING AT OUTPUT QUANTITIES RELEVANT TO SPECIFIC PROBLEM

tolerance is decreased until user decides that change in these output quantities is not important

## WHEN CORRECT SOLUTION IS NOT KNOWN, CONVERGENCE MUST BE TESTED BY COMPARING THE VALUES OF TWO SUCCESSIVE ITERATIONS

effect of relaxation factor is to reduce difference between iterations

SO REDUCTION OF RELAXATION FACTOR MUST
OFTEN BE ACCOMPANIED BY REDUCTION IN TOLERANCE, TO MAINTAIN SAME ACCURACY

## 4-3.3 Order Reduction

EFFICIENCY CAN OFTEN BE IMPROVED BY MINIMIZING NUMBER OF VARIABLES INVOLVED IN SOLUTION

SUPPRESSING EFFECT OF SELECTED VARIABLES CAN ALSO BE CONCEPTUALLY USEFUL

## SO ORDER REDUCTION AVAILABLE:

SET RESPONSE TO ZERO (PERHAPS OMIT VARIABLE AND CORRESPONDING EQUATION ENTIRELY)

QUASISTATIC REDUCTION OF DEGREES OF FREEDOM
order reduction specified in response piece for each element of each vector (superseded for any mode or transform)
specify zero, dynamic, or quasistatic
for trim, transient, and flutter tasks

## MODAL TRANSFORMATION OF STRUCTURAL DYNAMIC EQUATIONS

## ALLOWS ORDER REDUCTION ON BASIS OF FREQUENCY CONTENT OF EXCITATION, COMPARED TO NATURAL FREQUENCIES OF MODES

equations of motion and output

$$
\begin{gathered}
m \ddot{\xi}+k \xi=\phi_{e}^{T} f \\
x=\phi_{d} \xi
\end{gathered}
$$

partitioned into kept $\left(\xi_{k}\right)$ and neglected $\left(\xi_{l}\right)$

$$
\begin{gathered}
m_{k} \ddot{\xi}_{k}+k_{k} \xi_{k}=\phi_{e k}^{T} f \\
m_{l} \ddot{\xi}_{l}+k_{l} \xi_{l}=\phi_{e l}^{T} f \\
x=\phi_{d k} \xi_{k}+\phi_{d l} \xi_{l}
\end{gathered}
$$

for modes with large natural frequency, can neglect inertia relative to spring - quasistatic reduction

$$
\left(m_{l} s^{2}+k_{l}\right) \xi_{l}=m_{l}\left(s^{2}+\omega_{l}^{2}\right) \xi_{l} \cong k_{l} \xi_{l}
$$

good approximation if excitation at high frequencies is small
perhaps can just solve dynamic equation for $\xi_{k}$, so

$$
x \cong \phi_{d k} \xi_{k}
$$

but spring of high frequency mode can be small, leading to significant static response
most accurate to retain complete static response, in terms of static residual from truncated modes (equivalent to a mode-acceleration method)

$$
\begin{gathered}
\xi_{l}=k_{l}^{-1} \phi_{e l}^{T} f \\
x=\phi_{d k} \xi_{k}+\left(\phi_{d l} k_{l}^{-1} \phi_{e l}^{T}\right) f=\phi_{d k} \xi_{k}+G_{l} f
\end{gathered}
$$

when subsystem has rigid degrees of freedom, must also consider effect of inertial relief on residual

## 4-4 Physical System Pieces

## CORRESPOND TO PHYSICAL DESCRIPTION OF SYSTEM

## PIECES AVAILABLE TO CONSTRUCT SYSTEM (BY CLASS):

 COMPONENTtype $=$ available components
FRAME
INTERFACE
type $=$ structural dynamic
type $=$ input/output
OUTPUT
INPUT

THESE PIECES PRODUCE THE SYSTEM EQUATIONS

## 4-5 System Pieces: Components

COMPONENTS PERFORM ALL COMPUTATIONS ASSOCIATED WITH THE PHYSICS OF THE MODEL OF A SYSTEM SO COMPONENTS ARE FOCUS FOR MODELLING ISSUES including empiricism and approximations needed for practical model of many physical systems

> DEVELOPING AN IMPROVED MODEL REQUIRES DEVELOPING A NEW COMPONENT, WHICH WILL FIT INTO EXISTING ANALYSIS FRAMEWORK

## A COMPONENT CAN BE CONSIDERED AN OPERATOR THAT EVALUATES A VECTOR OR MATRIX

figure 3 illustrates functionality
at time $t$ component evaluates one of following vectors:
motion equation: $A_{n j}$
component output for input/output interface: $x_{n j}=B_{n j}$
component output for structural dynamic interface: axes motion $x_{n j}$
from degrees of freedom $\xi_{n i}$ (including frame motion) and from component input $f_{n i}$

COMPONENT CAN ALSO PERTURB THIS VECTOR (ANALYTICALLY OR NUMERICALLY), TO CONSTRUCT MATRIX COLUMN


Figure 4-3 Component functionality.

## IN GENERAL, EQUATIONS ARE TIME VARYING AND NONLINEAR CAN BE DIFFERENTIAL EQUATIONS (IF COMPONENT HAS DEGREES OF FREEDOM)

differential equation can be zero, first, or second order, as defined by dependence of $A_{n j}$ and $B_{n j}$ on derivatives of $\xi$ for structural dynamic no-residual interface, $x_{n j}$ can not depend on the velocity or acceleration of $\xi$ may not be assumed to be symmetric

OR IMPLICIT EQUATIONS (EVALUATING OUTPUT FROM INPUT)

## COMPONENT EQUATIONS CAN ALSO DEPEND ON FRAME MOTION $\beta$

frame motion is hierarchical, and each frame can be degrees of freedom (identified as rigid body motion of some component)
so frame dependence translates into dependence on system degrees of freedom: component rigid degrees of freedom that correspond to frame and all its parent frames

## INTERFACES BETWEEN COMPONENTS CAN BE STRUCTURAL DYNAMIC OR INPUT/OUTPUT KIND

COMPONENT DEFINITION INCLUDES COMPONENT INPUT AND COMPONENT OUTPUT
for a structural dynamic connection:
component input = force and moment at connection component output $=$ motion of axes at connection
component defines all input and output available to be used, must be connected to another system piece (interface, input, or output)

## STRUCTURAL DYNAMIC INPUT AND OUTPUT PAIR

 can be connected to only one other component but more than one connection can be defined at same point on component
## COMPONENT OUTPUT OF INPUT/OUTPUT KIND

can be connected to one or more interfaces and one or more output pieces

## COMPONENT INPUT OF INPUT/OUTPUT KIND

can be connected to only one interface or one input piece but interface variables and input variables can be combined either inside component, or by using a differential equation component
figure 4 shows use of component input and component output by interface, input, and output system pieces


Figure 4-4 Use of component input and component output.

## 4-5.1 Structural Dynamic Components

## STRUCTURAL DYNAMIC COMPONENTS CHARACTERIZED BY rigid body motion and frames mass, hence inertial and gravitational forces structural dynamic interfaces standard interface (component input and component output) defined for connections with aerodynamic components <br> STRUCTURAL DYNAMIC COMPONENTS DIFFER PRINCIPALLY IN MATTERS ASSOCIATED WITH ELASTIC MOTION

## A FRAME (PERHAPS INERTIAL) MUST BE IDENTIFIED

rigid motion of component measured relative to that frame

## STRUCTURAL DYNAMIC COMPONENT HAS RIGID BODY MOTION

 "CONSTRAINED COMPONENT": connected to frame, so no rigid body degrees of freedom"FRAME COMPONENT": rigid body degrees of freedom are the frame motion

OTHER CASES: rigid body degrees of freedom exist, and represent motion relative to the frame
figure 5 summarizes options for rigid body motion
ONLY A STRUCTURAL DYNAMIC COMPONENT CAN HAVE STRUCTURAL DYNAMIC INPUT AND OUTPUT
for connection to another component through a structural dynamic interface
structural dynamic connection made in common parent frame of two components


FRAME COMPONENT

motion $\mathrm{BP}=\mathrm{FP}=$ nominal + degree of freedom


Figure 4-5 Rigid body motion of structural dynamic component.

## 4-5.2 Aerodynamic Components

## AERODYNAMIC COMPONENTS INCLUDE WINGS AND WAKES

## WING OR BODY IS SURFACE MOVING THROUGH AIR

wing is thin surface, described by spanwise reference line (not necessarily straight), and two or more chordwise points at each span station

## INTERFACES BETWEEN STRUCTURE AND AIR OCCUR AT SURFACE OF WING OR BODY

interface is discretized (set of collocation points)
these collocation points must be defined as connection points on structural dynamic components
typically interface involves velocity and force at connection points

## WAKE COMPONENT SOLVES FOR MOTION OF AIR

wing and wake components can be separate or combined, depending on model and solution procedure mutual influence between aerodynamic components typically accounted for by wakes

## 4-6 System Pieces: Frames

## FRAMES HELP DERIVATION AND ANALYSIS OF PROBLEM

 BY PROVIDING DESCRIPTION OF MOTION APPROPRIATE TO SPECIFIC CONFIGURATIONPARTICULARLY FOR STRUCTURAL DYNAMIC COMPONENTS frame identified for a component, all motion measured relative that frame

## TO BE USEFUL, FRAME MOTION MUST BE PHYSICALLY RELEVANT

frame can be equated to rigid body motion of component (frame degrees of freedom)
component can be connected to frame (constrained component)
component motion relative frame can be small
components are coupled relative to their common frame
only motion relative that frame enters constraint equation; so can use body-axis velocity for system rigid motion

FRAME MOTION CAN BE PRESCRIBED, OR DEGREES OF FREEDOM

FRAME DEGREES OF FREEDOM
IMPLEMENTED BY IDENTIFYING FRAME AS RIGID BODY MOTION OF PARTICULAR COMPONENT ("FRAME COMPONENT")
so not necessary to introduce additional equations that tie frame to structure

## FRAMES ARE HIERARCHICAL

frame motion measured relative parent frame (perhaps inertial)

MOTION (DISPLACEMENT AND ROTATION) OF PHYSICAL POINT (PROBABLY ON SOME COMPONENT):
motion $=$ frame + frame $+\ldots+$ variable
OPERATION "+" IS ADDITION OF RIGID MOTION
first frame ("base frame") consists of motion relative to inertial space
base frame can be associated with an operating condition
second frame consists of motion relative to first frame, and so on
final variable measured relative to its frame
final variable will probably be combination of several motions
figure 6 illustrates use of frames to describe motion


Figure 4-6 Use of frames to describe motion.

## INERTIAL FRAME IS REFERENCE FOR ENTIRE SYSTEM <br> EARTH AXES ARE FIXED RELATIVE TO INERTIAL AXES

orientation of inertial frame relative to earth defined by direction of gravity
ground plane defined by downward normal, and height origin of inertial frame above ground level

## BASE FRAME (FREE FLIGHT)

THE BASE FRAME IS FIRST FRAME OF SYSTEM: ITS PARENT IS THE INERTIAL FRAME
"BASE FRAME" CAN BE ASSOCIATED WITH OPERATING CONDITION
linear motion must be represented by body-axes velocity or inertial axes displacement
angular motion must be represented by aircraft Euler angles

BASE FRAME CAN BE USED AS RIGID MOTION OF SYSTEM (FRAME DEGREES OF FREEDOM)
with elastic motion of system measured relative rigid motion of base frame
so only base frame requires operating condition

## MEAN AXES

FOR A FREE SYSTEM, OFTEN DESIRABLE TO DESCRIBE SYSTEM RIGID MOTION USING MOTION OF MEAN AXES
mean axes are defined such that relative to them the entire system has zero linear and angular momentum
zero linear momentum: origin of mean axes at system center-of-mass
for linear system, degrees of freedom of mean axes are zero frequency (rigid body) modes of free vibration modes
so mean axes and elastic modes are decoupled

## TYPICAL AIRCRAFT CONVENTIONS

inertial axes have $z$-axis down
system rigid motion is frame degrees of freedom (center-ofmass, mean axes if possible)
body-axes velocity and aircraft Euler angle representation of motion
rest position has $x$-axis in positive velocity direction (forward) yaw, pitch, and roll angles of aircraft Euler angle representation rotate the system from inertial axes to base frame

## SYSTEM RIGID MOTION FOR AIRCRAFT DYNAMICS

figure 7 illustrates frames and axes of system rigid motion CONVENTIONAL TO REPRESENT RIGID MOTION OF SYSTEM BY CENTER-OF-MASS, MEAN AXES
problem: not motion of physical point
solution: for linear normal modes component, rigid body motion is mean axes of component; usually can define system inertia so also mean axes of system

SYSTEM RIGID MOTION IN TRIM DEFINED BY OPERATING CONDITION
problem: complicated to use operating condition to describe trim motion of all components
solution: use operating condition to describe motion of base frame, and define all other motion relative to that frame

CONVENTIONAL TO REPRESENT SYSTEM RIGID MOTION BY BODY-AXIS VELOCITY COORDINATES
problem: complicated to obtain position relative inertial space when using body-axis velocity coordinates
solution: use body-axis velocity to describe motion of base frame, and define all other motion relative to that frame
(coupling components requires only their motion relative common frame, so position relative inertial space not needed)


Figure 4-7 System rigid motion.

## FRAMES FOR ROTORCRAFT SHELL

AIRFRAME FRAME (free flight)
aircraft convention for axes ( $z$ down, $x$ forward)
base frame (parent = inertial frame) frame degrees of freedom center-of-mass, mean axes of component conventions for definition of system inertia such that usually center-of-mass, mean axes of system body-axes velocity and aircraft Euler angle representation of motion
or inertial axes displacement
parent frame of rest of system
so operating condition only required for frame reference motion
and structural dynamic interfaces (relative common frame) do not require position relative inertial space and can use body-axes velocity representation

ROTOR FRAME (nonrotating hub, parent = airframe frame)
ROTATING FRAME (rotating hub, parent = rotor frame)
BLADE FRAME (rotating blade, parent = rotating frame)
frame degrees of freedom

## 4-7 System Pieces: Interfaces

AN INTERFACE IS A CONNECTION BETWEEN TWO OR MORE COMPONENTS

THERE ARE TWO TYPES OF INTERFACES:
STRUCTURAL DYNAMIC AND INPUT/OUTPUT

EACH INTERFACE DEFINES CONSTRAINT EQUATIONS, WHICH ARE SOLVED TO ELIMINATE SOME SYSTEM VARIABLES

STRUCTURAL DYNAMIC INTERFACE: ELIMINATE SYSTEM DEGREES OF FREEDOM $\xi_{b}$

INPUT/OUTPUT INTERFACE: ELIMINATE INTERFACE VARIABLES $f_{l}$

IN GENERAL, EQUATIONS ARE TIME VARYING AND NONLINEAR
possibly not symmetric even for structural dynamic systems
input/output interface equation can depend on derivatives
of system degrees of freedom

INTERFACE CONNECTS INPUT $f$ AND OUTPUT $x$ OF TWO OR MORE COMPONENTS
connection operation $B_{l}$ produces constraint equation, from $x$ and constraint variables $f_{l}$
connection operation $C_{l}$ produces constraint force $f$, from constraint variables $f_{l}$

## 4-7.1 Input/Output Interface

INPUT/OUTPUT CONNECTION DEFINED BY IDENTIFYING OUTPUT $x$ OF COMPONENT "A", AND INPUT $f$ OF ONE OR MORE COMPONENTS "B"

ALL COMPONENT INPUT AND OUTPUT INVOLVED MUST BE INPUT/OUTPUT KIND
"A" AND "B" CAN BE SAME COMPONENT

CONSTRAINT EQUATION IS SOLVED FOR CONSTRAINT VARIABLES $f_{l}$

$$
0=B_{l}=x_{A}-f_{l}
$$

causes component to evaluate $x_{A}$
THEN COMPONENT INPUT $f_{B}$ CAN BE EVALUATED FROM CONSTRAINT VARIABLES

$$
f_{B}=C_{l}=f_{l}
$$

figure 8 illustrates use of input/output interfaces


Figure 4-8 Use of input/output interfaces.

## 4-7.2 Structural Dynamic Interface

STRUCTURAL DYNAMIC INTERFACE ASSUMED HOLONOMIC AND INDEPENDENT

> HOLONOMIC MEANS THAT THE CONSTRAINT EQUATION CAN BE DEFINED IN TERMS OF DISPLACEMENTS (RATHER THAN VELOCITIES OR VIRTUAL DISPLACEMENTS)
> for nonholonomic interface, solution of constraint equation is path dependent (for example, rolling of a wheel on a surface)

INDEPENDENT MEANS THAT THE CONSTRAINT EQUATIONS ARE FULL RANK
dependent equations imply either redundant or inconsistent constraints

## STRUCTURAL DYNAMIC CONNECTION DEFINED BY IDENTIFYING STRUCTURAL DYNAMIC INTERFACE (CONSISTING OF INPUT $f$ AND OUTPUT $x$ AT SAME POINT) OF TWO COMPONENTS, "A" AND "B" <br> component can not be connected to itself <br> INTERFACE VARIABLE $f_{l}$ IS FORCE ACTING ON "A" SIDE OF INTERFACE

figure 9 illustrates use of structural dynamic interfaces

CONSTRAINT EQUATION SOLVED FOR SOME DEGREES OF FREEDOM $\xi_{b}$


Figure 4-9 Use of structural dynamic interfaces.

## CONSTRAINT EQUATIONS

$$
0=B_{l}=x_{A}-x_{B}
$$

(equations for angular motion constraint are exact, so $B_{l}$ is actually more complicated)

THEN COMPONENT INPUT $f$ (FORCE AND MOMENT) CAN BE EVALUATED FROM CONSTRAINT FORCES $f_{l}$

$$
\begin{aligned}
f_{A} & =C_{l}
\end{aligned}=+f_{l},
$$

( $f_{l}$ has only force if angular motion not constrained)

## CONNECTION KIND IS COMPLETE OR TORQUE

complete interface is true physical connection, torque interface is approximation

## TORQUE INTERFACE

SPECIAL (APPROXIMATE) STRUCTURAL DYNAMIC CONNECTION

## CONNECTS ONLY ROTATIONAL MOTION AND TORQUES

can act only on appropriate joints of component produces no net moment on true structural dynamic component; only involved in joint equations

## COMPLETE INTERFACE <br> INTERFACE CAN BE JOINT BETWEEN TWO COMPONENTS

general spring/damper/actuator joints implemented as part of all structural dynamic components
so only two joints implemented in interface: cantilever and pinned

CONSTRAINED COMPONENT (NOT SINGLE-POINT CONSTRAINT) USED TO ELIMINATE SYSTEM RIGID MOTION
so all structural dynamic interfaces are two-sided
POSITION AND ORIENTATION OF POINTS CONNECTED BY INTERFACE ARE DEFINED BY COMPONENT GEOMETRY

## CANTILEVER

connection with no relative motion; equates linear and angular motion of points on two components

## PINNED

ball joint connection; equates only linear motion of points on two components
figure 10 shows cantilever and pinned interfaces


PINNED

equate linear motion
$x^{A P / P}=x^{B P / P}$
(connection axes not equal)

Figure 4-10 Cantilever and pinned structural dynamic interfaces.

## USER MUST IDENTIFY SYSTEM DEGREES OF FREEDOM TO BE ELIMINATED BY CONSTRAINT EQUATIONS OF A STRUCTURAL DYNAMIC INTERFACE

PROGRAM CAN ELIMINATE RIGID DEGREES OF FREEDOM OF EITHER COMPONENT "A" OR COMPONENT "B"
so tree structure is simple: each interface should eliminate rigid motion of component on branch side of structure
with branch side identified as component " B ", interface forces are those acting on root side

## USER NEED ONLY BE CONCERNED WITH IDENTIFYING THOSE DEGREES OF FREEDOM ASSOCIATED WITH MULTIPLE LOAD PATHS

simple if elastic joint put at connection that creates multiple load path: eliminate joint degrees of freedom examples from rotorcraft shell:
pitch link - eliminate pitch bearing rotation, or some elastic torsion degree of freedom
snubber of bearingless hub - eliminate snubber joint degrees of freedom
figure 11 illustrates use of structural dynamic interfaces


Figure 4-11 Use of structural dynamic interfaces.

## 4-8 System Pieces: Output

AN OUTPUT PROVIDES AN EXTERNAL CONNECTION TO THE SYSTEM

OUTPUT KIND:
component degree of freedom, $\xi_{n}$
interface variable, $f_{l}$
input variable, $u_{m}$
frame degree of freedom, $\beta$
component output, $x$ (input/output kind only)

FOR ALL QUANTITIES EXCEPT COMPONENT OUTPUT $x$ :
ALREADY AVAILABLE IN SOLUTION, SO OUTPUT PIECE JUST CONTROLS WRITE PROCESS
separate solution for output is not necessary

## IDENTIFYING $y$ AS COMPONENT OUTPUT $x$ PRODUCES AN OUTPUT EQUATION

this equation must be solved in some part
only this output kind is available as output variable in flutter equations

## OUTPUT PIECE CONTROLS WRITE PROCESS

DEFINES QUANTITIES THAT ARE WRITTEN
such as time history or harmonics
AND THEIR DESTINATION
print, plot file, or printer-plot

## A TIME HISTORY IS EVALUATED FROM RESPONSE SOLUTION

whether that solution is in time domain or frequency domain

AT NUMBER OF TIME STEPS SPECIFIED BY OUTPUT PIECE

## FOR PERIODIC DATA, A COSINE/SINE FOURIER SERIES

 CAN BE EVALUATEDfrom preceding time history (even if solution is in frequency domain)

FOR NUMBER OF HARMONICS SPECIFIED BY OUTPUT PIECE
figure 12 illustrates the process


Figure 4-12 Calculation process for output piece.

## HARMONIC ANALYSIS

## PRODUCES COSINE/SINE FOURIER SERIES FROM DISCRETIZED DATA IN TIME DOMAIN

 INTERPOLATION OPTIONSFourier interpolation
Fourier series representation (all harmonics) gives discrete data points exactly, but behavior uncontrolled in between
linear interpolation: harmonics of a linear interpolation between data at discrete times

Fourier series representation (infinite number of harmonics) gives linear interpolation exactly
finite number of harmonics gives smoothed curve, close to discrete data points
figure 13 illustrates interpolation options

## FOURIER INTERPOLATION

O time history data
__ Fourier series representation


## LINEAR INTERPOLATION

O time history data
----- linear interpolation
—— Fourier series representation (finite number of harmonics)


Figure 4-13 Interpolation options of harmonic analysis.

## 4-9 System Pieces: Input

## AN INPUT PROVIDES AN EXTERNAL CONNECTION TO THE SYSTEM

required for trim and flutter tasks
a trim loop can change value of input (constant)
flutter can perturb input for linear equations
transient task needs components to generate prescribed control (input has constant value from trim solution)

## DEFINED BY LENGTH OF INPUT VECTOR AND NAMES OF ELEMENTS IN INPUT VECTOR

## CONNECTIONS OF INPUT TO COMPONENT INPUT $f$ ARE IDENTIFIED

component input must be input/output kind each input of component can be connected to only one source (interface variable $f_{l}$ or input vector $u_{m}$ )

TO OBTAIN COMPONENT INPUT FROM COMBINATION OF SEVERAL $u_{m}$ AND/OR $f_{l}$ :
combine within component
or use differential equation component (in static mode) to combine; result evaluated as input/output interface and sent to component that requires it

## 4-10 Environmental System Pieces

## PROVIDE STANDARD DESCRIPTION OF SYSTEM ENVIRONMENT

PIECES AVAILABLE TO CONSTRUCT SYSTEM (BY CLASS):
CASE
WIND
OPERATING CONDITION
PERIOD

WIND, OPERATING CONDITION, AND PERIOD SYSTEM PIECES
parameters available to all components
trim loop solution methods can change these parameters

## 4-11 Environment

## EARTH

## ASSUME FLAT, NONROTATING EARTH

earth axes fixed relative to inertial axes center-of-mass and center-of-gravity coincident no gravity-gradient forces

ACCELERATION PRODUCED BY GRAVITY
vector $g$, constant in inertial axes
direction of $g$ defines orientation of inertial frame on earth

## GROUND PLANE

defined by downward normal, and height origin of inertial frame above ground level
needed for rotor inflow and wake models when in-groundeffect

## AERODYNAMIC

SPEED OF SOUND $c_{s}$, DENSITY $\rho$, KINEMATIC VISCOSITY $\nu=\mu / \rho$ OF AIR (OR OTHER FLUID)
calculated from altitude for standard day
or from pressure altitude and temperature or from density and temperature or from density, sound speed, and viscosity

## 4-12 System Pieces: Case

## SYSTEM MUST HAVE ONE (AND ONLY ONE) CASE PIECE DEFINED

## INFORMATION

case description
selection of analysis tasks
control print of input data
timer and debug parameters
earth and aerodynamic environment parameters

## 4-13 System Pieces: Wind

## SYSTEM MUST HAVE ONE (AND ONLY ONE) WIND PIECE DEFINED

## PARAMETERS

wind speed
orientation of the wind/gust axes (G) relative inertial axes, defined by yaw and pitch angles
these parameters are used to calculate velocity of air relative inertial frame
trim loop solution methods can change these parameters

## WIND SPEED

wind speed positive for air from $+x$ axis direction ( $x$ is forward)

## WIND/GUST AXES

gust axes and wind axes are same
wind/gust axes can be
velocity axes (from a specified free-flight operating condition)
inertial axes
or input orientation
gust components use wind/gust axes to define change of air velocity: $u_{G}, v_{G}$, and $w_{G}$ components in $G$ axes

## OPTIONAL GROUND BOUNDARY LAYER

air velocity depends on height above ground
calculated using log law, modified log law, or power law wind speed is air velocity at reference height
effect of ground boundary layer included in: velocity relative air (aerodynamic interface) calculated by structural dynamic components
wake geometry distortion calculated by wing wake geometry component

## 4-14 System Pieces: Operating Condition

## SYSTEM MUST HAVE AT LEAST ONE OPERATING CONDITION PIECE DEFINED

## PARAMETERS

operating condition can be constrained or free flight speed
orientation of the body frame (F) relative inertial axes, defined by yaw, pitch and roll angles
turn rate
orientation of the velocity frame (V) relative inertial axes, defined by yaw and pitch angles
these parameters are used to define motion of "base frame"
trim loop solution methods can change these parameters

## FREE FLIGHT OPERATING CONDITION

## AIRCRAFT CONVENTIONS:

flight speed positive for body moving in $+x$ axis direction ( $x$ is forward) body axes or stability axes can be used to describe the motion
orientation defined by Euler angles, using yaw-pitch-roll sequence

TYPICALLY FREE SYSTEM WILL NOT USE WIND SPEED, AND GUST AXES ARE VELOCITY AXES

## CONSTRAINED OPERATING CONDITION

ONLY ORIENTATION OF BODY FRAME RELATIVE INERTIAL AXES USED
yaw, pitch, and roll angles
no flight speed or turn rate
A VELOCITY OF AIR RELATIVE TO BODY OBTAINED USING THE WIND

## 4-15 System Pieces: Period

## PARAMETER

rotational speed (constant)

## PERIOD KIND CAN BE BASE OR CHILD

BASE PERIOD
rotational speed defined trim loop solution methods can change this parameter

## CHILD PERIOD

parent period and gear ratio $r$ defined
(child rotational speed) $=r \times($ parent rotational speed $)$ not hierarchical (parent must be base period)

ANY SYSTEM PIECE CAN IDENTIFY A PERIOD SO CAN GET CORRESPONDING ROTATIONAL SPEED $\Omega$ DURING THE SOLUTION PROCESS

STANDARD RELATION BETWEEN AZIMUTH AND TIME IS USED:

$$
\psi=\Omega t+\psi_{0}
$$

REFERENCE AZIMUTH $\psi_{0}$ MUST BE DEFINED WHENEVER A PERIOD IS IDENTIFIED

## 4-16 Logical System Pieces

## DEFINE PROCEDURE FOR SOLVING SYSTEM EQUATIONS

 PART SOLVES SET OF EQUATIONS FOR RESPONSE, LOOP ITERATES BETWEEN PART SOLUTIONStype selects solution method

PIECES AVAILABLE TO CONSTRUCT TRIM SOLUTION (BY CLASS):
TRIM LOOP
type = no solution
type = successive substitution
type $=$ Newton Raphson
type $=$ regulator
TRIM PART
type $=$ no solution
type $=$ implicit
type $=$ static
type $=$ harmonic
type $=$ time finite element
TRANSFORM
MODES
RESPONSE
type $=$ rigid
type $=$ variable
WEIGHTS

PIECES AVAILABLE TO CONSTRUCT TRANSIENT SOLUTION (BY CLASS):

## TRANSIENT

TRANSIENT LOOP
type $=$ no solution
type $=$ successive substitution

## TRANSIENT PART

type $=$ no solution
type $=$ trim solution
type $=$ implicit
type $=$ integration

PIECES AVAILABLE TO CONSTRUCT FLUTTER SOLUTION (BY CLASS):

## FLUTTER

FLUTTER LOOP
FLUTTER PART
type $=$ no solution
type = interface
type $=$ differential equations

## 4-17 Trim Task

## EQUILIBRIUM SOLUTION FOR STEADY STATE OPERATING CONDITION

ASSUME:
SYSTEM ENVIRONMENT AND INPUT ARE CONSTANT OR PERIODIC

EQUILIBRIUM SOLUTION EXISTS
SOLUTION IS CONSTANT OR PERIODIC
possible to define systems for which these assumptions are not true

USUALLY IDENTIFY PARAMETER VALUES REQUIRED TO ACHIEVE A SPECIFIED OPERATING CONDITION (AN INVERSE PROBLEM)

PARAMETERS THAT CAN BE IDENTIFIED:
inputs, and operating condition variables (system mean rigid motion)

IDENTIFICATION REQUIRES:
output quantities achieve a target value, or aircraft force and moment be zero (mean equations of system rigid motion)

INVERSE SOLUTION IS IMPLEMENTED BY A TRIM LOOP

OR EQUATIONS CAN BE SOLVED FOR FIXED VALUES OF INPUTS AND OPERATING CONDITION
then free body will not be in equilibrium

## EXAMPLES FOR ROTORCRAFT

FREE FLIGHT
find inputs (pilot collective, cyclic, pedal controls) and system mean rigid motion (pitch and roll angles)
so mean equations of system rigid motion are zero (net mean force and moment on aircraft equal zero)

WIND TUNNEL
find inputs (collective and cyclic controls)
so output quantities achieve target values (thrust and flapping)

## FIXED CONTROLS

fixed input (collective and cyclic controls), in wind tunnel

## 4-17.1 Periodic Solution

WHEN EQUILIBRIUM SOLUTION IS PERIODIC, ASSUMED THAT PERIOD IS KNOWN

TYPICALLY OBTAINED FROM ROTATIONAL SPEED OF SOME STRUCTURAL DYNAMIC SUBSYSTEM, $T_{\text {ref }}=2 \pi / \Omega$

THERE CAN BE SEVERAL PERIODS $T_{i}$ SIMULTANEOUSLY INVOLVED IN THE SYSTEM BEHAVIOR

EXAMPLE: SINGLE MAIN-ROTOR AND TAIL-ROTOR HELICOPTER

SYSTEM PERIOD $T$, EQUAL TO COMMON INTEGER MULTIPLE OF ALL $T_{i}$, MIGHT STILL EXIST
with several rotating subsystems connected through gear train, the ratio of periods will be a rational number, and there will be a system period

BUT SUCH A PERIOD MAY BE TOO LARGE FOR A PRACTICAL TRIM ANALYSIS

TRANSIENT TASK CAN ANALYZE SUCH CASES

## APPROXIMATE SOLUTION WITH SEVERAL PERIODS PRESENT:

 SOLVE FOR RESPONSE AT EACH PERIOD SEPARATELY, IGNORING NON-HARMONIC VIBRATORY COUPLING BETWEEN PARTSCONSIDER PERIODIC AND TIME-INVARIANT PARTS

## PERIODIC PART

example: rotor
INHERENTLY INVOLVES PARTICULAR PERIOD $T_{I}$
so can be solved only for that period
assumption of periodicity requires that any input from other parts be at period $T_{I}$, or else only the mean value can be used

## TIME-INVARIANT PART

example: airframe
HAS NO INHERENT PERIOD
so can respond to input at all periods $T_{i}$
solving at each period separately is approximation if part equations are nonlinear

## 4-17.2 Rotorcraft Trim Task

## PARTITIONED, ITERATIVE SOLUTION PROCEDURE CONSTRUCTED BY ROTORCRAFT SHELL

figure 14a shows the loops and principal interface variables

## WAKE LOOP

computationally intensive task of obtaining wake influence coefficients (which depend on wake geometry) moved outside all other loops

## TRIM LOOP

inverse problem requires solution of algebraic equations for controls and other trim variables

## CIRCULATION LOOP

induced velocity calculated from integral equations, while rotor and airframe motion calculated from differential equations; large number of circulation variables

## MOTION LOOP

separate solution for rotor and airframe motion required to avoid interharmonic coupling (from rotating-to-nonrotating interface), and to handle case of two periods for mainrotor and tail-rotor configuration

## WAKE LOOP HAS UP TO THREE LEVELS

uniform inflow, nonuniform inflow with prescribed wake geometry, nonuniform inflow with free wake geometry
no test for convergence
figure 14b shows levels of wake loop


Figure 4-14a Basic loops of trim analysis (optional regulator loop not shown).


Figure 4-14b Levels of wake loop.

LOOPS AND THEIR SOLUTION METHODS:

| loop name | solution method |
| :--- | :--- |
| WAKE | successive substitution |
| TRIM | Newton-Raphson |
| REGULATOR | regulator |
| CIRCULATION | successive substitution |
| MOTION | successive substitution |
| AIRFRAME | no solution |

PART SOLUTION METHOD IS "HARMONIC" OR "TIME FINITE ELEMENT" FOR FOLLOWING:

ROTOR n HUB
ROTOR n BLADE m
AIRFRAME
DRIVE TRAIN
"IMPLICIT" METHOD USED FOR OTHER PARTS

## 4-18 System Pieces: Trim Loop

A LOOP CONTROLS EXECUTION OF PART SOLUTION AND WRITE MODULES, AND SOLVES NONLINEAR ALGEBRAIC EQUATIONS

LOOP IMPLEMENTS GENERAL ITERATION BETWEEN PART SOLUTIONS

ITERATION BETWEEN: parts solved in loops and all inner loops

ITERATION CONTROL AND CONVERGENCE TEST
OPERATIONS CONSTITUTE LOOP ALGORITHM
figure 15 outlines loop solution process

LOOP DESCRIBED BY IDENTIFYING PARTS, CHILD LOOP, AND WRITE MODULES

PARTS SOLVED IN LOOP (ITERATED WITH CHILD LOOPS), OR AT END OF LOOP
parts are solved in order defined
LOOPS ARE HIERARCHICAL loop can have at most one child and one parent; trim task can have more than one set of nested loops

WRITE MODULES PRODUCE OUTPUT, FOR:
SHELL
CONVERGENCE LOOP SOLUTION PART SOLUTION MODES
OUTPUT OF PART
OUTPUT
GRAPHICS

loop iteration<br>iteration control<br>solve parts in loop<br>child loop<br>test loop convergence<br>solve parts at end of loop<br>execute write modules at end of loop

Figure 4-15 Outline of loop solution process.

## METHODS IMPLEMENTED FOR TRIM LOOPS:

NO SOLUTION
SUCCESSIVE SUBSTITUTION (with stages)
NEWTON RAPHSON (with identification)
REGULATOR (with identification)

NO SOLUTION METHOD
NO ITERATION CONTROL OR CONVERGENCE TEST OPERATIONS

LOOP JUST ORGANIZES PART SOLUTIONS

## 4-18.1 Theory

## LOOP SOLVES NONLINEAR ALGEBRAIC EQUATIONS

CAN WRITE NONLINEAR ALGEBRAIC EQUATIONS IN TWO FORMS:
FIXED POINT, $x=G(x)$
ZERO POINT, $f(x)=0$
with vectors $x, G$, and $f$
function $G$ or $f$ consists of part and child loop operations
REQUIRE EFFICIENT AND CONVERGENT METHODS TO FIND SOLUTION $x=\alpha$

FOR NONLINEAR PROBLEMS, METHOD WILL BE ITERATIVE:
$x_{n+1}=F\left(x_{n}\right)$
operation $F$ depends on solution method
iteration will converge if $F$ is not too sensitive to errors in $x$ : converge if $\left|F^{\prime}(\alpha)\right|<1$
convergence is linear for $F^{\prime} \neq 0$, quadratic (at least) for $F^{\prime}=0$

ITERATIVE METHODS WILL HAVE RELAXATION FACTOR (AND OTHER PARAMETERS) TO IMPROVE CONVERGENCE, AND TOLERANCE TO MEASURE CONVERGENCE

## 4-18.2 Successive Substitution Method

## SUCCESSIVE SUBSTITUTION LOOP EXECUTES GENERAL ITERATION BETWEEN PART SOLUTIONS

used for wake, circulation, and motion loops of rotorcraft shell figure 16 outlines the method

## SUCCESSIVE SUBSTITUTION ALGORITHM SOLVES EQUATIONS OF FORM $x=G(x)$ (FIXED POINT)

let $y=G_{1}(x)$ represent result of parts solved in loop and $x=G_{2}(y)$ result of child loop solution
then $G=G_{2}\left(G_{1}\right)$, and $x$ is defined by how system equations are split between this loop and its child loops
$x$ is all quantities calculated by child loops, and used by parts solved in this loop
analysis finds $x$ from functionality of part equations

## ALGORITHM INCLUDES STAGES OR LEVELS: CYCLES OUTSIDE BASIC SUCCESSIVE SUBSTITUTION ITERATION

A PART SOLUTION CAN CHANGE ITS MODEL DEPENDING ON THE LOOP LEVEL

SO AN INITIALIZATION OR STARTUP PROCEDURE CAN BE IMPLEMENTED
used for wake loop

## level cycle

save: $x_{\text {old }}=x$
successive substitution iteration
relax: $x=\lambda x+(1-\lambda) x_{\text {old }}$
save: $x_{\text {old }}=x$
evaluate $x$ (solve parts in loop, child loops)
test convergence: error $=\left\|x-x_{\text {old }}\right\| \leq$ tolerance $\times$ weight

Figure 4-16 Outline of successive substitution method.

## THEORY

## ITERATION

direct iteration is simply $x_{n+1}=G\left(x_{n}\right)$, but $\left|G^{\prime}\right|>1$ for many practical problems
relaxed iteration uses $F=(1-\lambda) x+\lambda G$ :

$$
x_{n+1}=(1-\lambda) x_{n}+\lambda G\left(x_{n}\right)
$$

with relaxation factor $\lambda$; then convergence criterion is

$$
\left|1-\lambda+\lambda G^{\prime}\right|<1
$$

so $\lambda$ can be found to ensure convergence for any finite $G^{\prime}$

## CONVERGENCE TEST

correct solution $x=\alpha$ is not known, so convergence tested by comparing two successive iterations:

$$
\text { error }=\left\|x_{n+1}-x_{n}\right\| \leq \text { tolerance }
$$

typically error is absolute value of difference between iterations
effect of relaxation factor is to reduce difference between iterations, so reduction of $\lambda$ must be accompanied by reduction in tolerance, to maintain same convergence accuracy

PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF METHOD:

## MAXIMUM NUMBER OF ITERATIONS <br> TOLERANCE <br> RELAXATION FACTOR

METHOD FAILS IF $G^{\prime}(\alpha)=\infty$
typically then iteration oscillates about correct solution, magnitude of oscillation decreasing as $\lambda$ approaches zero
but at $\lambda=0$ the iteration is turned off, so correct solution can never be found
to analyze the system, necessary to change definition of problem:
change order that parts are solved in loop, thereby changing definition of $x$ and $G$
or change physical model that is source of sensitivity of $G$ to $x$
example: circulation loop, for hover near zero thrust

## 4-18.3 Newton-Raphson Method

NEWTON-RAPHSON LOOP EXECUTES GENERAL SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS
used for trim loop of rotorcraft shell
AVAILABLE TO IMPLEMENT SOLUTION OF INVERSE TRIM PROBLEM, IDENTIFYING PARAMETER VALUES REQUIRED TO ACHIEVE SPECIFIED OPERATING CONDITION
figure 17 outlines the method

NEWTON-RAPHSON ALGORITHM SOLVES EQUATIONS OF FORM $M(v)-M_{\text {target }}=0$ FOR VARIABLES $v$ (ZERO POINT) quantities $M$ can be an input/output interface variable $f_{l}$ some mean system rigid force or moment $F$
force $F$ corresponds to system rigid degrees of freedom; target is probably zero
variables $v$ can be
some input variable $u$
a parameter in a wind, operating condition, or period system piece
typically used to define rigid motion of system
initialize
evaluate $M$ (solve parts in loop, child loops)
test convergence: error $=\left|M_{j}-M_{\text {target } j}\right| \leq$ tolerance $\times$ weight $_{j}$
initialize derivative matrix $D$ to input matrix; $P=0$
calculate gain matrix: $C=\lambda D^{-1}$
iteration
identify derivative matrix
optional perturbation identification
perturb each element of $v: \delta v_{i}=\Delta \times$ weight $_{i}$
evaluate $M$ (solve parts in loop, child loops)
calculate $D, P=1 / \delta v_{i}^{2}$
calculate gain matrix: $C=\lambda D^{-1}$
optional recursive identification
update $D, P$
calculate gain matrix: $C=\lambda D^{-1}$
increment controls: $\delta v=-C\left(M-M_{\text {target }}\right)$
evaluate $M$ (solve parts in loop, child loops)
test convergence: error $=\left|M_{j}-M_{\text {target } j}\right| \leq$ tolerance $\times$ weight $_{j}$

Figure 4-17 Outline of Newton Raphson method.

## THEORY

## ITERATION

Taylor series expansion of $f(x)=M-M_{\text {target }}=0$ produces iteration operator $F=x-f / f^{\prime}$ :

$$
x_{n+1}=x_{n}-\left[f^{\prime}\left(x_{n}\right)\right]^{-1} f\left(x_{n}\right)
$$

which gives quadratic convergence; modified NewtonRaphson iteration is $F=x-C f$ :

$$
x_{n+1}=x_{n}-C f\left(x_{n}\right)=x_{n}-\lambda D^{-1} f\left(x_{n}\right)
$$

where $\lambda$ is relaxation factor, and derivative matrix $D$ is estimate of $f^{\prime}$
behavior of iteration depends on accuracy of $f^{\prime}$; here analysis can evaluate directly $f$, but not $f^{\prime}$; necessary to evaluate $f^{\prime}$ by numerical perturbation of $f$, and for efficiency derivatives may not be evaluated for each $x_{n}$
these approximations compromise convergence of method, so relaxation factor is introduced to compensate
convergence criterion is

$$
\left|1-C f^{\prime}\right|=\left|1-\lambda D^{-1} f^{\prime}\right|<1
$$

so iteration converges if $\lambda<2 D / f^{\prime}$

## INITIAL VALUES

Newton-Raphson method has good convergence when $x$ is sufficiently close to the solution, but frequently has difficulty converging elsewhere
so initial estimate $x_{0}$ that starts iteration is important parameter affecting convergence

## CONVERGENCE TEST

convergence of $x$ tested in terms of required value (zero) for $f$ :

$$
\text { error }=\|f\| \leq \text { tolerance }
$$

typically error is absolute value of $f$

## DERIVATIVE MATRIX IDENTIFICATION

derivative matrix $D$ obtained by identification process, either perturbation or recursive
perturbation identification can be performed at beginning of the loop, or at the beginning and again every $M_{\text {PID }}$ iterations
or never, using an input derivative matrix recursive identification can be performed never, or at each iteration (except when the perturbation identification is performed)
perturbation identification restarts the recursive identification

## PERTURBATION IDENTIFICATION

$D$ calculated from one-step or two-step finite-difference expression; each element $x_{i}$ of $x$ is perturbed, one at a time, giving the $i$-th column of $D$ :

$$
\begin{aligned}
D= & {\left[\begin{array}{lll}
\cdots & \frac{\partial f}{\partial x_{i}} & \cdots
\end{array}\right]=\left[\begin{array}{ll}
\cdots & \frac{f\left(x_{i}+\delta x_{i}\right)-f\left(x_{i}\right)}{\delta x_{i}} \\
\cdots
\end{array}\right] } \\
& \text { accuracy of } D \text { (hence convergence) affected by } \\
& \text { magnitude and sign of perturbation }
\end{aligned}
$$

## RECURSIVE IDENTIFICATION

iteration makes change $\Delta x$, which then produces change $\Delta f$; these increments contain new information about $f^{\prime}(x)$, which can be used to update $D$
weighted least-squares solution for $D$ can be used with equal weight on changes from each iteration, identification eventually turns itself off, since latest change is worth less and less compared to complete set of changes
latest information should be most important
so exponential window method used: weight on change from iteration $k$ steps in past is $\alpha^{k}$, where $0<\alpha<1$
adding one measurement (with weight $1 / \alpha$ ) to current estimate of $D$ produces recursive identification algorithm algorithm uses error variance matrix $P$; if calculation of $P$ diverges, perturbation identification can be performed more often, to reinitialize $P$
or Broyden's formula can be used
updated derivative matrix satisfies $D_{n} \Delta x_{n}=\Delta f_{n}$

PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF METHOD:

MAXIMUM NUMBER OF ITERATIONS<br>TOLERANCE<br>RELAXATION FACTOR<br>INITIAL VALUES<br>PERTURBATION IDENTIFICATION<br>how often<br>difference order<br>magnitude and sign of control step<br>RECURSIVE IDENTIFICATION<br>method<br>exponential window weight

## 4-18.4 Regulator Method

## REGULATOR LOOP EXECUTES OPTIMIZATION OF SYSTEM SOLUTION

algorithm similar to (and developed from) equations of automatic control system, but as a trim loop it functions as optimizer rather than modelling actual controller
controller can be part of system, defined using available components
such a controller must be defined in order to analyze its behavior in transient or flutter task, and it could be defined also for trim task

## REGULATOR ALGORITHM USES A SELF-TUNING REGULATOR

 SOLUTION METHOD TO FIND THE CONTROL $\theta$ TO MINIMIZE COST FUNCTION $J(z, \theta)$, CALCULATED FROM MEASUREMENTS $z$ :self-tuning regulator is system that combines recursive parameter estimation with linear feedback; cost function is

$$
J=\left(z_{n}-z_{\text {target }}\right)^{T} W_{z}\left(z_{n}-z_{\text {target }}\right)+\Delta \theta_{n}^{T} W_{\Delta \theta} \Delta \theta_{n}
$$

where $W_{z}$ and $W_{\Delta \theta}$ are diagonal weighting matrices
weighting matrices must also account for differences in units among the elements of $z$ and $\theta$
last term in $J$ limits rate of change of $\theta$, improving stability without changing converged value
measurements $z$ can be an input/output interface variable $f_{l}$ control $\theta$ can be some input variable $u$

## PROCESS, THEORY, PARAMETERS SIMILAR TO NEWTONRAPHSON METHOD

change in $\theta$ required to minimize $J$ found assuming a local linearization of $z(\theta)$ :

$$
z_{n}=z_{n-1}+T\left(\theta_{n}-\theta_{n-1}\right)
$$

number of measurements $z$ can be greater than number of controls $\theta$

## 4-19 System Pieces: Trim Part

METHODS IMPLEMENTED FOR TRIM PARTS:
NO SOLUTION
IMPLICIT
equations solved in passes
single value, periodic, not periodic, multiperiod
STATIC
solve differential equations
HARMONIC, TIME FINITE ELEMENT
solve differential equations
one or more periods; option filter response

PART DESCRIBED BY IDENTIFYING ITS MOTION, CONSTRAINT, AND OUTPUT EQUATIONS

EACH EQUATION CAN BE IN ONLY ONE PART (POSSIBLY NO-SOLUTION METHOD)

EACH PART CAN BE SOLVED IN ONE OR MORE LOOPS

# POSSIBLE FOR COMPONENT TO EXECUTE SOLUTION PROCEDURE WHEN IT EVALUATES COMPONENT OUTPUT <br> IMPLICIT SOLUTION FOR SINGLE VALUE CAN BE USED 

NO SOLUTION METHOD
EQUATIONS ASSIGNED TO PART ARE NOT SOLVED
corresponding variables will be zero
SO SUPPRESSES EFFECT OF THESE SYSTEM VARIABLES DURING TRIM TASK

## 4-19.1 Filtered Solution

SYSTEM CAN HAVE SYMMETRIES THAT IN CERTAIN OPERATING CONDITIONS IMPLY RESTRICTIONS ON FORM OF SOLUTION

CERTAIN OF THESE RESTRICTIONS ARE ENFORCED WHEN RESPONSE IS EVALUATED BY PART SOLUTION
at end of time step or end of iteration; if permitted by response definition

FOR ROTATING, AXISYMMETRIC SYSTEM (TYPICALLY $N$ IDENTICAL, EQUALLY SPACED SUBSYSTEMS):

CERTAIN HARMONICS OF SOLUTION SHOULD BE ZERO
so trim filter can be defined for harmonic or time finite element solution method

## OTHER SYMMETRIES

SYMMETRY ABOUT A PLANE (SUCH AS LATERAL SYMMETRY)
typically asymmetric motions should be zero; implement using standard order reduction

RELATIONS BETWEEN SOLUTIONS FOR VARIOUS SUBSYSTEMS
enforce by defining trim response of one subsystem as child of another subsystem (specified in response system piece)

## TRIM CHILD SOLUTION

## DEGREES OF FREEDOM (INCLUDING MODES) CAN ALSO

 BE DESIGNATED CHILD DEGREES OF FREEDOM FOR THE TRIM PART SOLUTIONafter the equations are solved, child solution replaced in trim part by parent (or averaged parent) solution

## GIMBAL OR TEETER DEGREES OF FREEDOM

certain harmonics of solution set to zero as well

THIS OPTION IS AVAILABLE FOR SYSTEMS THAT MUST BE SOLVED AS A WHOLE, EVEN THOUGH SYMMETRIES IMPLY IDENTICAL (WITH PHASE SHIFTS) MOTION OF SUBSYSTEMS

## 4-19.2 Implicit Solution Method

IMPLICIT METHOD DIRECTLY SOLVES CONSTRAINT AND OUTPUT EQUATIONS

PART HAS COMPONENT EVALUATE REQUIRED
COMPONENT OUTPUT VECTOR, AT SET OF TIME VALUES
component will use component input at current time, and perhaps at past times as well
set of time values can be specified as single value, periodic, not periodic, or multiperiod
figure 18 outlines the method

ORDER IN WHICH EQUATIONS ARE SOLVED:
SPECIFIED BY LOCATION OF PART IN LOOPS, AND WITHIN PART BY AN ASSIGNED SOLUTION PASS
since each interface variable will affect other constraint equations, important that equations be solved in proper order

[^0]Figure 4-18 Outline of implicit solution method.

## TIME VALUES

## SINGLE VALUE

solved at one time; often solution independent of time PERIODIC
solved at $J$ times in revolution; time defined in terms of azimuth $\psi_{j}=\Omega t_{j}+\psi_{0}$, with frequency $\Omega$ obtained from some period of system

NOT PERIODIC
solved at $K+1$ times, from $t_{B}$ to $t_{E}$ MULTIPERIOD
solved at $K+1$ times, from $t_{B}$ to $t_{E}$; time defined in terms of azimuth $\psi_{j}=\Omega t_{j}+\psi_{0}$, with frequency $\Omega$ obtained from some period of system; and solved at $J$ azimuth steps per revolution

## 4-19.3 Differential Equations

HARMONIC, TIME FINITE ELEMENT, AND STATIC SOLUTION METHODS SOLVE DIFFERENTIAL EQUATIONS

## MATRICES OF DIFFERENTIAL EQUATIONS

response $=$ nominal/reference + difference
nominal/reference specified by input
reference can be updated from difference
part solves for difference

MOTION AND CONSTRAINT EQUATIONS LINEARIZED ABOUT NOMINAL/REFERENCE OF RESPONSE

FOR TRIM, MATRICES ARE CONSTANT
AVERAGED OVER PERIOD,
OR EVALUATED AT ONE TIME
OPTIONALLY REFERENCE CAN BE UPDATED (FROM CURRENT SOLUTION FOR DIFFERENCE) AND MATRICES RECALCULATED DURING SOLUTION PROCESS
an update requires additional computation, but may improve convergence
best update strategy, minimizing computation while ensuring convergence, will be problem dependent

## OPTIONS FOR MATRIX UPDATE STRATEGY:

iterations in part solution are counted from beginning of system solution, or from beginning of part
always references are updated and matrices calculated for first iteration (system or part count)
they may be updated for second iteration
thereafter they may be updated never, or every $N$ iterations
where update interval $N$ may be constant, increased by specified number after each update, or increased by factor of 2 after each update any modes used by part may or may not be recalculated at each update

MOST EFFICIENT TO CALCULATE MATRICES JUST ONCE

UPDATE FOR SECOND ITERATION MAY BE NECESSARY TO GET EFFECT OF INTERFACE FORCES, SUCH AS CENTRIFUGAL FORCE OF ROTOR BLADE

## THEORY

PART HAS COMPONENT EVALUATE NONLINEAR MOTION AND CONSTRAINT EQUATIONS

$$
0=\widetilde{A}
$$

PRODUCT OF MATRICES AND RESPONSE DIFFERENCE ADDED TO BOTH SIDES OF NONLINEAR EQUATIONS

$$
H \xi=(\widetilde{A}+H \xi)=A
$$

no approximation yet - equations are still exact
linearized part of equations needed to solve nonlinear equations
inaccuracies in matrices will affect convergence
part solution method may introduce approximation (such as quasistatic reduction), or treat two sides differently

## DIFFERENTIAL EQUATIONS

$$
\begin{gathered}
{\left[\begin{array}{ccc}
H & \phi_{f 1}^{T} & \phi_{f 0}^{T} \\
\phi_{x 1} & G_{1} & 0 \\
\phi_{x 0} & 0 & 0
\end{array}\right]\left(\begin{array}{c}
\xi \\
f_{1} \\
f_{0}
\end{array}\right)=\left(\begin{array}{c}
A \\
B_{1} \\
B_{0}
\end{array}\right)} \\
y=B_{y}
\end{gathered}
$$

constraint equations and interface variables:
$f_{0}=$ structural dynamic interfaces
$f_{1}=$ input/output interfaces
$H$ is differential operator (mass, damping, spring)

## CONSTRAINT EQUATIONS SOLVED TO ELIMINATE PART VARIABLES

structural dynamic constraint equations solved to eliminate degrees of freedom $\xi_{b}$
assume interfaces independent, and $\xi_{b}$ properly identified, so constraint equations are full rank
so number of degrees of freedom is reduced (and solution for interface forces is decoupled from equations of motion)
input/output constraint equations solved to eliminate interface variables $f_{1}$
does not reduce number of degrees of freedom degrees of freedom $\xi$ partitioned into $\xi_{b}$ and $\xi_{c}$
$\eta_{1}=\xi_{c}$ ARE DEGREES OF FREEDOM REMAINING AFTER CONSTRAINT EQUATIONS ARE SOLVED

typically $\xi_{b}$ are component rigid body degrees of freedom, plus elastic or joint degrees of freedom for multiple load paths

so $\eta_{1}$ are remaining elastic and joint degrees of freedom, plus system rigid degrees of freedom

## MODAL TRANSFORM

## PART CAN HAVE ONE OR MORE MODE SETS

modal transform obtained from mass and spring of structural dynamic subset of equations of part
constant modal matrix $\Phi$, transforming degrees of freedom $\eta_{1}$
$\Phi$ defines mode shapes in terms of degrees of freedom $\eta_{1}$, which probably do not measure total displacement of structure
modal frequency, mass, and output are calculated and printed (to identify modes), but only $\Phi$ is used in solution

TRANSFORM $\eta_{1}=\Phi \theta$, AND MULTIPLY EQUATIONS FOR $\eta_{1} \mathrm{BY} \Phi^{T}$
$\theta$ are modal degrees of freedom $\left(\theta=\eta_{1}\right.$ if there is no modal transform)

## ADD MODAL STRUCTURAL DAMPING

diagonal viscous damping matrix $D$ with elements calculated from modal mass and spring, and input structural damping factor $g(d=g \sqrt{m k}$, so $g$ is twice critical damping ratio)

MODAL TRANSFORM (WITHOUT TRUNCATION) DOES NOT INTRODUCE APPROXIMATION, AS LONG AS $\Phi$ IS COMPLETE MATRIX

MODAL TRANSFORM USED BECAUSE ALLOWS MODAL TRUNCATION AND/OR ADDITION OF MODAL DAMPING

## ORDER REDUCTION

order reduction specified for elements of degrees of freedom $\xi$, or for modal degrees of freedom
so partition $\theta$ :
dynamic $\theta_{k} \quad$ solve complete equations quasistatic $\theta_{l} \quad$ truncate dynamic terms
zero $\theta_{z} \quad$ truncate dynamic and static terms
zero $\theta_{z}$ handled by omitting or ignoring equations, and setting response to zero
quasistatic reduction implemented by neglecting mass and damping of $\theta_{l}$ equations, then introducing transformation that decouples $\theta_{l}$ and $\theta_{k}$ in the spring matrix

## REDUCED EQUATIONS

part differential equations take form

$$
\begin{aligned}
h_{k k} \theta_{k} & =\widetilde{A}_{k} \quad \text { (dynamic) } \\
K_{l l} \tilde{\theta}_{l} & =A_{l \mathrm{QS}} \quad \text { (static) }
\end{aligned}
$$

forces required are obtained from $A, B_{1}$, and $B_{0}$
these equations solved for degrees of freedom $\theta_{k}$ and $\tilde{\theta}_{l}$ then response evaluated for $\xi$, $f_{0}$, and $f_{1}$

## PART SOLUTION METHOD MUST SOLVE THESE REDUCED EQUATIONS

## 4-19.4 Harmonic Solution Method

HARMONIC SOLUTION METHOD SOLVES DIFFERENTIAL EQUATIONS FOR PERIODIC RESPONSE, IN TERMS OF A FOURIER SERIES

## PERIOD SYSTEM PIECE IDENTIFIED TO SPECIFY FUNDAMENTAL FREQUENCY

FOR TIME INVARIANT PART, SOLUTION CAN BE OBTAINED FOR MORE THAN ONE PERIOD
figure 19 outlines the method

## HARMONIC SOLUTION METHOD SOLVES CONSTANTCOEFFICIENT DIFFERENTIAL EQUATIONS FOR RESPONSE DIFFERENCE

ASSUMED THAT SOLUTION IS PERIODIC, SO RESPONSE CAN BE REPRESENTED BY FOURIER SERIES frequency domain enforces periodicity of solution and allows use of large time step, if solution adequately described by small number of harmonics

IF ACTUAL EQUATIONS ARE TIME-VARYING (PERIODIC), MATRICES MUST BE AVERAGED
by assuming time-invariant matrices, equations can be solved separately for each harmonic
all interharmonic coupling is in forces acting on equations
initialize: $F_{\text {old }}=0$
evaluate implicit constraint equations
iteration
save response: $x_{\text {old }}=x$
calculate matrices
for each time value (over period)
evaluate equations: $F$ relax equations: $F=\lambda F+(1-\lambda) F_{\text {old }}$ save equations: $F_{\text {old }}=F$ calculate and harmonically analyze forces of reduced eqns solve equations for degrees of freedom $\theta$ calculate and filter response: $x$
test convergence: error $=\left\|x-x_{\text {old }}\right\| \leq$ tolerance $\times$ weight output
for each time value (over period)
evaluate equations: $B_{y}$
harmonically analyze equations
calculate and filter response: y
implement order reduction: $\xi, f_{0}, f_{1}, y, f_{l}$

Figure 4-19 Outline of harmonic solution method.

## TRIM FILTER OPERATIONS

RESPONSE CAN BE FILTERED WHEN EVALUATED AT END OF TIME STEP

## OPTIONS

suppress all but $p N$ harmonics, where $N$ is specified fundamental harmonic, and $p$ an integer; that is, suppress all harmonics not a multiple of $N$
for rotorcraft airframe, $N=$ number of blades
suppress all but $p N \pm 1$ harmonics
for gimbal or teeter motion of hub (in rotating frame), $N=$ number of blades
used to enforce true symmetries of system

## CAN ALSO FILTER DEGREES OF FREEDOM WHEN EQUATIONS ARE SOLVED

used for approximate solution, such as simulated gimbal or teeter hinge model

SET HARMONICS TO ZERO AND IGNORE CORRESPONDING EQUATIONS WHILE EQUATIONS ARE BEING SOLVED

## THEORY

## REDUCED EQUATIONS OF MOTION

differential equations for $\theta_{k}$, and static equations for $\theta_{l}$ :

$$
\begin{aligned}
h_{k k} \theta_{k} & =\widetilde{A}_{k} \quad \text { (dynamic) } \\
K_{l l} \tilde{\theta}_{l} & =A_{l \mathrm{QS}} \quad \text { (static) }
\end{aligned}
$$

equations have form $H u=R$
$H=H(d / d t)=$ differential operator (spring, damping, and mass) for $\theta_{k}$
$H=K=$ static operator (spring) for $\theta_{l}$
in general, $R$ depends on time and on $u$ and its derivatives, and problem is nonlinear

## FOURIER SERIES

azimuth and time related by $\psi=\Omega t+\psi_{0}$, with frequency $\Omega$ from part solution period assume cosine-sine Fourier series representation of solution for $u$ :

$$
u=u_{0}+\sum_{n=1}^{N} u_{n c} \cos n \psi+u_{n s} \sin n \psi
$$

all solution variables of part ( $\xi, f_{1}, f_{0}$, and $y$ ) are represented by Fourier series

## FORCE

force $R_{j}=R\left(\psi_{j}\right)$ evaluated at $J$ azimuth steps per revolution
harmonics $R_{n}$ obtained from $R_{j}$ over one period interpolation options (factor $K_{n}$ in harmonic analysis)

Fourier interpolation
linear interpolation (default): harmonics of linear interpolation between force at discrete times

## DAMPING IN MATRICES

matrices of equations of motion might not include all important damping sources
to improve convergence, estimate of damping can be added to both sides of equation for $\theta_{k}$
diagonal damping matrix $D$, calculated from diagonal mass and spring of $h_{k k}: d=2 \zeta \sqrt{m k}$, where $\zeta$ is input critical damping coefficient
no approximation (not numerical damping), since algorithm treats both sides of equation same
will affect convergence but not final result equation for harmonics (complex form):

$$
H_{n} u_{n}=\left[H(i n)+i n K_{n} D\right] u_{n}=(R+D \dot{u})_{n}=R_{n}
$$

damping term added to $R$ before harmonic analysis performed
factor $K_{n}$ (from interpolation option) included so added damping treated exactly same on both sides

## ITERATION

by assuming Fourier series representation of solution, differential equations are converted to algebraic equations, to be solved for harmonics
equations take form $x=G(x)$, where $x$ is force $R_{j}$ over a period
so successive substitution method applicable each iteration of algorithm finds solution over period, hence has a time loop
at each time step over period, evaluate force $R_{j}$ with relaxation factor for convergence of nonlinear problems

$$
R_{j}=\lambda R_{j}+(1-\lambda) R_{\text {jold }}
$$

old force is from previous iteration; $R_{\text {old }}$ initialized to zero at beginning of system solution, so with $\lambda \neq 1$ the part solution has asymptotic convergence for constant force $R$
then damping term added to $R_{j}$, and harmonics $R_{n}$ evaluated
solution for degrees of freedom is $u_{n}=H_{n}^{-1} R_{n}$
at end of each iteration (end of each period), convergence is tested

## CONVERGENCE TEST

correct solution for $u$ is not known, so convergence tested by comparing two successive iterations:

$$
\text { error }=\left\|u-u_{\text {old }}\right\| \leq \text { tolerance }
$$

typically error is rms of difference between iterations

PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF METHOD:

MAXIMUM NUMBER OF ITERATIONS
TOLERANCE
RELAXATION FACTOR
DAMPING
NUMBER OF AZIMUTH STEPS PER REVOLUTION

NUMBER OF HARMONICS
MATRIX CALCULATION
perturbation order
magnitude and sign of perturbation
MATRIX UPDATE STRATEGY

## 4-19.5 Time Finite Element Solution Method

TIME FINITE ELEMENT SOLUTION METHOD SOLVES
DIFFERENTIAL EQUATIONS FOR PERIODIC RESPONSE, IN TERMS OF A FOURIER SERIES (AS SHAPE FUNCTIONS)
harmonics of equations of motion solved for harmonics of response, using Newton-Raphson algorithm

PERIOD SYSTEM PIECE IDENTIFIED TO SPECIFY FUNDAMENTAL FREQUENCY

FOR TIME INVARIANT PART, SOLUTION CAN BE OBTAINED FOR MORE THAN ONE PERIOD
figure 20 outlines the method

PROCESS SIMILAR TO HARMONIC METHOD IN MANY RESPECTS WITH NEWTON-RAPHSON ITERATION

TIME FINITE ELEMENT SOLUTION METHOD SOLVES CONSTANTCOEFFICIENT DIFFERENTIAL EQUATIONS FOR RESPONSE DIFFERENCE

ASSUMED THAT SOLUTION IS PERIODIC, SO RESPONSE CAN BE REPRESENTED BY FOURIER SERIES (AS SHAPE FUNCTIONS)

IF ACTUAL EQUATIONS ARE TIME-VARYING (PERIODIC), MATRICES MUST BE AVERAGED
initialize: $F_{\text {old }}=0$; evaluate $\widehat{R} ; C$ from $H, P=0$
evaluate implicit constraint equations
iteration
save response: $x_{\text {old }}=x$
calculate matrices
identify derivative matrix
optional perturbation identification
perturb each element of $u: \delta u_{i}=\Delta \times$ weight $_{i}$
evaluate $\widehat{R}$ (without relaxation of $F$ )
calculate $D, P=1 / \delta u_{i}^{2}$
calculate gain matrix: $C=\lambda_{N} D^{-1}$
optional recursive identification
update $P, C$
increment degrees of freedom: $\delta u=-C \widehat{R}$
evaluate $\widehat{R}$
calculate and filter response: $x$
for each time value (over period)
evaluate equations: $F$
relax equations: $F=\lambda F+(1-\lambda) F_{\text {old }}$
save equations: $F_{\text {old }}=F$
calculate and harmonically analyze forces of reduced eqns calculate equation residual $\widehat{R}$
test convergence: error $=\left\|x-x_{\text {old }}\right\| \leq$ tolerance $\times$ weight
output
for each time value (over period)
evaluate equations: $B_{y}$
harmonically analyze equations
calculate and filter response: $y$
implement order reduction: $\xi, f_{0}, f_{1}, y, f_{l}$

Figure 4-20 Outline of time finite element solution method.

## THEORY

## REDUCED EQUATIONS OF MOTION

differential equations for $\theta_{k}$, and static equations for $\theta_{l}$ :

$$
\begin{aligned}
h_{k k} \theta_{k} & =\widetilde{A}_{k} \quad \text { (dynamic) } \\
K_{l l} \tilde{\theta}_{l} & =A_{l \mathrm{QS}} \quad \text { (static) }
\end{aligned}
$$

equations have form $H u=R$ or $\widehat{R}=R-H u=0$

$$
H=H(d / d t)=\text { differential operator (spring, }
$$ damping, and mass) for $\theta_{k}$

$$
H=K=\text { static operator (spring) for } \theta_{l}
$$

## TIME FINITE ELEMENTS

Hamilton's equation for periodic structural dynamic system

$$
0=\delta \int_{0}^{T} L d t=\int_{0}^{T} \delta u^{T} \widehat{R} d t
$$

finite element method expands response as $u(t)=h(t)^{T} q$

$$
\begin{aligned}
& h=\text { shape functions in time } \\
& q=\text { finite element variables }
\end{aligned}
$$

equations to be solved for $q$ are then

$$
0=\int_{0}^{T} h^{T} \widehat{R} d t
$$

can divide period into $N_{t}$ elements, with polynomial shape functions over each element
disadvantage: many $q$ variables required even for motion that only contains few harmonics
so here use harmonics for shape functions
$q=$ coefficients in Fourier series for $u$
solve harmonics of equations of motion for $q$, using Newton-Raphson method

## FOURIER SERIES

azimuth and time related by $\psi=\Omega t+\psi_{0}$, with frequency $\Omega$ from part solution period
assume cosine-sine Fourier series representation of solution for $u$ :

$$
u=u_{0}+\sum_{n=1}^{N} u_{n c} \cos n \psi+u_{n s} \sin n \psi
$$

all solution variables of part ( $\xi, f_{1}, f_{0}$, and $y$ ) are represented by Fourier series

## FORCE

force $R_{j}=R\left(\psi_{j}\right)$ evaluated at $J$ azimuth steps per revolution
harmonics $R_{n}$ obtained from $R_{j}$ over one period interpolation options (factor $K_{n}$ in harmonic analysis)

Fourier interpolation
linear interpolation (default): harmonics of linear interpolation between force at discrete times
equation residuals (complex form):

$$
\widehat{R}_{n}=R_{n}-H_{n} u_{n}
$$

## NEWTON-RAPHSON ITERATION

by introducing time finite elements, differential equations are converted to algebraic equations, to be solved for harmonics
equations take form $f(x)=0$, where $x$ are harmonics of motion and $f$ are harmonics of residual equations
so Newton-Raphson method applicable
Newton-Raphson algorithm
$x_{m+1}=x_{m}-C f\left(x_{m}\right)=x_{m}-\lambda_{N} D^{-1} f\left(x_{m}\right)$
with relaxation factor $\lambda_{N}$ (defined separately from relaxation factor for forces)
and derivative matrix $D$ (estimate of $f^{\prime}$ )
derivative matrix $D$
initialize using matrix $H_{n}$ for cases without perturbation identification; include damping estimate in order to avoid singularities at resonant harmonics; gives iteration similar to harmonic method
optional perturbation identification at beginning of part or system solution; optionally repeated every $M_{\text {PID }}$ iterations
optional recursive identification exponential window or Broyden

## ITERATION

each iteration of algorithm finds solution over period, hence has a time loop
at each time step over period, evaluate force $R_{j}$ with relaxation factor for convergence of nonlinear problems

$$
R_{j}=\lambda R_{j}+(1-\lambda) R_{\text {jold }}
$$

then harmonics $R_{n}$ are evaluated, and residual equations $\widehat{R}_{n}$ calculated
solution for degrees of freedom updated using $\delta u=-C \widehat{R}$
at end of each iteration (end of each period), convergence is tested

## CONVERGENCE TEST

correct solution for $u$ is not known, so convergence tested by comparing two successive iterations:

$$
\text { error }=\left\|u-u_{\text {old }}\right\| \leq \text { tolerance }
$$

typically error is rms of difference between iterations

PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF METHOD:

MAXIMUM NUMBER OF ITERATIONS<br>TOLERANCE<br>RELAXATION FACTORS<br>DAMPING (FOR INITIAL DERIVATIVE MATRIX)<br>PERTURBATION IDENTIFICATION<br>how often<br>difference order<br>mag and sign of degree of freedom step<br>RECURSIVE IDENTIFICATION<br>method<br>exponential window weight<br>NUMBER OF AZIMUTH STEPS PER REVOLUTION<br>NUMBER OF HARMONICS<br>MATRIX CALCULATION<br>perturbation order<br>magnitude and sign of perturbation<br>MATRIX UPDATE STRATEGY

## 4-20 System Pieces: Transform

PROVIDES STANDARD MEANS TO DEFINE TRANSFORMATIONS OF SYSTEM VARIABLES AND EQUATIONS

DEGREES OF FREEDOM, MODES, OUTPUT VARIABLES, OR INPUT VARIABLES
for each transformed variable, can specify order reduction (zero, dynamic, or quasistatic)

IN TRIM AND TRANSIENT AND FLUTTER TASKS

TRANSFORMS IMPLEMENTED:
MULTIBLADE COORDINATES
ROTATING-TO-NONROTATING
SYMMETRIC/ANTISYMMETRIC

## MULTIBLADE COORDINATE TRANSFORM

INTENDED FOR ROTATING SYSTEMS WITH AXISYMMETRIC GEOMETRY BUT ARBITRARY MOTION
most useful for configurations with $N$ equally-spaced rotating subsystems
used for rotor blades
can be applied to configuration with blades that are not equally-spaced or not identical; if blade-to-blade differences are small, still expect to obtain benefits of transform

ELIMINATES PERIODIC COEFFICIENTS $(N \geq 3)$
FROM COUPLING OF ROTATING AND NONROTATING SYSTEM
for coupled rotor and airframe motions BY AVERAGING WEAKLY PERIODIC SYSTEM
for aerodynamics of edgewise-moving rotor
SO HAVE TIME-INVARIANT EQUATIONS FOR FLUTTER AND TRANSIENT ANALYSES

ELIMINATES 1/REV BLADE MOTION IN FLIGHT DYNAMICS PROBLEMS
transformed (nonrotating) degrees of freedom vary more slowly than rotating degrees of freedom

SO CAN PERHAPS USE LARGER TIME STEP IN TRANSIENT ANALYSIS

## THEORY

rotating system with $N$ equally spaced, identical blades
$m$-th blade at azimuth $\psi_{m}=\psi+m \Delta \psi, \Delta \psi=2 \pi / N$ for $m=1$ to $N$
$\psi=\Omega t+\psi_{0}$ is prescribed, reference azimuth
vector of rotating frame variables (length $N$, one for each blade): $x_{\text {rot }}=\left(x_{m}\right)$
vector of corresponding nonrotating variables or multiblade coordinates (also length $N$ ):

$$
x_{\mathrm{non}}=\left(\begin{array}{c}
x_{0} \\
x_{n c} \\
x_{n s} \\
x_{N / 2}
\end{array}\right)
$$

with $n=1$ to $(N-1) / 2$ for $N$ odd;
or $n=1$ to $(N-2) / 2$ for $N$ even
$x_{0}$ is collective coordinate;
$x_{1 c}$ and $x_{1 s}$ are cyclic coordinates
transformation: $x_{\text {rot }}=T x_{\text {non }}$
$m$-th row of $T$ is

$$
\left.\begin{array}{l}
t_{m}=\left(\begin{array}{lll}
1 & \cos k \psi_{m} & \sin k \psi_{m}
\end{array}-1^{m}\right.
\end{array}\right) .
$$

## ROTATING-TO-NONROTATING TRANSFORM

 INTENDED FOR VARIABLES SUCH AS DEGREES OF FREEDOM OF ROTATING-FRAME GIMBAL OR UNIVERSAL JOINT
## THEORY

rotating frame variables $x_{\text {rot }}=\left(\begin{array}{ll}x_{A} & x_{B}\end{array}\right)^{T}$ nonrotating frame variables $x_{\text {non }}=\left(\begin{array}{ll}x_{c} & x_{s}\end{array}\right)^{T}$
typically motion or force variables about two perpendicular axes in plane of rotation

$$
\psi=\Omega t+\psi_{0} \text { is prescribed, reference azimuth }
$$

transformation: $x_{\text {rot }}=T x_{\text {non }}$

$$
\binom{x_{A}}{x_{B}}=\left[\begin{array}{rr}
\cos \psi & \sin \psi \\
-\sin \psi & \cos \psi
\end{array}\right]\binom{x_{c}}{x_{s}}
$$

## SYMMETRIC/ANTISYMMETRIC TRANSFORM

 INTENDED FOR SYSTEMS WITH PLANE OF SYMMETRY ONLY USED IN FLUTTER TASK analysis of flutter equations can be performed separately for symmetric and antisymmetric subsets of the system used for tilting proprotor aircraft
## THEORY

right and left variables (lateral symmetry): $x=\left(\begin{array}{ll}x_{R} & x_{L}\end{array}\right)^{T}$ symmetric and antisymmetric variables: $y=\left(\begin{array}{ll}y_{S} & y_{A}\end{array}\right)^{T}$ transformation: $x=T y$

$$
\binom{x_{R}}{x_{L}}=\left[\begin{array}{rr}
1 & 1 \\
1 & -1
\end{array}\right]\binom{y_{S}}{y_{A}}
$$

## 4-21 System Pieces: Modes

## PROVIDES STANDARD MEANS TO DEFINE MODAL TRANSFORMATION OF STRUCTURAL DYNAMIC DEGREES OF FREEDOM

## FOR USE BY PART SOLUTION PROCEDURE

constant modal matrix $\Phi$ to transform reduced degrees of freedom and equations
can specify order reduction for each modal variable (zero, dynamic, or quasistatic)

IN TRIM, TRANSIENT, AND FLUTTER TASKS

## MODAL TRANSFORMATION

MODAL TRANSFORMATION FOLLOWED BY MODAL
TRUNCATION CAN MAKE ANALYSIS MORE EFFICIENT (IF ACCURACY IS RETAINED)

BY REDUCING NUMBER OF DEGREES OF FREEDOM INVOLVED IN DYNAMICS
modal transformation also allows introduction of modal damping

## FOR LINEARIZED PROBLEM (FLUTTER TASK)

modal truncation reduces size of system, and helps interpretation of results

FOR NONLINEAR PROBLEM (TRIM AND TRANSIENT TASKS)
still must evaluate all equations and forces of components and interfaces, so size of problem not reduced
transform does restrict states to just those excited by environment

MODAL TRANSFORMATION HELPS
CONVERGENCE OF TRIM TASK WITH ELASTIC ROTOR BLADE MODEL

## FOR EACH NEW PROBLEM, MUST DETERMINE NUMBER OF MODES REQUIRED FOR ACCURATE SOLUTION

if truncation does not improve efficiency or convergence, and do not need modal damping, no reason to use modes

## ANALYSIS PRINTS MODAL FREQUENCIES AND MASSES, AND MODE SHAPES FROM OUTPUT VARIABLES

used to identify modes
none of printed information is used in part solution
part solution only uses modal matrix $\Phi$ (mode shapes for part degrees of freedom, not for output variables)
figure 21 illustrates part degrees of freedom and output variables in mode set

typical beam component shown

Figure 4-21 Part degrees of freedom and output variables in mode set.

## MODE SETS

## TRIM, TRANSIENT, OR FLUTTER PART CAN USE ONE OR MORE MODE SETS

## IDENTIFY MOTION, CONSTRAINT, AND OUTPUT EQUATIONS

motion and constraint equations from only structural dynamic components and interfaces, subset of part equations
linearized about current reference solution
output equations only used to normalize and print mode shapes

## AS IMPLEMENTED, MODES MUST BE FOR CONSTRAINED SUBSYSTEM

so mode set must include constraint equations that eliminate any rigid degrees of freedom of subsystem

NORMALIZATION
MODES ARE ORDERED BY FREQUENCY
OUTPUT VARIABLES $y_{q}$ OF MODE SET USED TO NORMALIZE AND PRINT MODES SHAPE
typically $y_{q}$ includes displacement at key points of subsystem

## 4-22 System Pieces: Response

## PROVIDES STANDARD MEANS TO DEFINE CHARACTERISTICS OF SYSTEM VARIABLES

## RESPONSE REQUIRED FOR

COMPONENT DEGREES OF FREEDOM $\xi$
FRAMES $\beta$
INTERFACE VARIABLES $f_{l}$
INPUT VARIABLES $u_{m}$
OUTPUT VARIABLES $y_{q}$
defined by vector

RESPONSE SPECIFICATION INCLUDES:
NOMINAL, REFERENCE, AND INITIAL DIFFERENCE
including rest position, rotating frame, operating condition
REPRESENTATION OF RIGID MOTION
CHILD RESPONSE IN TRIM
WEIGHTS
convergence test, perturbation, mode normalization
ORDER REDUCTION
zero, dynamic, or quasistatic
for trim, transient, and flutter tasks
(superseded by mode or transform specification)
SYMMETRY (FOR FLUTTER TASK)
symmetric, antisymmetric, or both

## RESPONSE TYPES: RIGID AND VARIABLE <br> CONVENTIONS FOR DESCRIPTION OF MOTION

## 4-22.1 Response Representation

## FORM OF REPRESENTATION OF EACH SYSTEM VARIABLE:

total $=$ reference + difference

CONSIDERING RIGID MOTION VARIABLES, ALSO INTRODUCE NOMINAL:
total $=$ nominal + (reference + difference $)$
sum of reference and difference always scalar
sum of nominal and degree of freedom may involve kinematics of rigid motion

RIGID RESPONSE REPRESENTATION REQUIRED FOR RIGID BODY MOTION OF COMPONENTS, AND FOR FRAMES

RIGID MOTION IS MEASURED RELATIVE TO FRAME, AND FRAMES ARE HIERARCHICAL

COMPLETE REPRESENTATION OF MOTION RELATIVE SOME PARENT FRAME (ULTIMATELY, INERTIAL SPACE):

$$
\text { motion }=\text { frame }+ \text { frame }+\ldots+\text { total }
$$

each frame and total has separate rigid response definition

## NOMINAL, REFERENCE, AND DIFFERENCE

SOLUTION PROCEDURES SOLVE FOR DIFFERENCE
FIND MOTION RELATIVE TO SOME PRESCRIBED NOMINAL AND REFERENCE

CHOOSE NOMINAL AND REFERENCE TO HELP SOLUTION PROCESS

## USES OF NOMINAL AND REFERENCE:

to keep difference small, by accounting for rest position and any large rotation or displacement
analysis does not assume that difference is small, but many solution procedures work better if it is
to keep difference periodic, so solution procedure need be applied only over one revolution
to avoid singularities in representation of rotations to introduce specified operating condition

## DISTINGUISH BETWEEN NOMINAL AND REFERENCE BECAUSE CAN BE DIFFERENT FORMS OF MOTION

reference required since it can be updated with latest solution for difference (because sum is scalar)
nominal can be needed to account for large rigid motion, or steady state velocity or rotation, or so the difference will be constant or periodic

## USER (OR ROTORCRAFT SHELL) SELECTS NOMINAL AND REFERENCE KIND

available conventions for nominal and reference must be appropriate for problems to be handled by analysis

DIFFERENCE KIND AND OTHER PROPERTIES OF RESPONSE FOLLOW FROM CHOICE OF PART SOLUTION PROCEDURE from trim part solution, difference can be
single value, periodic, not periodic, or multiperiod time domain or harmonics one period, or primary and secondary periods with part filter active or not
from transient part solution, difference can be time domain structured for implicit or integration solution possibly just trim response

## UPDATED REFERENCE

PART SOLUTIONS MAY LINEARIZE SYSTEM EQUATIONS ABOUT NOMINAL AND REFERENCE

CONVERGENCE MAY BE IMPROVED IF REFERENCE IS UPDATED FROM CURRENT SOLUTION FOR DIFFERENCE, AND MATRICES RECALCULATED (MODES PROBABLY RECALCULATED ALSO)

UPDATE CONVENTIONS IMPLEMENTED:
for trim task:
trim reference updated as required by part solution procedure, by adding mean part of current difference value
reference of all response variables updated at end of trim task
for transient task:
updated trim reference is reference in transient task equations can be linearized about current transient solution
for flutter task:
equations are usually linearized about total trim solution (nominal, reference, and difference)

## 4-22.2 Response Evaluation

TRIM RESPONSE MUST ACCOMMODATE SYSTEM WITH MORE than one period

TRIM PARTS (SUCH AS HARMONIC SOLUTION METHOD) CAN SOLVE EQUATIONS ASSUMING THAT SYSTEM HAS KNOWN PERIOD, SO EQUILIBRIUM SOLUTION IS KNOWN TO BE PERIODIC

IF DIFFERENT SUBSYSTEMS HAVE DIFFERENT PERIODS, THEN APPROXIMATIONS ARE REQUIRED TO ENFORCE ASSUMPTION OF PERIODICITY WITHIN PART SOLUTION
produces approximate solution, for reasonable effort can use transient task to get exact solution SO DIFFERENTIAL EQUATIONS OF TRIM PART IDENTIFIED AS TIME INVARIANT OR PERIODIC

## PERIODIC PART (ROTOR)

CAN BE SOLVED ONLY FOR RESPONSE AT ITS OWN PERIOD
uses component input $=$ mean value + vibratory motion at same period

TIME INVARIANT PART (AIRFRAME)
CAN BE SOLVED FOR RESPONSE AT SEVERAL PERIODS separate solution for each period acting on part (primary period, and optionally one or more secondary periods)
uses component input = mean value (primary period only) + vibratory motion at each period

## 4-22.3 Child Response in Trim

SYSTEM CAN HAVE SYMMETRIES THAT IN CERTAIN OPERATING CONDITIONS IMPLY RELATION BETWEEN SOLUTIONS FOR VARIOUS SUBSYSTEMS

CAN BE ENFORCED BY DEFINING RESPONSE OF ONE SUBSYSTEM AS CHILD OF ANOTHER SUBSYSTEM

ONLY NECESSARY TO SOLVE FOR PARENT RESPONSE, SO CAN SIGNIFICANTLY REDUCE COMPUTATION REQUIRED
child response used for blades of rotor
SYMMETRIES CAN ALSO IMPLY RESTRICTIONS ON FORM OF PART SOLUTION
can be enforced at end of part solution, by order reduction and filter operations

## SYMMETRIES

AXISYMMETRIC SYSTEM (TYPICALLY $N$ IDENTICAL, EQUALLY SPACED SUBSYSTEMS, PERHAPS ROTATING)
motion can be identical except for phase shift between child and parent
typically also have certain harmonics of solution equal zero (trim filter)

SYMMETRY ABOUT PLANE (SUCH AS LATERAL SYMMETRY)
motion can be mirrored across plane, hence identical except for sign changes between child and parent
also antisymmetric motions should be zero (order reduction)

## TRIM RESPONSE CAN BE IDENTIFIED AS CHILD OF SOME OTHER RESPONSE

## NO TRIM SOLUTION FOR CHILD

equations in no-solution type part, or not in any loop
IDENTIFY PARENT
not hierarchical

## SPECIFY PHASE SHIFT

parent can not have secondary periods
SPECIFY SIGN CHANGE

## RESPONSE EVALUATION

CHILD RESPONSE AT TIME $t$ EVALUATED FROM PARENT RESPONSE (DIFFERENCE, REFERENCE, AND NOMINAL) AT TIME $t_{e}$
with phase shift, $t_{e}=t+\Delta t$
$\Delta t=\Delta \psi / \Omega$ for periodic or multiperiod response

## PERHAPS WITH SIGN CHANGES

for rigid response type: sign of angular motion about $x$, $y$, or $z$-axis is changed; and signs of linear motion about other two axes are changed

## 4-22.4 Response Specification

RESPONSE TYPES: RIGID AND VARIABLE<br>REFERENCE FOR RESPONSE CONVENTIONS: CAMRAD II Documentation, Volume I, Theory; Chapter "Response"

## VARIABLE RESPONSE TYPE

CONVENTIONS FOR NOMINAL AND REFERENCE:
ROTATING VARIABLE (depends on a period) REST POSITION
NONE

ROTATING VARIABLE
NOMINAL MOTION IS CONSTANT ANGULAR VELOCITY FOR SCALAR QUANTITY (NOT RIGID BODY ROTATION)

REST POSITION
NOMINAL MOTION IS CONSTANT DISPLACEMENT

NONE
NO NOMINAL OR REFERENCE; DIFFERENCE IS TOTAL RESPONSE
intended to provide efficient storage and access, for quantities that do not require nominal or reference

## RIGID RESPONSE TYPE

## CONVENTIONS FOR NOMINAL AND REFERENCE:

BASE FRAME (depends on an operating condition)
ROTATING FRAME (depends on a period)
REST POSITION

BASE FRAME NOMINAL/REFERENCE
FOR RESPONSE OF A "BASE FRAME" (PARENT FRAME IS INERTIAL)
defined in terms of operating condition (which can be changed by solution procedures), and can provide constant-velocity motion
motion of system body axes F, relative to inertial axes I
operating condition defines nominal and reference, which is mean motion of $F$ relative $I$
difference contains remainder of motion, found by the part solution (with zero mean for trim task)

## OPTIONS:

free, body axes: no nominal, so degrees of freedom represent total motion of $F$ relative I
body-axes velocity or inertial-axes displacement and aircraft Euler angles
free, stability axes: degrees of freedom represent motion of velocity axes relative inertial axes (V relative I); so nominal is constant orientation of $F$ relative V body-axes velocity or inertial-axes displacement and aircraft Euler angles
constrained: no nominal, so degrees of freedom represent total motion of $F$ relative I
constant reference linear motion
and aircraft Euler angles, Rodrigues parameters, or arbitrary Euler angles

## ROTATING FRAME

NOMINAL MOTION IS ROTATION BY ANGLE $\psi=\Omega t+\psi_{0}$ ABOUT DESIGNATED AXIS

## REST POSITION

NOMINAL MOTION IS CONSTANT DISPLACEMENT
AND ROTATION

## REPRESENTATION OF RIGID MOTION

FOR RIGID RESPONSE TYPE, DEGREE OF FREEDOM IS RIGID MOTION:
linear motion $x^{B A / A}$ ( B relative A , in A axes), and derivatives
angular motion $C^{B A}$, and derivatives
linear motion only, or both linear and angular motion
constrained component has no rigid motion; point mass component needs only linear motion; other components will have both linear and angular motion

NEED SCALAR, THREE-PARAMETER REPRESENTATION OF LINEAR MOTION $q$ AND ANGULAR MOTION $p$

## REPRESENTATION OF LINEAR MOTION

inertial axes displacement
$q=x^{B A / A}=$ displacement measured in A axes inertial acceleration $=\dot{v}+\tilde{\omega} v=C^{B A} \ddot{q}$
body axes displacement
$q=x^{B A / B}=$ displacement measured in B axes inertial acceleration $=\dot{v}+\tilde{\omega} v=\ddot{q}+2 \widetilde{\omega} \dot{q}+\tilde{\omega} q+\tilde{\omega} \tilde{\omega} q$
body axes velocity
$\dot{q}=v^{B A / B}=$ velocity measured in B axes
inertial acceleration $=\dot{v}+\widetilde{\omega} v=\ddot{q}+\widetilde{\omega} \dot{q}$

## BODY AXES VELOCITY REPRESENTATION OF LINEAR MOTION

 CAN ONLY BE USED WITH BASE FRAME NOMINAL KIND used for airframe framedifficulties arise because displacement $x$ not directly obtained from $q$, but rather by integral of $\dot{q}$ (which will be path dependent)

$$
\begin{aligned}
& \dot{q}=v^{B A / B}=C^{B A} \dot{x} \dot{x}^{B A / A} \\
& \text { so } x^{B A / A}=\int^{t} C^{A B} \dot{q} d t
\end{aligned}
$$

## COMPLETE EVALUATION OF DISPLACEMENT IS NOT

 IMPLEMENTEDdo not integrate total $\dot{q}$ to obtain $x$
for trim or transient, contribution of reference calculated, but difference neglected
for flutter, approximate perturbation for small trim velocity

SO BODY AXES VELOCITY REPRESENTATION OF LINEAR MOTION CAN NOT BE USED IF DISPLACEMENT RELATIVE INERTIAL FRAME IS IMPORTANT
use inertial axes displacement instead
correct, but unconventional representation for aircraft rigid body motion

## REPRESENTATION OF ANGULAR MOTION

 aircraft Euler angles (Tait-Bryan)$$
\begin{aligned}
& p=(\phi \theta \psi)^{T}=\text { rotation angles about } x, y, \text { and } z \text { axes } \\
& \text { rotation }=C^{B A}=X_{\phi} Y_{\theta} Z_{\psi}
\end{aligned}
$$

Rodrigues parameters (Euler-Rodrigues)

$$
\begin{aligned}
& p=2 u \tan (\psi / 2)=\text { rotation by angle } \psi \text { about axis } u \\
& \text { rotation }=C^{B A}=I-R \tilde{p}
\end{aligned}
$$

arbitrary Euler angles

$$
\begin{aligned}
& p=(\phi \theta \psi)^{T}=\text { rotation angles about } u, v \text {, and } w \text { axes } \\
& \quad u, v \text {, and } w=\text { positive or negative } x, y \text {, or } z \text { axis } \\
& \text { rotation }=C^{B A}=U_{\phi} V_{\theta} W_{\psi}
\end{aligned}
$$

## 4-22.5 Convergence Test

ITERATIVE SOLUTION METHOD FOR PART OR LOOP MAY REQUIRE THAT CONVERGENCE BE TESTED

BY COMPARING CURRENT VALUE $x$ OF VARIABLE WITH
OLD VALUE $x_{\text {old }}$ FROM PREVIOUS ITERATION

## GENERAL FORM OF CONVERGENCE TEST:

$$
\text { error }=\left\|x-x_{\text {old }}\right\| \leq \text { tolerance } \times \text { weight }
$$

tolerance is single number, from part or loop data
weighting factor ("weight") is defined for each element of vector, as part of response data
error is norm of difference between current and old solution for single value: error $=\left|x-x_{\text {old }}\right|$ for other difference kind: error $=\operatorname{rms}\left(x-x_{\text {old }}\right)$

RELATIVE VALUES OF CONVERGENCE WEIGHTS AFFECT WHAT VARIABLE SEEMS TO DRIVE CONVERGENCE OF LOOP OR PART hence affect interpretation of tolerance value of tolerance required for accurate solution is based on physical results

## 4-22.6 Perturbation

PART OR LOOP SOLUTION METHOD MAY REQUIRE THAT VARIABLE $x$ BE PERTURBED

TYPICALLY IN ORDER TO LINEARIZE SOME SUBSET OF SYSTEM

## GENERAL FORM OF PERTURBATION:

$$
\delta x=\Delta \times \text { weight }
$$

perturbation amplitude $\Delta$ is from part or loop data weighting factor ("weight") is defined for each element of vector, as part of response data
separate values of $\Delta$ are used for displacement, velocity, and acceleration perturbations

## VARIABLE PERTURBATION AFFECTS ACCURACY OF MATRICES CALCULATED BY FINITE DIFFERENCE APPROXIMATION

relative values of perturbation weights must be chosen so all perturbations have similar behavior (all too small or all too large) with a common perturbation amplitude

## 4-23 System Pieces: Weights

## PROVIDES STANDARD PERTURBATION AND CONVERGENCE

 WEIGHTS FOR RESPONSESTANDARD WEIGHTS CALCULATED FROM REFERENCE QUANTITIES:
length $L$, angular velocity $\Omega$, linear velocity $V$, force $F$

ITERATIVE PART OR LOOP SOLUTION METHODS:
MAY REQUIRE THAT CONVERGENCE BE TESTED MAY REQUIRE THAT VARIABLE BE PERTURBED
analysis automatically identifies which variables a loop or part must perturb and test

FOR EACH RESPONSE SYSTEM PIECE, NEED TO SPECIFY WEIGHTS THAT CAN BE USED TO PERTURB THE VARIABLE AND TO TEST ITS CONVERGENCE

OTHER SYSTEM PIECES CAN ALSO REQUIRE WEIGHTS

## WEIGHT USED IS PRODUCT OF STANDARD WEIGHT AND INPUT FACTOR

typically factor is 1 ., but available to handle special cases
IF WEIGHTS SYSTEM PIECE IS NOT IDENTIFIED FOR A VARIABLE, THEN INPUT VALUE OF WEIGHT IS USED

VARIABLE KIND MUST BE IDENTIFIED IN ORDER TO OBTAIN APPROPRIATE STANDARD WEIGHT

| variable kind | standard weight |
| :--- | :--- |
| basic | 1 |
| unit | 1 |
| angular displacement (deg) | $1 / 57.3$ |
| angular displacement (rad) | $\Omega / 100$ |
| angular velocity | $\Omega^{2} / 100$ |
| angular acceleration | $L / 100$ |
| linear displacement | $V / 100$ |
| linear velocity | $\Omega V / 100$ |
| linear acceleration | $F / 100$ |
| force | $L F / 1000$ |
| moment | $V F / 1000$ |
| power | $L V / 10000$ |
| circulation | $\left(50 F / L^{2}\right) / 100$ |
| dynamic pressure | $v, q, d v / d t, \omega$ |
| aerodynamic interfaces of structural $d y n a m i c ~ c o m p o n e n t s ~$ |  |
| velocity (10 elements) | $r, d r / d t$ |
| position (6 elements) | force, moment |
| force (6 elements) |  |
| aerodynamic | magnitude and position |
| circulation peaks |  |

## 4-24 Transient Task

NUMERICALLY INTEGRATE SYSTEM EQUATIONS, FROM TRIM SOLUTION, FOR PRESCRIBED EXCITATION

TRANSIENT SOLUTION BEGINS AT TIME $t_{B}$, AND ENDS AT TIME $t_{E}$ EACH PART SOLVES ITS EQUATIONS OVER $t_{B}$ to $t_{E}$, USING RESPONSE TIME STEP $\Delta t$
solution at this time step saved, and thus available to other parts
part solution procedure may use smaller time increment $\delta t$, because of accuracy or convergence considerations
save response at step size that reflects content of solution

RESPONSE HAS TRIM VALUE AT $t_{B}$ and at earlier times, if required

## RESTART CAPABILITY FOR TRANSIENT TASK TRANSIENT SOLUTION CAN BE INTERRUPTED

at time $t_{X}$ in numerical integration part
or at specified iteration and level in successive substitution loop

## AT THE INTERRUPTION:

complete solution saved to restart file;
transient output produced, using solution obtained so far; transient task is terminated

## RESTARTED JOB:

jump to interrupt point in specified part or loop;
restore solution by reading restart file;
transient task continues
results same as if transient task completed in single job
possible to interrupt again a restarted transient task

## PURPOSE OF RESTART CAPABILITY IS TO ALLOW PROGRESS OF TRANSIENT TASK TO BE ASSESSED BY EXAMINATION AT INTERMEDIATE POINT IN SOLUTION

none of the analysis input can be changed for restarted job (except specification of next interrupt)
transient analysis is completely specified by initial job

## 4-24.1 Rotorcraft Transient Task

## TRANSIENT SOLUTION PROCEDURE CONSTRUCTED BY ROTORCRAFT SHELL WITH NO LOOPS

solve transient problem all in one part
no solution loop ROTORCRAFT integration part ROTORCRAFT

## 4-25 System Pieces: Transient Loop

METHODS IMPLEMENTED FOR TRANSIENT LOOPS:
NO SOLUTION
SUCCESSIVE SUBSTITUTION (with stages)

LOOP DESCRIBED BY IDENTIFYING PARTS, CHILD LOOP, AND WRITE MODULES

PARTS SOLVED IN LOOP (ITERATED WITH CHILD LOOPS), OR AT END OF LOOP
parts are solved in order defined

## LOOPS ARE HIERARCHICAL

loop can have at most one child and one parent
transient task can have more than one set of nested loops
WRITE MODULES PRODUCE OUTPUT, FOR:
SHELL
CONVERGENCE LOOP SOLUTION
PART SOLUTION
MODES
OUTPUT OF PART
OUTPUT
GRAPHICS

LOOP ALGORITHM, THEORY, PARAMETERS SIMILAR TO TRIM LOOPS

## 4-26 System Pieces: Transient Part

METHODS IMPLEMENTED FOR TRANSIENT PARTS:
NO SOLUTION
TRIM SOLUTION
IMPLICIT
equations solved in passes
INTEGRATION
solve differential equations
numerical integration algorithms

PART DESCRIBED BY IDENTIFYING ITS MOTION, CONSTRAINT, AND OUTPUT EQUATIONS

EACH EQUATION CAN BE IN ONLY ONE PART (POSSIBLY NO-SOLUTION METHOD)

EACH PART CAN BE SOLVED IN ONE OR MORE LOOPS

## EACH PART SOLVES ITS EQUATIONS OVER TRANSIENT TIME

 RANGE $t_{B}$ to $t_{E}$, USING RESPONSE TIME STEP $\Delta t$solution at this time step saved, and thus available to other parts different parts can use different time steps

PART THAT PERFORMS NUMERICAL INTEGRATION USES TIME INCREMENT $\delta t$

ACCURACY OR CONVERGENCE CONSIDERATIONS DETERMINE SIZE OF $\delta t$

WHILE CONTENT OF SOLUTION DETERMINES SIZE OF $\Delta t$ step $\Delta t$ must be multiple of $\delta t$

## NO SOLUTION METHOD

EQUATIONS ASSIGNED TO PART ARE NOT SOLVED
corresponding variables will be zero
SO SUPPRESSES EFFECT OF THESE SYSTEM VARIABLES DURING TRANSIENT TASK

## TRIM SOLUTION METHOD

EQUATIONS ASSIGNED TO PART ARE NOT SOLVED
corresponding variables will have value from trim solution

## 4-26.1 Implicit Solution Method

## IMPLICIT METHOD DIRECTLY SOLVES CONSTRAINT AND OUTPUT EQUATIONS

## PROCESS AND THEORY SIMILAR TO TRIM PART

## TIME VALUES

response time steps
solved at $K$ times, from $t_{B}+\Delta t$ to $t_{E}$

## 4-26.2 Differential Equations

INTEGRATION SOLUTION METHOD SOLVES DIFFERENTIAL EQUATIONS

PROCESS AND THEORY SIMILAR TO TRIM PART

## TRANSFORM AND REDUCTION

CAN USE MODAL TRANSFORM FOR STRUCTURAL DYNAMIC DEGREES OF FREEDOM OF PART DEGREES OF FREEDOM OR MODES CAN BE TRANSFORMED
multiblade coordinates
rotating-to-nonrotating
SOLUTION FOR SELECTED VARIABLES CAN BE QUASISTATIC, WITH OR WITHOUT A RESIDUAL

POSSIBLE TO OMIT SELECTED VARIABLES FROM SOLUTION

## MATRICES OF DIFFERENTIAL EQUATIONS

MOTION AND CONSTRAINT EQUATIONS LINEARIZED ABOUT CURRENT TRANSIENT SOLUTION

EQUATIONS CAN BE TIME INVARIANT OR TIME-VARYING
optionally, matrices can be averaged over period
OPTIONALLY REFERENCE CAN BE UPDATED (FROM CURRENT SOLUTION FOR DIFFERENCE) AND MATRICES RECALCULATED DURING SOLUTION PROCESS
matrices that depend on time-varying transform
(multiblade coordinates or rotating-to-nonrotating) are always updated at each integration step

IF EQUATIONS ARE NONLINEAR, MIGHT RECALCULATE MATRICES WHENEVER REFERENCE CHANGES

IF EQUATIONS ARE TIME-VARYING, MIGHT RECALCULATE MATRICES EACH TIME STEP

GENERALLY MORE EFFICIENT TO EXECUTE UPDATE LESS OFTEN

## 4-26.3 Integration Solution Method

## INTEGRATION METHOD SOLVES DIFFERENTIAL EQUATIONS FOR TRANSIENT RESPONSE, BY NUMERICAL INTEGRATION

 INTEGRATION ALGORITHMS:linear Newmark nonlinear Newmark linear Wilson nonlinear Wilson linear HHT nonlinear HHT
"linear" and "nonlinear" refer to form of algorithm, each can be applied to linear or nonlinear problem

## LINEAR NEWMARK ALGORITHM IS GOOD CHOICE

figure 22 outlines the method

## AT EACH TIME STEP AN ITERATION IS REQUIRED FOR NONLINEAR PROBLEMS

## THEORY

## REDUCED EQUATIONS OF MOTION

differential equations for $\theta_{k}$, and static equations for $\theta_{l}$ :

$$
\begin{aligned}
h_{k k} \theta_{k} & =\widetilde{A}_{k} \\
K_{l l} \widetilde{\theta}_{l} & =A_{l \mathrm{QS}}
\end{aligned}
$$

equations for $\theta_{l}$ directly inverted at each time step
equations to be integrated take form $H u=R$
$H=H(d / d t)=$ differential operator (spring, damper, and mass)
in general, $R$ depends on time and on $u$ and its derivatives, and problem is nonlinear
initialize: $F_{\text {old }}=0, \theta_{\text {old }}=\operatorname{trim}$
time step $k$ (response $\Delta t$ )
initialize response: $x_{k}=x_{k-1}$
time increment $i$ (integration $\delta t$ )
calculate matrices
initialize degrees of freedom $\theta$
iteration
save response: $x_{\text {old }}=x$
evaluate equations: $F$
relax equations: $F=\lambda F+(1-\lambda) F_{\text {old }}$
save equations: $F_{\text {old }}=F$
calculate forces of reduced equations
integrate equations for degrees of freedom $\theta$
calculate response: $x$
test convergence: error $=\left\|x-x_{\text {old }}\right\| \leq$ tolerance $\times$ weight
save degrees of freedom $\theta$
output
for each time value (response $\Delta t$ )
evaluate equations: $y=B_{y}$
implement order reduction: $\xi, f_{0}, f_{1}, y, f_{l}$

Figure 4-22 Outline of integration solution method.

## FINITE-DIFFERENCES

introducing finite-difference approximation for time derivatives converts differential equations at time $t^{i}$ into algebraic equations

$$
\widetilde{K}^{i} u^{i}=\widetilde{R}^{i}
$$

solved for displacement $u^{i}$, and then derivatives $\dot{u}^{i}$ and $\ddot{u}^{i}$ evaluated using finite-difference expressions for nonlinear problem, solve $\widehat{R}=R-H u=0$
finite-difference approximation for derivatives converts to algebraic equations:

$$
0=\widehat{R}\left(u^{i}\right)=(R-H u)^{i}=\widetilde{R}^{i}-\widetilde{K}^{i} u^{i}
$$

solved for displacement $u^{i}$ at time $t^{i}$
Newton-Raphson solution (with relaxation) is

$$
\widehat{K}_{n}^{i} \Delta u^{i}=\lambda \widehat{R}_{n}^{i}
$$

where $\widehat{K}$ is derivative matrix (partial derivative of $-\widehat{R}$ with respect to $u$ )
with approximation $\widehat{K} \cong \widetilde{K}$, equation for nonlinear case becomes:

$$
\widetilde{K}_{n}^{i} \Delta u^{i}=\lambda \widehat{R}_{n}^{i}
$$

solved for change $\Delta u$ from iteration $n$ to iteration $n+1$, all at time $t^{i}$
same algorithm obtained considering relaxed, successive substitution solution of $\widetilde{K} u=\widetilde{R}$
for generality, relaxation applied to force $R$ rather than $u$

## ITERATION

algorithm has time loop, over transient time range
at each time increment, there is an iteration
at beginning of iteration, solution for $u^{i}$ initialized using $u^{i-1}$
evaluate force $R^{i}$
with relaxation factor for convergence of nonlinear problems

$$
R^{i}=\lambda R^{i}+(1-\lambda) R_{\mathrm{old}}^{i}
$$

old force is from previous iteration (initialized to zero at beginning of part solution)
reduced equations integrated, solving for degrees of freedom $u^{i}$ and derivatives
at end of each iteration, convergence is tested
when iteration converged, $u$ and $R$ saved for use during next time increment

## RELAXATION CAN ALSO BE APPLIED TO SELECTED IMPLICIT CONSTRAINT EQUATIONS

used to obtain convergence of circulation with vortex wake model

## CONVERGENCE TEST

correct solution for $u^{i}$ is not known, so convergence tested by comparing two successive iterations:

$$
\text { error }=\left\|u-u_{\text {old }}\right\| \leq \text { tolerance }
$$

typically error is absolute value of difference between iterations

## INITIALIZATION OF ITERATION AT $t^{i}$

at start of iteration, solution $u^{i}$ must be initialized, using $u^{i-1}$
options:
zero
constant
constant displacement
constant acceleration
zero acceleration
choice will affect only convergence of iteration
in practice, this initialization option is not usually important

## INTEGRATION ALGORITHMS

ALGORITHM MUST SOLVE FOR $u$ AND ITS DERIVATIVES FROM $H(M, C$, and $K)$ AND $R$, ALL AT TIME $t^{i}$

## EXPLICIT METHODS NOT CONSIDERED (ONLY

 CONDITIONALLY STABLE)
## LINEAR NEWMARK

one-step, one-leg method formulated for linear problems Newmark constant average acceleration method (trapezoidal rule)
unconditionally stable, and has no attenuation of high frequency modes

## LINEAR WILSON

one-step, two-leg method formulated for linear problems linear acceleration method (Wilson- $\theta$ ), with $\theta=1.4$
unconditionally stable for $\theta>1.37$, and has attenuation of high frequency modes

## LINEAR HILBER-HUGHES-TAYLOR

 one-step, two-leg method formulated for linear problems HHT $\alpha$-method (extension of Newmark method); numerical dissipation to damp out spurious participation of high frequency modesunconditionally stable, and has attenuation of high frequency modes
typically $\alpha=-0.05$ or -0.10
NONLINEAR NEWMARK, WILSON, OR HILBER-HUGHESTAYLOR
methods formulated for nonlinear problems

PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF METHOD:

MAXIMUM NUMBER OF ITERATION TOLERANCE RELAXATION FACTORS DAMPING<br>NUMERICAL INTEGRATION METHOD HHT NUMERICAL DISSIPATION<br>INITIALIZATION METHOD<br>SIZE OF INTEGRATION TIME STEP<br>MATRIX CALCULATION<br>perturbation order<br>magnitude and sign of perturbation<br>averaging<br>MATRIX UPDATE STRATEGY

## DISTURBANCES AT BEGINNING OF TRANSIENT, $t_{B}$

derivatives of $u$ are evaluated using solution at previous time step (finite-differences)
at beginning of transient $\left(t_{B}+\delta t\right)$, need $u=\theta_{k}$ at $t_{B}$
obtained from trim solution
trim and transient solution can use different degrees of freedom, different modes, different transforms
such solution and model differences can produce initial disturbance in transient solution

## 4-27 Flutter Task

## DIFFERENTIAL EQUATIONS LINEARIZED ABOUT TRIM

 LINEARIZE SYSTEM EQUATIONS, ABOUT TRIM SOLUTION linearization implemented using loops and parts ANALYZE RESULTING DIFFERENTIAL EQUATIONSFLUTTER EQUATIONS CAN BE TIME INVARIANT OR PERIODIC TIME INVARIANT EQUATIONS OBTAINED FOR TRULY TIME INVARIANT SYSTEM

OR BY AVERAGING EQUATIONS OF PERIODIC SYSTEM
first (outermost) loop can average system equations over period, producing constant-coefficient approximation
child loops must average equations for subsystem that has period not equal system period averaged equations are approximation
if system is strongly periodic, constant-coefficient approximation may not well represent the true behavior
averaging can also suppress interactions between subsystems coupled by loops

PERIODIC EQUATIONS ARE ANALYZED USING FLOQUETLYAPONOV THEORY
time invariant equations can be analyzed as if they are periodic
way to check accuracy of integration required for periodic equations

## EXAMPLES FOR ROTORCRAFT

periodic coefficients from aerodynamics of edgewise flow
with multiblade coordinates and three or more blades, averaged coefficients good enough up to advance ratio of about $\mu=0.5$
coupled dynamics of two-bladed rotor and shaft motion strong $1 /$ rev periodicity, so Floquet theory is required FOR USE BY EXTERNAL ANALYSIS

## 4-27.1 Analysis of Flutter Equations

FLUTTER LOOPS AND PARTS PRODUCE LINEAR DIFFERENTIAL EQUATIONS DESCRIBING SYSTEM RESPONSE

$$
\begin{array}{r}
A_{2} \ddot{x}+A_{1} \dot{x}+A_{0} x=B_{0} v \\
y=C_{2} \ddot{x}+C_{1} \dot{x}+C_{0} x+D_{0} v
\end{array}
$$

$x=$ vector of degrees of freedom
$v=$ vector of controls
$y=$ vector of sensors
subset of $x, \dot{x}, \ddot{x}$, and $v$ can be designated as output quantities
coefficient matrices can be constant, or periodic in time
figure 23 outlines analysis process

## ANALYSIS TASKS

FOR TIME INVARIANT EQUATIONS (TRUE OR AVERAGED)
eigenanalysis
time history response
frequency response
rms gust response
FOR PERIODIC EQUATIONS
eigenanalysis
time history response

```
time-invariant equations
    evaluate equations (loops and parts)
    analyze equations
        eigenanalysis
        time history response
        frequency response
        rms gust response
periodic equations
    time loop (one period)
        evaluate equations (loops and parts)
        store equations
    eigenanalysis
        time loop (one period)
        interpolate equations
        analyze equations
    time history response
        time loop
        interpolate equations
        analyze equations
```

Figure 4-23 Outline of flutter task.

## REDUCTION OF EQUATIONS

## DEGREES OF FREEDOM, OUTPUT VARIABLES, OR INPUT VARIABLES CAN BE ELIMINATED FROM EQUATIONS

options:
complete equations (eliminate only designated variables)
symmetric equations
antisymmetric equations

## QUASISTATIC REDUCTION OF DEGREES OF FREEDOM

 options: full dynamics (no quasistatic reduction) eliminate designated degrees of freedom eliminate all but system rigid degrees of freedom neglect mass and damping of quasistatic degrees of freedom $x_{l}$; then introduce transformation that decouples spring matrixSO NINE SUBSETS OF FLUTTER EQUATIONS CAN BE ANALYZED

## STATE VARIABLE FORM

## EQUATIONS TRANSFORMED TO STANDARD FIRST ORDER FORM

degree of freedom that has both mass and spring terms is second order, so states are displacement and velocity
remaining degrees of freedom are first order, with single state each (displacement or velocity)

## STATE VECTOR

velocity of all degrees of freedom, and displacement of second order degrees of freedom

$$
x=\left(\begin{array}{l}
\dot{x}_{2} \\
\dot{x}_{1} \\
x_{2}
\end{array}\right)
$$

$x_{1}=$ first order degrees of freedom
$x_{2}=$ second order degrees of freedom
although for first order degrees of freedom with zero mass, state is actually displacement

EQUATIONS OF MOTION

$$
\begin{aligned}
& \dot{x}=A x+B v \\
& y=C x+D v
\end{aligned}
$$

## TIME INVARIANT EQUATIONS

## EIGENANALYSIS

eigenvalues (poles)
frequency, damping, time constant
state eigenvectors
output eigenvectors
zeros for each input/output pair residues and static response

## TIME HISTORY RESPONSE

for each input/output pair (unit amplitude of input)
input time history options:
impulse, step, ramp
cosine impulse, sine doublet
square impulse, square doublet
triangular impulse, triangular doublet general piecewise linear
time history calculation:
from modes (eigenvalues and eigenvectors): independent of time step
from numerical integration: time step affects accuracy
modified trapezoidal method
4-th order Runge-Kutta method

## FREQUENCY RESPONSE

for each input/output pair (unit amplitude of input)
transfer function calculation:
from matrices: probably most accurate
from modes (eigenvalues and eigenvectors):
probably fastest
from poles and zeros: probably least accurate
frequencies:
static response from matrices (zero frequency)
calculated range (linear or log scale)
input list

## RMS GUST RESPONSE

input $=$ vector of gust components
rms response not properly defined for system with unstable modes
gust spectra: Dryden and von Kármán
rms gust magnitude
gust correlation length or time constant
rms response calculation:
by stochastic method (from modes), with Markov process gust model (Dryden spectrum only)
by integrating transfer function (matrices, modes, or poles and zeros)

## PERIODIC EQUATIONS

## EVALUATE EQUATIONS

evaluated over one period, stored for analysis
time step based on harmonic content of coefficients
interpolated to times required by numerical integration (for state transition matrix or time history)
time step based on dynamics of system
interpolation methods:
linear
Fourier (default)
Fourier-linear

## EIGENANALYSIS

eigenvalues
z-plane (state transition matrix)
s-plane
eigenvectors of state transition matrix
state transition matrix $\phi$ calculated by integrating $\dot{\phi}=A \phi$
over one period
numerical integration: time step affects accuracy modified trapezoidal method 4-th order Runge-Kutta method

## TIME HISTORY RESPONSE

for each input/output pair (unit amplitude of input) input time history options:
impulse, step, ramp
cosine impulse, sine doublet square impulse, square doublet triangular impulse, triangular doublet general piecewise linear time history calculation:
from numerical integration: time step affects accuracy
modified trapezoidal method
4-th order Runge-Kutta method

## 4-27.2 Rotorcraft Flutter Task

## PARTITIONED PROCEDURE CONSTRUCTED BY ROTORCRAFT

 SHELL TO GENERATE EQUATIONSFOR ENTIRE ROTORCRAFT
OR JUST FOR INDEPENDENT BLADE

## SEPARATE LOOP FOR EACH SUBSYSTEM:

ROTORCRAFT
ROTOR n
ROTOR $n$ INFLOW
ROTOR n HUB
ROTOR n BLADE m
AIRFRAME
DRIVE TRAIN
partitioning system makes it possible to examine equations of each subsystem
if rotors turn at different speeds, then always average equations for second (and other) rotor, airframe, and drive train
"DIFFERENTIAL EQUATIONS" PART METHOD FOR FOLLOWING:
ROTOR n INFLOW
ROTOR n HUB
ROTOR n BLADE m
AIRFRAME
DRIVE TRAIN
"interface" parts used for most input/output constraint equations, to minimize size of equations handled at each step

## 4-28 System Pieces: Flutter Loop

FLUTTER LOOP OPERATIONS:
COMBINE EQUATIONS OF PARTS AND CHILD LOOPS
TRANSFORM DEGREES OF FREEDOM, OUTPUT, AND INPUT
multiblade coordinate
rotating-to-nonrotating
symmetric/antisymmetric
REDUCED EQUATIONS
eliminate degrees of freedom, output, input
AVERAGE EQUATIONS OVER PERIOD
QUASISTATIC REDUCTION OF EQUATIONS
degrees of freedom; with or without residual
any variables identified for reduction (zero or quasistatic) that remain in final loop equations will be handled by analysis of flutter equations

## 4-29 System Pieces: Flutter Part

METHODS IMPLEMENTED FOR FLUTTER PARTS:
NO SOLUTION
INTERFACE
produce linearized equations
DIFFERENTIAL EQUATIONS
produce linearized equations

PART DESCRIBED BY IDENTIFYING ITS MOTION, CONSTRAINT, AND OUTPUT EQUATIONS

EACH EQUATION CAN BE IN ONLY ONE PART (POSSIBLY NO-SOLUTION METHOD)

EACH PART CAN BE SOLVED IN ONLY ONE LOOP (OR NO LOOP)

## MOTION, CONSTRAINT, AND OUTPUT EQUATIONS LINEARIZED

 ABOUT TRIM RESPONSEassume trim solution produces equilibrium solution, which therefore cancels from flutter equations

## CONSTRAINT EQUATIONS ARE SOLVED TO ELIMINATE SYSTEM VARIABLES

when system is divided into parts, necessary to make additional assumptions about the form of equations
so constraint equations of parts can be solved independently, and part equations then coupled by loops

# NO SOLUTION METHOD EQUATIONS ASSIGNED TO PART ARE NOT LINEARIZED corresponding variables will then have zero perturbation SO SUPPRESSES EFFECT OF THESE SYSTEM VARIABLES DURING FLUTTER TASK 

## PARAMETERS CONTROLLING ACCURACY OF LINEARIZATION: MATRIX CALCULATION <br> perturbation order <br> magnitude and sign of perturbation

Chapter 5

## COMPONENTS THEORY

GENERAL REFERENCE FOR CHAPTER: CAMRAD II Documentation, Volume II, Components Theory

## 5-1 Components

FOR CONFIGURATION GENERALITY, SPLIT SYSTEM INTO PIECES, WITH CONNECTIONS BETWEEN

COMPONENTS ARE PIECES THAT PERFORM ALL COMPUTATIONS ASSOCIATED WITH THE PHYSICS OF THE MODEL OF A SYSTEM

## COMPONENTS AVAILABLE TO CONSTRUCT SYSTEM:

rigid body
linear normal modes
finite element beam
rod/cable

transmission
reference frame
filter
reference plane
differential equation
programmable
transfer function
Fourier series
prescribed control
gust
rigid airframe aerodynamics
airframe flow field
lifting line wing
rigid wing
wing inflow
rotor inflow
rotor dynamic wake
wing wake
wing wake geometry rotor wake geometry
wing performance rotor performance rotorcraft performance
helicopter tail boom computational fluid dynamics
plugin
figure 1 shows use of components to construct rotorcraft


Figure 5-1 Rotorcraft model (simplified).

COMPONENT CAN HAVE DEGREES OF FREEDOM $\xi_{i}$, DEPENDENCE ON DEGREES OF FREEDOM MEANS THAT EQUATIONS OF MOTION EXIST, AND USUALLY A DIFFERENTIAL EQUATION FORMULATION IS POSSIBLE

## COMPONENT CAN DEPEND ON FRAME MOTION $\beta$

COMPONENT HAS INPUT $f_{i}$ AND OUTPUT $x_{j}$ COMPONENT INPUT IS OF STRUCTURAL DYNAMIC KIND OR INPUT/OUTPUT KIND
can be connected to an interface or to a system input piece

COMPONENT OUTPUT IS OF STRUCTURAL DYNAMIC KIND OR INPUT/OUTPUT KIND
can be connected to an interface or to a system output piece

FOR STRUCTURAL DYNAMIC INTERFACE, INPUT AND OUTPUT OCCUR IN PAIRS
input is vector of force and moment at the connection output is motion of axes at the connection

## HENCE THE COMPONENT PRODUCES MOTION AND OUTPUT EQUATIONS, DEPENDING ON DEGREES OF FREEDOM, FRAME MOTION, AND INPUT:

$$
\begin{aligned}
0 & =A_{j}\left(\xi_{i}, \beta, f_{i}\right) \\
x_{j} & =B_{j}\left(\xi_{i}, \beta, f_{i}\right)
\end{aligned}
$$

in general, nonlinear and time varying
if component does not have degrees of freedom, motion equations do not exist

## A COMPONENT CAN BE CONSIDERED AN OPERATOR THAT EVALUATES A VECTOR OR MATRIX

figure 2 illustrates functionality
at time $t$ component evaluates one of following vectors:
motion equation: $A_{j}$
component output for input/output interface: $x_{j}=B_{j}$
component output for structural dynamic interface: axes motion $x_{j}$
from degrees of freedom $\xi_{i}$ (including frame motion) and from component input $f_{i}$

COMPONENT CAN ALSO PERTURB THIS VECTOR (ANALYTICALLY OR NUMERICALLY), TO CONSTRUCT MATRIX COLUMN


Figure 5-2 Component functionality.

## 5-2 Structural Dynamic Components

## ALL STRUCTURAL DYNAMIC COMPONENTS SHARE COMMON

 CHARACTERISTICS, INCLUDING:rigid body motion and frame mass, hence inertial and gravitational forces joints
structural dynamic interfaces
aerodynamic interfaces
applied load interfaces
controls and sensors
figure 3 illustrates standard functionality of structural dynamic components

## COMPONENT MOTION DESCRIBED BY

RIGID BODY MOTION, WHICH CAN ALWAYS BE LARGE PLUS ELASTIC MOTION, WHICH IS MEASURED RELATIVE RIGID MOTION

STRUCTURAL DYNAMIC COMPONENTS DIFFER IN MODELLING OF ELASTIC MOTION

## STANDARD RELATIONS USE EXACT KINEMATICS

each component has large rigid body motion exact kinematics for all structural dynamic interfaces


Figure 5-3 Functionality of structural dynamic components.

## 5-2.1 Kinematics

FRAME F (PERHAPS INERTIAL) IDENTIFIED FOR STRUCTURAL DYNAMIC COMPONENT

## ALL MOTION OF COMPONENT MEASURED RELATIVE THAT FRAME

## COMPONENT HAS RIGID BODY MOTION, DESCRIBED BY MOTION OF BODY AXES B

"CONSTRAINED COMPONENT"
this motion connected to the frame; rigid body motion is not degrees of freedom
"FRAME COMPONENT"
rigid body degrees of freedom are the frame motion
OTHER CASES
rigid body degrees of freedom exist, and represent motion relative to the frame
figure 4 summarizes options for rigid body motion

## RIGID BODY MOTION CAN BE JUST LINEAR DEGREES OF FREEDOM

ignoring angular motion appropriate for point mass (no moments of inertia or center-of-gravity offset) with only pinned interfaces


FRAME COMPONENT
component frame F = body axes B

motion $\mathrm{BP}=\mathrm{FP}=$ nominal + degree of freedom

OTHER CASES


Figure 5-4 Rigid body motion of structural dynamic component.

## CONVENTION TO DESCRIBE GEOMETRY AND KINEMATICS INTERFACES OCCUR AT CONNECTIONS (C) ON STRUCTURE

so all loads act at connections
GEOMETRY SPECIFIED IN TERMS OF LOCATIONS (E)
figure 5 summarizes construction of motion at a connection

## POSITION OF CONNECTION DESCRIBED IN STEPS:

$\mathrm{I} \rightarrow \mathrm{P} \rightarrow \mathrm{F} \rightarrow \mathrm{B} \rightarrow \mathrm{E} \rightarrow \mathrm{J} \rightarrow \mathrm{C}$ position of each step consists of orientation and displacement (and derivatives if required) of an axis system, measured relative axes of preceding step addition of steps involves addition of axes rigid motion, not scalar summation
each step may itself consist of several motions
any subset of motion can be calculated, as required by component analysis

## FOR EACH COMPONENT, CAN DEFINE:

any number of locations
any number of joints at a location
any number of connections at a location, or at a joint any number of uses of a connection, including interfaces and sensors
with some restrictions (for example, aerodynamic interfaces can not be at a joint)

C $=$ connection
$\mathrm{J}=$ joint
$\mathrm{E}=$ location
$B=$ body motion
$\mathrm{F}=$ component frame
$\mathrm{P}=$ parent frame
I = inertial frame

$$
\mathrm{Cl}=\mathrm{CJ}+\mathrm{JE}+\mathrm{EB}+\mathrm{BF}+\mathrm{FP}+\mathrm{PI}
$$

$\mathrm{PI}=\sum \mathrm{P}_{i} \mathrm{P}_{i-1}=\sum$ (nominal + dof)
$\mathrm{FP}=$ (nominal + dof); or (dof + nominal)
$\mathrm{BF}=$ (nominal + dof); or (dof + nominal); or 0 (frame degrees of freedom); or constant BF (constrained component)
$E B=E L+L B ;$ or constant EB (no elastic)
$\mathrm{JE}=\mathrm{JD}+\mathrm{DE}$; or 0 (no joint)
$\mathrm{CJ}=$ constant CJ ; or constant CE (no joint)

Figure 5-5a Summary of motion at a connection.


Figure 5-5b Summary of motion at a connection.

## STEPS IN DESCRIPTION OF POSITION ON STRUCTURE

## INERTIAL FRAME I

general reference for all motion

## PARENT FRAME P

some parent frame of the component
Pl is sum of all frame motions between P and inertial frame
each parent frame might be frame degrees of freedom (rigid body degrees of freedom of some other component)

## COMPONENT FRAME F

designated frame of component
FP is motion of frame relative its parent
FP is sum of nominal and degree-of-freedom motions (in either order, standard rigid-type response)

## BODY MOTION B

rigid body motion of component
BF is motion of component body axes relative to component frame

BF is sum of nominal and degree-of-freedom motions (in either order, standard rigid-type response)
frame component: body axes of this component are the frame ( $\mathrm{B}=\mathrm{F}$ ); so frame motion FP provides rigid body degrees of freedom of component, while motion BF is zero
constrained component: body axes of this component are fixed relative to the frame; constant position of body is specified by $z^{B F / F}$ and $C^{B F}$ ( B relative F , in F axes)

## ELASTIC MOTION E

geometric location and elastic motion
EB is position of location relative body axes, $z^{E B / B}$ and $C^{E B}$ ( E relative B , in B axes)

EB is sum of elastic degrees of freedom and constant position ( $\mathrm{EB}=\mathrm{EL}+\mathrm{LB}$ )
analysis can suppress effects of elastic motion or elastic force at location (an approximation)

JOINT MOTION J
joint motion of component (only present if joint exists at the location)

JE is position of joint relative location axes, $z^{J E / E}$ and $C^{J E}$ (J relative E, in E axes)

JE is sum of joint degrees of freedom and constant position (JE = JD + DE)

CONNECTION C
position of connection on component
CJ is constant position of connection relative joint axes, $z^{C J / J}$ and $C^{C J}$ (C relative J , in J axes)
if connection not at joint, then CJ is actually position relative location axes (C relative E, in E axes)

## 5-2.2 Joints

## CONVENTION TO DEFINE JOINTS

JOINT MOTION OCCURS AFTER RIGID BODY AND ELASTIC MOTION, WITH NO MASS ON CONNECTION SIDE OF JOINT
so joint equations of motion obtained from just balance of forces acting on the joints
there must be at least one interface producing forces on the joint

MORE COMPLEX MOTIONS MODELLED BY USING SEVERAL COMPONENTS

## JOINT KINEMATICS DESCRIBE POSITION OF JOINT AXES

 RELATIVE TO A LOCATION (JE)JE is sum of joint degrees of freedom and constant position $(J E=J D+D E)$
constant position of joint (for zero joint variables) relative location axes is specified by $z^{D E / E}$ and $C^{D E}$ ( D relative $E$, in E axes)
then joint motion occurs, described by $x^{J D / D}$ and $C^{J D}$ (and derivatives)

## JOINT CAN CONSIST OF

just linear motion
just angular motion
linear followed by angular motion
angular followed by linear motion

## CONFIGURATIONS FOR LINEAR MOTION

1-linear (slide, prism, linear hinge)
2-linear (plane)
3-linear (space)
described by one to three variables (with gain factor)
axes can be arbitrarily specified

## CONFIGURATION FOR ANGULAR MOTION

hinge (hinge, revolute)
2-hinge (universal)
3-hinge (hinge plus universal)
hinge (Rodrigues parameters)
gimbal (2-Rodrigues)
rod end (3-Rodrigues, ball, spherical, pinned)
hinge (third variable) then gimbal
gimbal then hinge (first variable)
described by one to three variables (with gain factor; first variable is last rotation)
axes can be arbitrarily specified
Rodrigues parameters are convenient way to generate joint with all rotations treated same

## LINEAR OR ANGULAR DISPLACEMENT IN JOINT EQUALS JOINT VARIABLE $\theta$ TIMES GAIN FACTOR

CONVENTIONS WILL HANDLE COMBINED MOTIONS
such as screw, rack and pinion, or transmission

## EACH JOINT VARIABLE $\theta$ CAN BE DEGREE OF FREEDOM, PRESCRIBED, OR CONTROLLED: <br> DEGREE OF FREEDOM

joint degree of freedom can be shared by more than one variable, of one or more than one joint
each joint degree of freedom (involving one or more joint variables) has equation of motion obtained by equilibrium of loads on the joints:
$f_{\text {spring }}=f_{\text {interfaces }}$
reaction $f_{\text {spring }}$ obtained from spring/damper/ actuator model
load $f_{\text {interfaces }}$ obtained by summing effects of all interface loads acting on all joints involved in this degree of freedom

## PRESCRIBED

joint motion can be constant velocity, rotating variable, or constant position

## CONTROLLED

joint motion can be obtained from a control vector (component input), implementing a displacement actuator if motion is time varying, this control vector should include joint velocity and acceleration, as well as joint displacement
force or offset actuator can be implemented using spring/damper/actuator convention for joint degree of freedom
controlled joint can not be used in flutter task
joint degree of freedom with offset actuator can be used instead (and the degree of freedom can be quasistatic)
controlled joint can be used in trim and transient tasks, as long as control variable is solved in separate part

## STRUCTURAL DYNAMIC TORQUE INTERFACE ONLY ALLOWED ON CERTAIN JOINTS

JOINTS WHERE LAST MOTION (FIRST VARIABLE) IS ANGULAR ROTATION ABOUT AN AXIS

## 5-2.3 Spring/Damper/Actuator

## STANDARD SPRING/DAMPER/ACTUATOR MODEL USED TO PROVIDE REACTION FORCE AS REQUIRED BY COMPONENT USED FOR JOINTS, ROD/CABLES, OR TRANSMISSION BRANCHES

component can use reaction force in only one context, depending on component type

FOR EACH DEGREE OF FREEDOM $\theta$ (SCALAR ELEMENT OF DEGREE-OF-FREEDOM VECTOR):

SCALAR REACTION FORCE $f$ IS GENERATED
$f_{\text {spring }}=f(\theta, \dot{\theta}, c)$
$c=$ optional control (element of control vector) for an actuator model

FORCE EVALUATED AS SUM OF SPRING, DAMPER, TABLE, AND BIAS TERMS

$$
f_{\text {spring }}=K\left(\theta-\theta_{0}\right)+C\left(\dot{\theta}, \theta-\theta_{0}\right)+T\left(\dot{\theta}, \theta-\theta_{0}, s\right)-c_{0}
$$

with offset $\theta_{0}$ and bias $c_{0}$ offset $\theta_{0}$ usually value of $\theta$ for zero load; bias $c_{0}$ gives load for zero displacement
$s$ is optional switch in table
optionally can support only compression, or only tension

## ACTUATOR ADDED BY INTRODUCING CONTROL $c$

force actuator, $c_{0}$ replaced by $\left(c_{0}+c\right)$
offset actuator, $\theta_{0}$ replaced by $\left(\theta_{0}+c\right)$
for very large stiffness, offset actuator should produce $\theta=\theta_{0}+c$, which is a displacement actuator joint has separate implementation for displacement actuator
figure 6 summarizes the actuator models

## FOR LINEAR SPRING AND LINEAR DAMPER:

$$
\begin{array}{ll}
\hline f_{\text {spring }}=K \theta+C \dot{\theta}-\left(K \theta_{0}+c_{0}\right) & \text { no actuator } \\
f_{\text {spring }}=K \theta+C \dot{\theta}-\left(K \theta_{0}+c_{0}+K c\right) & \text { offset actuator } \\
f_{\text {spring }}=K \theta+C \dot{\theta}-\left(K \theta_{0}+c_{0}+c\right) & \text { force actuator }
\end{array}
$$

DISPLACEMENT ACTUATOR (controlled joint)


OFFSET ACTUATOR


FORCE ACTUATOR


$$
\begin{aligned}
\theta & =\text { variable to be controlled } \\
\mathrm{c} & =\text { control (component input) } \\
\mathrm{F} & =\text { force } \\
\mathrm{R} & =\text { reaction } \\
W & =\text { spring (no load length } \theta_{0} \text { ) }
\end{aligned}
$$

Figure 5-6 Summary of actuator models.

## SPRING HAS LINEAR AND ELASTOMERIC TERMS

DAMPER HAS LINEAR, ELASTOMERIC, AND HYDRAULIC TERMS

$$
\begin{aligned}
K(\theta) & =K_{\operatorname{lin}} \theta+K_{\text {elast }} \\
C(\dot{\theta}, \theta) & =C_{\text {lin }} \dot{\theta}+C_{\text {elast }}+C_{\text {hyd }}
\end{aligned}
$$

used with an offset $\theta_{0}$
linear terms have constant spring and damping coefficients

## ELASTOMERIC SPRING/DAMPER DESCRIBED BY

 POLYNOMIALS IN DISPLACEMENT $\theta$ AND RATE $\dot{\theta}$ :$$
\begin{aligned}
K_{\text {elast }} & =\operatorname{sign}(\theta)\left(P_{e k d}(|\theta|)+P_{e k r}(|\dot{\theta}|)\right) \\
C_{\text {elast }} & =\operatorname{sign}(\dot{\theta})\left(P_{e c d}(|\theta|)+P_{e c r}(|\dot{\theta}|)\right)
\end{aligned}
$$

each polynomial is fifth-order (six terms)
$P_{e k d}$ term is nonlinear spring
$P_{\text {ecr }}$ term is nonlinear viscous damping
$P_{e c d}$ term is friction damping
necessary to identify coefficients of this time domain model from measured force and motion data (usually for harmonic motion at one or two frequencies)

HYDRAULIC DAMPER DESCRIBED BY POLYNOMIALS IN RATE $\dot{\theta}$ :

$$
C_{\mathrm{hyd}}=\operatorname{sign}(\dot{\theta}) \min \left(P_{h a}(|\dot{\theta}|), P_{h b}(|\dot{\theta}|)\right)
$$

each polynomial is fifth-order (six terms)
hydraulic damper typically has

$$
\left.\begin{array}{rl}
P_{h a} & =c_{a} \dot{\theta}^{2}
\end{array}=f_{\max }\left(\dot{\theta} / \dot{\theta}_{\max }\right)^{2}\right)
$$

MORE GENERAL BEHAVIOR CAN BE IMPLEMENTED BY OBTAINING FORCE $T(\dot{\theta}, \theta)$ FROM A TWO-DIMENSIONAL TABLE
or $T(\dot{\theta}, \theta, s)$ from a three-dimensional table, including switch parameter $s$ (perhaps $s=$ time)

NONLINEAR SPRING/DAMPER CAN BE DESCRIBED IN TERMS OF EQUIVALENT LINEAR SPRING AND LINEAR DAMPER

VALUES OF EQUIVALENT $K$ AND $C$ DEPEND ON AMPLITUDE AND FREQUENCY OF MOTION
such behavior depends on finite amplitude of motion, so can not be obtained by local linearization of force

SO WHEN PERTURB SPRING/DAMPER/ACTUATOR, CAN OPTIONALLY USE

$$
\begin{aligned}
K(\theta) & =K_{\text {equiv }} \theta \\
C(\dot{\theta}, \theta) & =C_{\text {equiv }} \dot{\theta} \\
T(\dot{\theta}, \theta) & =0
\end{aligned}
$$

for the trim, transient, or flutter task

## 5-2.4 Interfaces

STRUCTURAL DYNAMIC INTERFACE
ONLY STRUCTURAL DYNAMIC COMPONENT CAN HAVE STRUCTURAL DYNAMIC INPUT AND OUTPUT

FOR CONNECTION TO ANOTHER COMPONENT THROUGH STRUCTURAL DYNAMIC INTERFACE
interface occurs at connection (C)
INTERFACE KIND CAN BE COMPLETE OR TORQUE

COMPLETE INTERFACE: TRUE PHYSICAL CONNECTION
CONNECTS TWO COMPONENTS RELATIVE THEIR COMMON PARENT FRAME (P)
component input is force and moment acting on the connection
component output is motion of axes at the connection, relative common frame axes ( $x^{C P / P}$ and $C^{C P}$, and their derivatives)

## TORQUE INTERFACE

DEALS WITH ONLY ROTATIONAL MOTION AND TORQUES (APPROXIMATION)
can be used only on appropriate angular joint produces no net moment on true structural dynamic component; only involved in joint equations

## AERODYNAMIC INTERFACE

## STANDARD INPUT/OUTPUT INTERFACE (COMPONENT INPUT AND/OR COMPONENT OUTPUT)

FOR INTERFACES WITH AERODYNAMIC COMPONENTS
interface occurs at connection on structure (which can not be at a joint)

## AERODYNAMIC COMPONENT TYPICALLY INVOLVES SET OF COLLOCATION POINTS

task of calculating geometry of aerodynamic collocation points belongs to structural dynamic component, which knows about physical configuration
calculating velocity of collocation point relative to air also belongs in structural dynamic component, since requires kinematics of connection
connection that is collocation point typically has aerodynamic interfaces for:
interference and gust velocities (input)
velocity relative air (output)
position (output)
resulting aerodynamic load (input)

## POSSIBLE COMPONENT INPUT <br> force ( $F$ ) <br> force and moment ( $F$ and $M$ ) <br> interference velocity $\left(v_{A}\right)$ <br> gust velocity $\left(v_{G}\right)$ <br> gust angular velocity $\left(\omega_{G}\right)$

at connection; in body, frame, or parent frame axes

## POSSIBLE COMPONENT OUTPUT

velocity relative the air ( $v, q, \dot{v}, \omega$ )
position relative origin of axes ( $r$ and $\dot{r}$ )
or subset of these quantities
at connection; in connection, body, frame, or parent frame axes
velocity relative the air calculated using all interference and gust velocities (input) at this connection; and wind velocity, including ground boundary layer
time derivatives of interference and gust velocities are not considered
figure 7 illustrates calculation of velocity relative the air


Figure 5-7 Aerodynamic interface to calculate velocity relative the air.

## APPLIED LOAD INTERFACE

## APPLIED FORCE CAN ACT ON COMPONENT

interface occurs at connection on structure

FORCE ACTS AT CONNECTION
SPECIFY AXIS OF FORCE IN C FRAME
$u=x, y$, or $z$-axis direction
MAGNITUDE OF FORCE IS GAIN $a$ TIMES VALUE $f$ OF SOME ELEMENT OF CONTROL VECTOR (COMPONENT INPUT)

SO APPLIED FORCE IS $F^{C}=u a f$
OR APPLIED MOMENT IS $M^{C}=u a f$

## 5-2.5 Controls and Sensors

## CONTROLS

SET OF CONTROL VECTORS AVAILABLE TO COMPONENT
COMPONENT INPUT, FOR CONNECTION TO SYSTEM INPUT PIECE OR TO INPUT/OUTPUT INTERFACE
a particular control vector and element can be used once, more than once, or not at all

STANDARD FEATURES OF STRUCTURAL DYNAMIC COMPONENT CAN USE CONTROLS FOR:

JOINTS
ACTUATORS
APPLIED LOAD INTERFACES
COMPONENT-SPECIFIC USES ARE PERMITTED

SENSORS
SET OF SENSOR VECTORS CAN BE DEFINED FOR COMPONENT

COMPONENT OUTPUT, FOR CONNECTION TO SYSTEM OUTPUT PIECE OR TO INPUT/OUTPUT INTERFACE

## STANDARD SENSORS MEASURE: <br> motion <br> motion of axes at a point velocity relative air <br> aerodynamic interface motion <br> degrees of freedom motion of torque interface <br> structural dynamic interface constraint force and moment <br> torque <br> reaction <br> total load at point joint degree of freedom weight and load factor <br> power <br> joint <br> joint degree of freedom <br> VALUE OF ANY SENSOR CAN BE MULTIPLIED BY SCALE FACTOR

automatic scaling for value in g's, degrees, Mach number, or horsepower

THERE MAY ALSO BE COMPONENT-SPECIFIC SENSORS AVAILABLE

## 5-2.6 Structural Dynamic Components

## ALL STRUCTURAL DYNAMIC COMPONENTS HAVE STANDARD FEATURES

differ in modelling of elastic motion

## COMPONENT MOTION DESCRIBED BY

rigid body motion, which can always be large (exact kinematics)
plus elastic motion, which is measured relative rigid motion
exact kinematics for all structural dynamic interfacesSTRUCTURAL DYNAMIC COMPONENTS IMPLEMENTEDRIGID BODY COMPONENT no elastic motion
LINEAR NORMAL MODES COMPONENT elastic motion represented by free normal modes
FINITE ELEMENT BEAM COMPONENT elastic motion represented by beam deflection (axial, bending, torsion)
ROD/CABLE COMPONENT elastic motion represented by flexible rod or cable between two points

## 5-3 Linear Normal Modes Component

## STRUCTURAL DYNAMIC COMPONENT

ELASTIC MOTION DESCRIBED BY FREE-VIBRATION NORMAL MODES
typically modes obtained from large finite-element analysis
if component is constrained, then constrained modes can be used

RIGID MOTION DESCRIBES CENTER-OF-MASS, MEAN-AXES MOTION OF BODY

SO RIGID BODY DEGREES OF FREEDOM ARE ORTHOGONAL TO FREE-VIBRATION MODES
elastic motion is small (consistent with using linear modes), measured relative to rigid motion

## AERODYNAMIC SPRING, DAMPING, AND CONTROL TERMS INCLUDED

 approximation to effects of aerodynamics on elastic motion
## MODAL SENSORS INCLUDED

to calculate quantities proportional to modal deflection (loads, stress, strain)

## LINEAR NORMAL MODES <br> MODES CALCULATED FOR FREE BODY, INCLUDING RIGID MOTION

small motion is assumed, consistent with linear modes modes are orthogonal
no effect of frame motion on elastic modes
no effect of elastic motion on inertial properties of rigid modes
rigid modes represent motion of center-of-mass, mean-axes of body

MODEL EXTENDED BY REPLACING RIGID MODES BY LARGE RIGID BODY MOTION OF BODY AXES B
origin of body axes must be at center-of-mass
but not necessary to use principal axes
elastic modes represent motion relative to the body axes
elastic and rigid equations are still fully decoupled

## MODE DESCRIBED BY:

modal mass, spring, structural damping
mode shapes at connections where forces and moments are applied

3 linear and 3 angular displacements
defined at locations (in body or undeflected location axes)
input using namelist arrays, or table file aerodynamic spring and damping, and modal force of control
scaled with air density and velocity relative air

## FOR REFERENCE IN PREPARING INPUT PARAMETERS, MODAL EQUATION OF MOTION HAS FOLLOWING FORM (DIMENSIONAL):

$$
M\left[\ddot{q}+g \omega \dot{q}+\omega^{2} q\right]=\xi^{T} F+\gamma^{T} M+1 / 2 \rho V^{2}\left[F_{q \delta} \delta-F_{q \dot{q}}(\dot{q} / V)-F_{q q} q\right]
$$

## 5-4 Finite Element Beam Component

## STRUCTURAL DYNAMIC COMPONENT

ELASTIC MOTION DESCRIBED BY AXIAL, BENDING, AND TORSION DEFLECTION OF A BEAM

RIGID MOTION IS MOTION OF ONE END OF BEAM DO NOT USE NODAL COORDINATES AS DEGREES OF FREEDOM
elastic motion measured relative to rigid motion elastic motion is moderate for second-order model; rigid motion can always be large

## GENERAL GEOMETRY OF BEAM

component has straight beam axis any number of locations (on beam axis)
any number of joints at a location
any number of connections at a location or at a joint
joints and connections are typically in a beam section: in a plane normal to bent beam axis
any number of interfaces and sensors at a connection
figure 8 illustrates use of beam component


Figure 5-8 Use of beam component.

## MODEL ROTOR BLADE BY BREAKING STRUCTURE INTO SEVERAL BEAM SEGMENTS

"nodes" are points where segments are joined
in CAMRAD II terminology: connections at joint/location on each structural dynamic component, with structural dynamic interface between
insert node to handle beam axis (perhaps the elastic axis) with curvature, kinks, or jumps
beam segment has straight beam axis
insert node to handle properties that vary rapidly along blade span (major jumps in properties at nodes)

Gaussian integration along beam segment (implying polynomial approximation to variation of properties, accurate only if variation is sufficiently smooth)
continuous shape functions for elastic deflection within beam segment (can not accurately represent large changes in curvature or slope)
if very short beam segments are required to accommodate properties, then beam theory is probably not applicable


## BEAM MODEL

 MATERIALEuler-Bernoulli beam theory for isotropic material, with elastic axis; or
beam theory for anisotropic or composite materials effects of cross-section warping included in section properties
for anisotropic model, effects of transverse shear deformation included in section properties structural model assumes strain is small

## GEOMETRY

kinematics of elastic motion are exact, almost-exact, or second-order
almost-exact model still uses second-order approximation for extension and torsion produced by bending
second-order model retains only second-order effects of elastic motion in potential and kinetic energies (hence moderate deflection)
cross-section rotation produced by bending must be less than 90 deg within beam component
so very large elastic motion must be modelled using several beam components

STRAIGHT BEAM AXIS (UNDEFLECTED), LENGTH $\ell$ BEAM AXIS IS ELASTIC AXIS FOR ISOTROPIC MODEL beam axis is positive $x$-axis of B frame, beam extending from $x=0$ to $x=\ell$
motion BF describes orientation and displacement of beam end, relative the frame
nominal part of BF typically is rest position of beam relative frame
figure 9 illustrates the configuration

## LOCATIONS, JOINTS, AND CONNECTIONS

locations are on beam axis at axial station $x$ undeflected location axes parallel body axes elastic motion produces additional displacement and rotation of location axes
so E axes are bent and twisted section axes, with origin on beam axis
connection has constant position (offset and rotation) relative location axes
or joint has constant position relative location axes, and connection has constant position relative joint axes


Figure 5-9 Beam configuration.

## BEAM PROPERTIES

## ARBITRARY LOCUS OF CENTER-OF-GRAVITY ARBITRARY LOCUS OF TENSION CENTER

properties defined relative beam axes
displacements $y$ and $z$ measured from beam axis (the $x$ axis)
pitch angles are measured from $x-y$ plane, positive for rotation about $x$-axis
structural properties
$\theta_{C}$ : pitch of structural principal axes, relative $x-y$ plane; pitch angle can be large
$y_{C}$ and $z_{C}$ : offset of tension center (modulus-weighted centroid) from beam axis, relative principal axes (at $\theta_{C}$ )
$k_{P}$ : modulus-weighted radius of gyration, about the beam axis
$E A, E I_{y y}, E I_{z z}, G J, k_{T}$ (isotropic)
$S_{u u}, S_{\phi u}, S_{w u}, S_{v u}, S_{\phi \phi}, S_{w \phi}, S_{v \phi}, S_{w w}, S_{v w}, S_{v v}$ (anisotropic)
inertial properties
$\theta_{I}$ : pitch of inertial principal axes, relative $x-y$ plane; pitch angle can be large
$y_{I}$ and $z_{I}$ : offset of center of gravity (mass-weighted centroid) from beam axis, relative principal axes (at $\theta_{I}$ )

$$
m, I_{\theta}, I_{P}
$$

Gaussian integration of properties along beam length input data use piecewise linear definition of properties

## BEAM DEFLECTION

axial deflection ( $u$ ), then bending deflection ( $v$ then $w$ ), then torsion ( $\phi$ )
axial and torsion degrees of freedom exclude motion produced by bending kinematics
so zero axial degree of freedom is solution for large EA
and zero torsion degree of freedom is solution for large $G J$
elastic deflections represented by polynomial shape functions
maximum 3 shape functions for bending
maximum 4 shape functions for axial or torsion
typically use $u v w \phi=3222$ for rotor blades (total 15 degrees of freedom, 6 rigid and 9 elastic)
unit deflection ( 1 ft or $\mathrm{m}, 1$ radian) at end of beam coefficients of shape functions are elastic degrees of freedom include model of structural damping for each elastic degree of freedom

## SENSORS

## SECTION LOAD

component specific sensor
torsion and bending moments, axial tension, and shear forces at a location (axial station $x_{L}$ ); acting at tension center, in structural principal axes
calculated from deflection, or by force balance

## DEFLECTION METHOD

section load from stiffness and elastic displacement at $x_{L}$
bending moment $=E I$ times curvature torsion moment $=G J$ times slope
accuracy depends on accuracy of representation of curvature or slope (product of degrees of freedom and shape functions)
at step in stiffness should be corresponding step in curvature or slope, so load remains continuous with small number of shape functions not possible to simulate such a step well
theory does not imply continuity of curvature on two sides of node

## FORCE BALANCE METHOD

section load from difference between applied forces and inertial forces acting on beam segment to one side of $x_{L}$ sensor gives at beam ends same result as nodal reaction
can capture steps in section load produced by discrete forces and moments on beam aerodynamic loads integrated as distributed forces and moments on beam

## NODAL REACTION

can also calculate beam reaction using standard sensor for structural dynamic load
about point at tension center, in structural principal axes
accuracy only depends on tolerance in solution for equilibrium of beam
but need node (structural dynamic interface) at sensor point

SA-349 Flight Test, $\mathrm{C}_{\mathrm{T}} / \sigma=.065, \mu=.14$


Three-Bladed Rotor at $\mathrm{C}_{\mathrm{T}} / \sigma=.065, \mu=.14$
uniform blade properties


Three-Bladed Rotor at $\mathrm{C}_{\mathrm{T}} / \sigma=.065, \mu=.14$
blade stiffness $100 \%$ to $\mathrm{r}=.6,125 \%$ to $\mathrm{r}=.8,75 \%$ to $\mathrm{r}=1.0$


Three-Bladed Rotor at $\mathrm{C}_{\mathrm{T}} / \sigma=.065, \mu=.14$ blade stiffness $100 \%$ to $\mathrm{r}=.5,125 \%$ to $\mathrm{r}=.7,75 \%$ to $\mathrm{r}=1.0$


## 5-5 Rod/Cable Component

STRUCTURAL DYNAMIC COMPONENT
ELASTIC MOTION DESCRIBED BY SET OF FLEXIBLE RODS OR CABLES, EACH CONNECTING TWO POINTS
figure 10 illustrates configuration

RIGID MOTION IS MOTION OF CENTRAL BODY, WHICH CONTAINS ALL THE MASS

ONE END OF ROD/CABLE CONNECTED TO LOCATION ON CENTRAL BODY

ELASTIC DEGREES OF FREEDOM ARE POSITION OF OTHER END OF ROD/CABLE
measured relative to body location (hence relative to rigid motion)


Figure 5-10 Rod/cable configuration.

## ROD/CABLE DOES NOT SUPPORT MOMENTS

ENDS ARE PINNED; SO AT ROD/CABLE LOCATION:
no cantilever structural dynamic interfaces no interface can produce moment no joints

SPRING/DAMPER/ACTUATOR CONNECTS ROD/CABLE ENDS, TRANSMITTING FORCE THAT IS ALWAYS DIRECTED BETWEEN TWO POINTS

CABLE MODELLED BY ASSUMING THAT SPRING CAN NOT SUPPORT COMPRESSION LOAD
bending dynamics of cable are neglected

## COMPONENT SPECIFIC SENSORS

to measure motion, length, reaction, power (of rod/cable)

## 5-6 Transmission Component

TRANSMISSION COMPONENT MODELS SUBSYSTEM THAT TRANSMITS ROTATIONAL MOTION AND TORQUES

APPROXIMATION TO ACTUAL DEVICE
ROOT AND ONE OR MORE BRANCHES (OR NO BRANCHES), CONNECTED THROUGH GEAR TRAIN
figure 11 shows the configuration
figure 12 illustrates functionality

NOT TRUE STRUCTURAL DYNAMIC COMPONENT, LACKING RIGID BODY MOTION AND FRAME

SHARES SOME STANDARD FEATURES OF STRUCTURAL DYNAMIC COMPONENT
inertia (rotational only)
structural dynamic interfaces (torque only)
applied load interfaces (torque only)
controls
sensors (component-specific only)
BUT EXCLUDING JOINTS, GRAVITATIONAL FORCES, AND AERODYNAMIC INTERFACES

STANDARD SPRING/DAMPER/ACTUATOR MODEL USED FOR ELASTIC BRANCHES


Figure 5-11 Configuration of transmission component.


Figure 5-12 Functionality of transmission component.

## COMPONENT MOTION DESCRIBED BY

RIGID ROTATION $\theta_{0}$
describes rotation of entire transmission, without elastic restraint root motion is only rigid

BRANCH MOTION CAN INCLUDE ELASTIC ROTATION $\theta_{i}$, MEASURED RELATIVE RIGID ROTATION

GEAR RATIOS RELATE ROTATION OF ROOT AND BRANCH TO DEGREES OF FREEDOM

## ROOT AND BRANCHES

AT END OF ROOT AND END OF EACH BRANCH, THERE IS: rotation $\phi$ torques produced by rotational inertia $I$ and rotational damping $D$
interface torque $Q$ (structural dynamic interface or control torque)

## VARIABLE $\theta_{0}$ DESCRIBING RIGID MOTION CAN BE

 degree of freedomprescribed: constant velocity, rotating variable, or constant position
controlled: motion from control vector, implementing displacement actuator
(as for joint variables)

MOTION OF EACH BRANCH CAN BE RIGID, OR ELASTIC WITH DEGREE OF FREEDOM $\theta_{i}$
equation of motion for $\theta_{i}$ from equilibrium of loads on branch: $f_{\text {spring }}=f_{\text {interfaces }}$
reaction $f_{\text {spring }}$ from spring/damper/actuator model load $f_{\text {interfaces }}=Q$ from interface load acting on branch

## SENSORS

SET OF SENSOR VECTORS, TO MEASURE motion (degrees of freedom, torque interface) structural dynamic interface constraint (torque) reaction (branch) power (branch)
governor

## GOVERNOR MEASURES RIGID ROTATION ERROR

$\theta_{0}$ relative reference rotation
proportional (rotational speed error) and integral (rotation angle error) terms, with constant gains

## 5-7 Reference Frame Component

REFERENCE FRAME COMPONENT PROVIDES ACCESS TO FRAME MOTION

ALSO CAN SUM AND TRANSFORM VECTOR QUANTITIES, TYPICALLY LOADS OR MOTION
related to structural dynamic components, but does not model physical entity
figure 13 illustrates functionality

## FRAMES:

COMPONENT FRAME IS REFERENCE FRAME F
parent frame P can be specified
axes T can be defined, by rotation from frame $F$ (TF can be constant or variable)

## COMPONENT FUNCTIONS:

PROVIDE ACCESS TO FRAME MOTION
motion of frame axes
velocity relative air
position and orientation relative ground unit vectors of frame

SUM AND TRANSFORM LOAD VECTORS (FORCE AND MOMENT)
from frame F to parent frame P; or to axes T
SUM AND TRANSFORM MOTION VECTORS
sum, average, or weighted sum
from frame $F$ to parent frame $P$, or to axes $T$ from parent frame $P$ to frame $F$, or to axes $T$


Figure 5-13 Functionality of reference frame component.

## 5-8 Filter Component

## FILTER COMPONENT IMPLEMENTS FILTER OPERATION ON INPUT CAN FUNCTION AS LOW-PASS FILTER, RESOLVER, OR DIFFERENTIATOR

FREQUENTLY REQUIRED TO OBTAIN MEAN VALUE OF QUANTITY
figure 14 illustrates functionality

## FILTER OPERATIONS:

mean
min/max: minimum, maximum, mid value, half peak-to-peak square: mean square, rms, amplitude, cyclic product harmonic: cosine, sine, magnitude, phase derivative: velocity, acceleration
mean, cosine, and sine operations are linear
all other operations are nonlinear

## FILTER MODEL

OPERATION CAN BE IMPLEMENTED BY IMPLICIT MODEL, DIFFERENTIAL EQUATION MODEL, OR DIRECT MODEL
different model kinds can be used for trim, transient, and flutter tasks


Figure 5-14 Functionality of filter component.

## IMPLICIT

operates on response solution within a window uses standard capability of analysis to apply filter when evaluating component input
can not be used for flutter task

## DIFFERENTIAL EQUATION

uses first-order lag to filter the input
only available for the mean, cosine, and sine operations

## DIRECT

performs no operation on the input available for cases where filter is not required or not appropriate for all three tasks

## MODEL OPTIONS DEPEND ON OPERATION AND TASK:

| task | linear <br> operations | nonlinear <br> operations |
| :--- | :--- | :--- |
| trim | all models | implicit |
| transient | all models <br> flutter | differential equation or direct | | implicit |
| :--- |
| direct |

for nonlinear operations, filter operation is not applied in flutter task (and results will probably not be meaningful)

## 5-9 Reference Plane Component

REFERENCE PLANE COMPONENT IDENTIFIES ORIENTATION OF A PLANE

## ORIENTATION DEFINED BY SMALL ROTATION FROM SOME NOMINAL PLANE

figure 15 illustrates functionality

PLANES:
NOMINAL PLANE ( $x-y$ PLANE IN T AXES) HAS FIXED ORIENTATION RELATIVE COMPONENT FRAME (F AXES) orientation is small rotation from nominal plane (large angles would require nonlinear identification method)

IDENTIFICATION METHODS:
PLANE THROUGH SET OF POSITION VECTORS
OR HARMONICS OF NORMAL DISPLACEMENT OF POSITION VECTORS

OR HARMONICS OF SENSOR(S) THAT MEASURE NORMAL DISPLACEMENT
identified orientation can be filtered, to produce mean position

## FILTER MODEL

OPERATION CAN BE IMPLEMENTED BY IMPLICIT MODEL, DIFFERENTIAL EQUATION MODEL, OR DIRECT MODEL as for filter component


Figure 5-15 Functionality of reference plane component.

## 5-10 Differential Equation Component

DIFFERENTIAL EQUATION COMPONENT IMPLEMENTS STATIC, FIRST ORDER, OR SECOND ORDER DIFFERENTIAL EQUATIONS COMPONENT CAN ALSO FUNCTION TO SIMPLY ADD SCALAR QUANTITIES
figure 16 illustrates functionality

## EQUATIONS

VARIABLES
$\xi=$ degree of freedom vector
$x=$ component output vector
$v=$ differential equation input vector
$v$ is combination of component input:
can concatenate elements of input vectors
or sum, average, or weighted sum of vectors

STATIC EQUATIONS

$$
x=D_{s} v
$$

input form static, first order, or second order
optionally $D_{s}=I$

## FIRST ORDER DIFFERENTIAL EQUATIONS

$$
\begin{aligned}
\dot{\xi} & =A \xi+B v \\
x & =C \xi+D v
\end{aligned}
$$

input form first order or second order


Figure 5-16 Functionality of differential equation component.

## SECOND ORDER DIFFERENTIAL EQUATIONS

$$
\begin{aligned}
& A_{2} \ddot{\xi}+A_{1} \dot{\xi}+A_{0} \xi=B_{0} v \\
x= & C_{2} \ddot{\xi}+C_{1} \dot{\xi}+C_{0} \xi+D_{0} v
\end{aligned}
$$

input form must be second order

MATRICES INPUT USING NAMELIST ARRAYS, OR TABLE FILE

## STATIC EQUATION

STATIC EQUATION $x=D_{s} v$ CAN BE USED TO ADD COMPONENT INPUT, IN GENERAL WITH MATRIX OF WEIGHTS
implements scalar addition of quantities
reference frame component used for vector addition of loads and motion

DEFINING $v$ AS CONCATENATED CONTROL VECTORS, AND $D_{s}$ AS CORRESPONDING CONCATENATED MATRICES, GIVES

$$
x=D_{s} v=\left[\begin{array}{lll}
\cdots & D_{i} & \cdots
\end{array}\right]\left(\begin{array}{c}
\vdots \\
f_{i} \\
\vdots
\end{array}\right)
$$

common application of this component

## OUTPUT TRANSFORMATION

## OPTIONALLY A NONLINEAR TRANSFORMATION OF THE OUTPUT VARIABLES CAN BE IMPLEMENTED

$x=F\left(x_{\text {lin }}\right)$, where $x_{\text {lin }}$ is calculated from the linear equations
transformation evaluated by linear interpolation (with no extrapolation) from set of values $F_{n}$ at $x_{n}$

## 5-11 Programmable Component

## PROGRAMMABLE COMPONENT IMPLEMENTS USER-DEFINED CALCULATIONS

component equations can be static, first order, or second order
BASIC VERSION EQUIVALENT TO DIFFERENTIAL EQUATION COMPONENT

## OTHER VERSIONS CONSTRUCTED BY MODIFYING SUBROUTINES CALLED BY PROGRAMMABLE COMPONENT

figure 17 illustrates functionality

## EQUATIONS

## VARIABLES

$\xi=$ degree of freedom vector
$x=$ component output vectors
$f=$ component input vectors
$v=$ differential equation input vector
$v$ is combination of component input $f$ :
can concatenate elements of input vectors;
or sum, average, or weighted sum of vectors

## STATIC EQUATIONS

$$
\begin{aligned}
F & =D_{s} v \\
x & =U_{B}(F, v, f, t)
\end{aligned}
$$

$U_{B}$ is user-defined function; $U_{B}=F$ for basic version input form of matrix static, first order, or second order; or matrices (hence $F$ ) can be ignored


Figure 5-17 Functionality of programmable component.

## FIRST ORDER EQUATIONS

$$
\begin{aligned}
E & =A \xi+B v-\dot{\xi} \\
F & =C \xi+D v \\
0 & =U_{A}(E, \dot{\xi}, \xi, v, f, t) \\
x & =U_{B}(F, \dot{\xi}, \xi, v, f, t)
\end{aligned}
$$

$U_{A}$ and $U_{B}$ are user-defined function; $U_{A}=E$ and $U_{B}=F$ for basic version
input form of matrices first order or second order; or matrices (hence $E$ and $F$ ) can be ignored

## SECOND ORDER EQUATIONS

$$
\begin{aligned}
E & =B_{0} v-\left(A_{2} \ddot{\xi}+A_{1} \dot{\xi}+A_{0} \xi\right) \\
F & =C_{2} \ddot{\xi}+C_{1} \dot{\xi}+C_{0} \xi+D_{0} v \\
0 & =U_{A}(E, \ddot{\xi}, \dot{\xi}, \xi, v, f, t) \\
x & =U_{B}(F, \ddot{\xi}, \dot{\xi}, \xi, v, f, t)
\end{aligned}
$$

$U_{A}$ and $U_{B}$ are user-defined function; $U_{A}=E$ and $U_{B}=F$ for basic version
input form of matrices second order; or matrices (hence $E$ and $F$ ) can be ignored

## MORE THAN ONE OUTPUT VECTOR CAN BE DEFINED

basic version of component gives $x=F$ for first output vector, and $x=0$ for all others

## MATRICES INPUT USING NAMELIST ARRAYS, OR TABLE FILE <br> if matrices are ignored, the equations of motion and output equations are entirely user-defined

## USER-DEFINED CALCULATIONS

USER-DEFINED CALCULATIONS IMPLEMENTED BY MODIFYING SUBROUTINES CALLED BY PROGRAMMABLE COMPONENT

| subroutine | operation |
| :--- | :--- |
| UPGMRD | read parameters |
| UPGMWT | write parameters |
| UPGMTB | tables required |
| UPGMIN | initialize |
| UPGMB0 | function $U_{B}$ (static) |
| UPGMB1 | function $U_{B}$ (first order) |
| UPGMB2 | function $U_{B}$ (second order) |
| UPGMA1 | function $U_{A}$ (first order) |
| UPGMA2 | function $U_{A}$ (second order) |

SUBROUTINE NAMES AND COMMON NAMES BEGINNING WITH LETTER " $u$ " ARE RESERVED FOR USER
exception: common UNITCM not available to user

INPUT PARAMETER SPECIFIES KIND OF USER-DEFINED CALCULATIONS TO BE PERFORMED

## 5-12 Transfer Function Component

## TRANSFER FUNCTION COMPONENT IMPLEMENTS FIRST ORDER DIFFERENTIAL EQUATIONS

DEFINED BY SINGLE (SCALAR) TRANSFER FUNCTION $H$ : $w=H v$
figure 18 illustrates functionality

## EQUATIONS

VARIABLES
transfer function input $v$ (scalar) $=$ weighted sum elements of component input vectors
component output vector = gain times transfer function output $x$ (scalar)

TRANSFER FUNCTION
$H=$ product of poles, zeros, gains, and lags
time lag modelled as one pole and one zero
poles must be unique
number of poles $\geq$ number of zeros $\geq 0$
CONSTRUCT EQUIVALENT FIRST ORDER DIFFERENTIAL EQUATION
analysis is in time domain, not frequency domain


Figure 5-18 Functionality of transfer function component.

## 5-13 Fourier Series Component

## FOURIER SERIES COMPONENT GENERATES TIME HISTORY FROM HARMONICS

figure 19 illustrates functionality

## FOURIER SERIES

COMPONENT EVALUATES $c$ IN TIME DOMAIN, FROM HARMONICS:

$$
c=G\left[f_{0}+\sum_{n=1}^{M}\left(f_{n c} \cos n \psi+f_{n s} \sin n \psi\right)\right]
$$

$G=$ constant gain value
$c$ can be calculated using subset of harmonics available

## COMPONENT INPUT

 component input $f$ is vector of harmonics:$$
f^{T}=\left(\begin{array}{lllll}
f_{0} & \cdots & f_{n c} & f_{n s} & \cdots
\end{array}\right)
$$

derivatives of $f$ may also be available

## COMPONENT OUTPUT

$c$ is scalar, but component output can include time derivatives


Figure 5-19 Functionality of Fourier series component.

## 5-14 Prescribed Control Component

## PRESCRIBED CONTROL COMPONENT GENERATES TRANSIENT TIME HISTORY

FOR USE IN TRANSIENT TASK
figure 20 illustrates functionality

## TIME HISTORY

COMPONENT EVALUATES $c$ FROM PRESCRIBED FUNCTION OF TIME:

$$
c=G F\left(t-t_{B}\right)
$$

$G=$ constant gain value
$t_{B}=$ time at which transient begins

## TIME HISTORY OPTIONS (FUNCTION $F(\tau)$ )

step, ramp
cosine impulse, sine doublet
square impulse, square doublet
triangular impulse, triangular doublet
general piecewise linear random (uniform, normal, or filtered)

## COMPONENT OUTPUT

component output can be $c$ (scalar), perhaps including derivatives
or component output can be vector of displacements (c times constant scale factor for each vector element)


Figure 5-20 Functionality of prescribed control component.

## 5-15 Gust Component

## GUST COMPONENT GENERATES AERODYNAMIC GUST VELOCITY GUST VELOCITY CALCULATED AT COLLOCATION POINTS ON SYSTEM

figure 21 illustrates functionality

## AERODYNAMIC GUST

GUST MODELS:
elementary (uniform, angular, or quadratic); prescribed (function of one position coordinate); tabular (function of 2, 3, or 4 of the 4 space/time coordinates)

## COMPONENT OUTPUT

gust velocity $v^{I}$, and perhaps angular velocity $\omega^{I}$ (perturbation velocities of air, in inertial axes)

## GUST AMPLITUDE (COMPONENT INPUT)

gust amplitude $v^{G}$ (and perhaps amplitude of angular velocity or gradients)
perturbation velocities of air, in negative wind/gust axes
figure 22 shows definition and sign conventions with aircraft conventions for wind/gust axes: $u_{G}$ positive aft, $v_{G}$ positive from right, $w_{G}$ positive up
gust amplitude used by all models; multiplied by shape factor for prescribed or tabular model


Figure 5-21 Functionality of gust component.


Figure 5-22 Velocity of air produced by wind and gust.

## GUST FIELD

in general, velocity in gust field depends on collocation point position in wind/gust axes, and on time
measured from some origin: typically origin inertial frame, perhaps origin of system base frame (moving with system)
optionally convected with wind velocity $W$

## CONVECTED GUST

free body moves through gust field with flight velocity $V$; gust field can be convected by wind velocity $W$
hence spatial variation of gust velocity produces time variation at system
figure 23 illustrates geometry

## UNIFORM GUST MODEL

VELOCITY FROM INPUT GUST AMPLITUDE
transformed from wind/gust axes to inertial axes
gust velocity varies with time, but uniform throughout space
velocity does not depend on collocation point position

## ANGULAR GUST MODEL

VELOCITY AND ANGULAR VELOCITY AT COLLOCATION POINT, FROM INPUT GUST AMPLITUDE


Figure 5-23 Geometry and sign conventions for convected gust.

## GRADIENT GUST MODEL

## VELOCITY FROM INPUT GRADIENT GUST AMPLITUDE TERMS; INCLUDING UNIFORM, LINEAR, AND QUADRATIC VARIATION IN SPACE

## PRESCRIBED GUST MODEL

VELOCITY AT COLLOCATION POINT, FROM GUST
AMPLITUDE TIMES SPATIAL SHAPE FACTOR
shape factor function of distance along one axis $(x, y$, or $z$-axis of wind/gust axes)
shape function options:
step, ramp
cosine impulse, sine doublet
square impulse, square doublet
triangular impulse, triangular doublet
general piecewise linear

## TABULAR GUST MODEL

## VELOCITY AT COLLOCATION POINT, FROM GUST

AMPLITUDE TIMES SHAPE FACTOR
separate shape factor for each of three gust velocity components
shape factors depend on two, three, or four of the four space/time coordinates in wind/gust axes ( $x, y, z, t$ )
obtained from 2D or 3D or 4D table
if gust amplitude has value of 1 , then table gives velocity of air relative inertial frame, in negative wind/gust axes

## 5-16 Aerodynamic Components

AERODYNAMIC COMPONENTS INCLUDE WINGS AND WAKES
WING AND WAKE COMPONENTS CAN BE SEPARATE OR COMBINED, DEPENDING ON THE MODEL AND SOLUTION PROCEDURE

WAKE GEOMETRY AND WAKE-INDUCED VELOCITY CAN BE CALCULATED BY SEPARATE COMPONENTS
figure 24 illustrates typical relation between aerodynamic components

## WINGS AND BODIES

WING OR BODY IS SURFACE MOVING THROUGH AIR (WING IS THIN SURFACE)

INTERFACES BETWEEN STRUCTURE AND AIR OCCUR AT SURFACE

INTERFACE IS DISCRETIZED: SET OF COLLOCATION POINTS
collocation points must be defined as connection points on structural dynamic components

GENERALLY INTERFACE BETWEEN STRUCTURAL DYNAMIC AND AERODYNAMIC COMPONENTS IS IN TERMS OF VELOCITY AND FORCE (DISCRETIZED) AT COLLOCATION POINTS
follows from boundary conditions of aerodynamic problem


Figure 5-24 Typical relation between aerodynamic components.

## INFLOW AND WAKES

## WAKE COMPONENT SOLVES FOR MOTION OF AIR

in principle, there is complete mutual interaction between wings and bodies of system (calculated from wakes)
in practice, some paths of influence may be neglected

## ONE WAKE PER WING

## COMPONENT MAY DEAL WITH ALL WAKES IN WING SET

by convention, mutual interference between wings in a set is completely calculated
one or more rotors is a wing set
momentum theory deals with entire rotor
calculation of mutual interference between aerodynamic components depends on how interfaces (in terms of interference velocity) are constructed

## INFLOW OR WAKE COMPONENT CALCULATES WAKEINDUCED VELOCITY AT A COLLOCATION POINT

collocation points can be on wing generating the wake, on another aerodynamic component, or at any other point in flow field
structural dynamic components refer to this velocity as interference velocity $v_{A}$, in the standard aerodynamic interface
in general, inertial frame is only common parent frame for wake geometry and all collocation points, so inertial axes used in wake analysis

MODEL CAN BE UNIFORM INFLOW ("INFLOW"
COMPONENTS) OR NONUNIFORM INFLOW ("WAKE" COMPONENTS)
vortex wake model typically used to calculate nonuniform induced velocity
or approximate wake model based on ideal-wing theory can be used
approximate model called "uniform inflow", although induced velocity might still have simple variation over surface of rotor disk

UNIFORM INFLOW SOLUTION GENERALLY USED TO INITIALIZE NONUNIFORM INFLOW SOLUTION, FOR BETTER CONVERGENCE

## NONUNIFORM INFLOW COMPONENTS ("WAKE")

CALCULATE INDUCED VELOCITY FROM INTEGRAL EQUATION

BASED ON DISCRETIZED VORTEX WAKE MODEL
integral equation arises from integration over the wake age, so velocity depends on loading at past times

TRIM AND TRANSIENT SOLUTION METHODS CAN EVALUATE INDUCED VELOCITY BY INTEGRATING OVER WAKE
but integral equation can not be linearized for flutter task
for flutter task, need set of differential equations equivalent to integral equation

## UNIFORM INFLOW COMPONENTS ("INFLOW")

> GLOBAL, QUASISTATIC REPRESENTATIONS OF WAKE FOR ROTOR, USING EMPIRICAL MODELS BASED ON MOMENTUM OR VORTEX THEORY
approximations usually require additional corrections in wing component (such as tip loss factor)

## QUASISTATIC MODEL IS DESIGNED TO BE USED WITH MEAN (FILTERED) VALUES OF ROTOR LOADING AND VELOCITY

so model can be used in all solution tasks, by using appropriate filters for trim, transient, and flutter models

## COMPONENT CAN ALSO INCLUDE DYNAMIC INFLOW MODEL

global, low frequency representation of wake
finite state model of unsteady aerodynamic effects of wake, relating parameters defining induced velocity and aerodynamic loading distributions on rotor, by means of ordinary differential equations
typically low order models (in time and space) for perturbations from trim solution, based on simplified representations of rotor aerodynamics
quasistatic version of dynamic inflow can be used, but frequently time lags are important

## WAKE GEOMETRY

## WAKE GEOMETRY DESCRIBES POSITION OF WAKE VORTICITY IN SPACE

undistorted geometry obtained from motion of wing wake element is convected by wind, from position in air at which it was created
this geometry is distorted by self-induced velocity of all wakes in system, and by wings and bodies

## DISTORTED WAKE GEOMETRY CALCULATED FOR SET OF WINGS

not necessarily same set as for calculation of induced velocity
specifying set (one or more rotors) defines what mutual interaction between wakes is accounted for in wake geometry

## WAKE GEOMETRY MODEL CAN BE RIGID, PRESCRIBED, OR FREE

rigid model calculates wake geometry distortion from convection by mean induced and interference velocity at wing
prescribed wake geometry obtained from empirical model, based on measurements
free wake geometry obtained by calculation
wake geometry model may depend solution status (wake loop level of trim task)

## AERODYNAMIC SOLUTION

## PARTITIONED SOLUTION PROCEDURE

solution often implemented using separate wing, wake, and wake geometry components
and calculations of single component can be performed in separate steps
so different wing sets can be used, depending on models available and mutual interaction accounted for
and different solution procedures and time steps can be used

## TYPICALLY:

## WAKE GEOMETRY COMPONENTS OBTAIN WAKE POSITION FROM MOTION AND LOADING

## THEN WAKE COMPONENTS CALCULATE INFLUENCE COEFFICIENTS FROM WAKE GEOMETRY, AND THEN INDUCED VELOCITY FROM INFLUENCE COEFFICIENTS AND LOADING

THEN WING COMPONENTS OBTAIN LOADING FROM INDUCED VELOCITY

## TRIM TASK CAN USE PARTITIONED SOLUTION PROCEDURE parts solve equations and loops iterate between part solutions

typically have wake geometry loop and circulation loop

## WAKE GEOMETRY LOOP

FOR EFFICIENCY, MOVE COMPUTATIONALLY INTENSIVE CALCULATIONS OUTSIDE INNER LOOPS (IF ALLOWED BY WEAK COUPLING)

CALCULATION OF WAKE GEOMETRY AND INFLUENCE COEFFICIENTS CAN BE MOVED OUTSIDE THE TRIM ITERATION
possible when coupling between wake geometry and rest of solution is relatively weak
hence when rotor is trimmed to specified speed, orientation, and lift

## WAKE GEOMETRY LOOP DIVIDES SYSTEM INTO CALCULATION OF WAKE GEOMETRY AND INFLUENCE COEFFICIENTS, AND SOLUTION OF REST OF EQUATIONS

successive substitution iteration, with stages or levels:
level 1 is uniform inflow
level 2 is nonuniform inflow with rigid or prescribed wake geometry
level 3 is nonuniform inflow with free wake geometry
for better convergence, uniform inflow used to initialize nonuniform inflow, and rigid wake geometry used to initialize free wake geometry
whether results of level 3 or level 2 are required depends on importance of free wake geometry and nonuniform inflow to the problem

AT EACH LEVEL, ITERATION IS NECESSARY IF ANYTHING IS BEING CALCULATED BY INNER LOOPS THAT WILL CHANGE WAKE GEOMETRY
such as speed; mean inflow; rotor lift or circulation; or inboard circulation peak locations
with trim solution inside wake geometry loop, may not be necessary to iterate on wake geometry at each level
if rotor is at fixed collective (rather than trimmed to specified lift), then overall geometry of wake not known in advance, only after rotor loading calculated
so if wake geometry significantly influences rotor loading, iteration required in wake geometry loop and need relaxation factor on wake geometry

## CIRCULATION LOOP

ITERATION BETWEEN WAKE OR INFLOW COMPONENTS (WHICH CALCULATE INDUCED VELOCITY FROM LOADING)

AND COMPONENTS OF REST OF SYSTEM (WHICH CALCULATE THE LOADING FOR FIXED INDUCED VELOCITY)
so can solve wake components (integral equations) and structural dynamic components (differential equations) by appropriate methods
and to reduce number of variables being solved in part

SUCCESSIVE SUBSTITUTION ITERATION
relaxation factor of $\lambda=0.10$ or 0.05 often required for nonuniform inflow models

## FOR HOVERING ROTOR NEAR ZERO THRUST, CIRCULATION ITERATION DOES NOT CONVERGE

modified inflow calculation required to achieve convergence

## 5-17 Rigid Airframe Aerodynamics Component

RIGID AIRFRAME AERODYNAMICS COMPONENT CALCULATES AERODYNAMIC FORCES AND MOMENTS ACTING ON A RIGID BODY

CONSISTING OF WING-BODY, HORIZONTAL TAIL, AND VERTICAL TAIL

THE "RIGID BODY" IS TYPICALLY A SINGLE STRUCTURAL DYNAMIC COMPONENT
figure 25 illustrates functionality

## AERODYNAMIC MODEL

QUASISTATIC MODEL, USING SMALL NUMBER OF COLLOCATION POINTS
intended for static and low frequency motion of airframe designed to be used with mean (filtered) values of component input


Figure 5-25 Functionality of rigid airframe aerodynamics component.

## ELASTIC MOTION INVOLVES LOCALIZED, HIGH FREQUENCY OSCILLATIONS

wake model and noncirculatory loads needed for high frequency aerodynamics are not included
component model not consistent with elastic motion
so velocity at collocation points should be calculated without elastic motion, and aerodynamic loads should not be included in elastic equations of motion of airframe

## THREE OR FOUR COLLOCATION POINTS ON THE AIRFRAME

wing-body, horizontal tail, and vertical tail (and perhaps a collocation point for stability derivatives)
velocity relative air required at collocation points
collocation point is point-of-action of corresponding force and moment
velocities, angular motion, and loads are in body axes of airframe

## AERODYNAMIC PROPERTIES

aerodynamic properties of airframe defined in velocity axes
airframe angle of attack $\alpha$ and sideslip angle $\beta$ define orientation of velocity axes relative body axes
figure 26 shows conventions for velocity axis forces
angle of attack + for linear velocity in $z$-axis direction
sideslip + for linear velocity in $y$-axis direction
yaw + for rotation about $z$-axis
horizontal tail can have cant angle $\phi_{H T}$ (+ to left, so at 90 degrees equivalent to vertical tail)
vertical tail can have cant angle $\phi_{V T}$ (+ to right, so at 90 degrees equivalent to horizontal tail)

## AIRFRAME AERODYNAMIC CONTROLS

flaperon $\delta_{f}$ (positive for down deflection, increasing wingbody lift)
elevator $\delta_{e}$ (positive for down deflection, increasing horizontal tail lift)
aileron $\delta_{a}$ (positive for down deflection on right wing, causing a negative roll moment)
rudder $\delta_{r}$ (positive for down deflection, increasing vertical tail lift)


Figure 5-26 Velocity axis loads.

## AERODYNAMIC LOAD CAN BE CALCULATED USING NONLINEAR MODEL OR STABILITY DERIVATIVES (LINEARIZED) MODEL

## TRIM TASK USES NONLINEAR MODEL

nonlinear loads calculated from simple equations and/or tables
can also be used in transient and flutter tasks separate properties can be used for wing-body, horizontal tail, and vertical tail
so can have separate aerodynamic interference effects at three collocation points or tail loads can be included in wing-body properties

## LINEARIZED MODEL EVALUATES LOADS FROM STABILITY DERIVATIVES

can only be used in transient and flutter tasks perturbation (total minus trim) loads calculated from perturbation velocity conventional aircraft model, useful for tiltrotor configuration

## NONLINEAR MODEL

VELOCITY AXIS LOADS ACTING ON WING-BODY (WB), HORIZONTAL TAIL (HT), AND VERTICAL TAIL (VT)
wing-body drag, side, and lift forces: $D, Y$, and $L$ (along $x, y$, and $z$ axes respectively, acting at wing-body collocation point)
wing-body roll, pitch, and yaw moments: $M_{x}, M_{y}$, and $M_{z}$ (about $x, y$, and $z$ axes respectively, with origin at wingbody collocation point)
horizontal tail drag and lift forces: $D$ and $L$
vertical tail drag and lift forces: $D$ and $L$
loads defined in terms of force or moment divided by dynamic pressure $q$

TEN LOADS ( $6 \mathrm{WB}, 2 \mathrm{HT}, 2 \mathrm{VT}$ ), EACH OBTAINED FROM ONE OF FOLLOWING SOURCES

## equations

two-dimensional table
function of $\alpha$ or $\beta$, and Mach number $M$
three-dimensional or four-dimensional tables
function of $\alpha, \beta, M$, flaperon, elevator, aileron, rudder, or flap (any combination of three or four independent variables)

## MODEL INCLUDES EFFECTS OF AERODYNAMIC INTERFERENCE FROM WING-BODY ON THE TAIL

tail angle of attack change $\epsilon$ from wing body lift, including time lag from $\dot{\alpha}_{W B}$
tail sideslip change $\sigma$ from wing-body roll rate

## EQUATIONS

FOR WING BODY:

$$
\begin{aligned}
&\left(\begin{array}{c}
M_{y} / q \\
D / q \\
L / q
\end{array}\right)=\left(\begin{array}{c}
M_{0} / q+\left(M_{\alpha} / q\right)\left(\alpha_{W B}+i_{W B_{M}}\right) \\
f_{W B}+\left(g_{W B}-f_{W B}\right) \sin ^{2}\left(\alpha_{W B}+i_{W B_{D}}\right)+D_{I}(L / q)^{2} \\
L_{0} / q+\left(L_{\alpha} / q\right)\left(\alpha_{W B}+i_{W B_{L}}\right)
\end{array}\right) \\
&+\left(\begin{array}{c}
M_{\delta_{e}} / q \\
D_{\delta_{e}} / q \\
L_{\delta_{e}} / q
\end{array}\right) \delta_{e}+\left[\begin{array}{cc}
M_{\delta_{f}} / q & M_{\delta_{F}} / q \\
D_{\delta_{f}} / q & D_{\delta_{F}} / q \\
L_{\delta_{f}} / q & L_{\delta_{F}} / q
\end{array}\right]\binom{\delta_{f}}{\delta_{F}} \\
&\left(\begin{array}{c}
M_{x} / q \\
M_{z} / q \\
Y / q
\end{array}\right)=\left(\begin{array}{c}
N_{x 0} / q+\left(N_{x_{\beta}} / q\right) \beta_{W B} \\
N_{z 0} / q+\left(N_{z_{\beta}} / q\right) \beta_{W B} \\
Y_{0} / q+\left(Y_{\beta} / q\right) \beta_{W B}
\end{array}\right)+\left(\begin{array}{c}
N_{x_{\delta_{r}} / q} / q \\
N_{z_{\delta_{r}}} / q \\
Y_{\delta_{r}} / q
\end{array}\right) \delta_{r}+\left[\begin{array}{ccc}
V N_{x_{p}} / q & V N_{x_{r}} / q & N_{x_{\delta_{a}}} / q \\
V N_{z_{p}} / q & V N_{z_{r}} / q & N_{z_{\delta_{a}}} / q \\
V Y_{p} / q & V Y_{r} / q & Y_{\delta_{a}} / q
\end{array}\right]\left(\begin{array}{c}
p / V \\
r / V \\
\delta_{a}
\end{array}\right)
\end{aligned}
$$

elevator and rudder terms in wing-body loads should only be used if tail loads are not present
last two terms in $D / q$ both give $\alpha^{2}$ dependence second term represents stall drag third term is induced drag EXPECTED SIGNS OF DERIVATIVES: $Y_{\beta}, N_{z_{\beta}}$, $N_{x_{\delta a}}$ negative

FOR HORIZONTAL TAIL:

$$
\binom{D / q}{L / q}=\binom{f_{H T}+\left(g_{H T}-f_{H T}\right) \sin ^{2}\left(\alpha_{H T}+i_{H T_{D}}\right)}{L_{0} / q+\left(L_{\alpha} / q\right)\left(\alpha_{H T}+i_{H T_{L}}\right)}+\binom{D_{\delta_{e}} / q}{L_{\delta_{e}} / q} \delta_{e}
$$

FOR VERTICAL TAIL:
$\binom{D / q}{L / q}=\binom{f_{V T}+\left(g_{V T}-f_{V T}\right) \sin ^{2}\left(\alpha_{V T}+i_{V T_{D}}\right)}{L_{0} / q+\left(L_{\alpha} / q\right)\left(\alpha_{V T}+i_{V T_{L}}\right)}+\binom{D_{\delta_{r}} / q}{L_{\delta_{r}} / q} \delta_{r}$
incidence angles $i$ measured relative body axes, such that zero load obtained for $\alpha=-i$
lift and moment stall modelled by using truncated angle of attack

$$
\alpha_{e}=\operatorname{sign}(\alpha) \min \left(|\alpha|, \alpha_{\max }\right)
$$

## TWO-DIMENSIONAL TABLE

equations still needed for rate, flaperon, aileron, and flap terms; and for elevator and rudder terms

FOR WING-BODY:

$$
\begin{gathered}
\left(\begin{array}{c}
M_{y} / q \\
D / q \\
L / q
\end{array}\right)=\left(\begin{array}{c}
S c C_{M} \\
S C_{D} \\
S C_{L}
\end{array}\right)+\left(\begin{array}{c}
M_{\delta_{e}} / q \\
D_{\delta_{e}} / q \\
L_{\delta_{e}} / q
\end{array}\right) \delta_{e}+\left[\begin{array}{cc}
M_{\delta_{f}} / q & M_{\delta_{F}} / q \\
D_{\delta_{f}} / q & D_{\delta_{F}} / q \\
L_{\delta_{f}} / q & L_{\delta_{F}} / q
\end{array}\right]\binom{\delta_{f}}{\delta_{F}} \\
\left(\begin{array}{c}
M_{x} / q \\
M_{z} / q \\
Y / q
\end{array}\right)=\left(\begin{array}{c}
S b C_{\ell} \\
S b C_{n} \\
S C_{y}
\end{array}\right)+\left(\begin{array}{c}
N_{x_{\delta_{r}}} / q \\
N_{z_{\delta_{r}} / q} / q \\
Y_{\delta_{r}} / q
\end{array}\right) \delta_{r}+\left[\begin{array}{ccc}
V N_{x_{p}} / q & V N_{x_{r}} / q & N_{x_{\delta_{a}} / q} \\
V N_{z_{p}} / q & V N_{z_{r}} / q & N_{z_{\delta_{a}} / q} / \\
V Y_{p} / q & V Y_{r} / q & Y_{\delta_{a}} / q
\end{array}\right]\left(\begin{array}{c}
p / V \\
r / V \\
\delta_{a}
\end{array}\right)
\end{gathered}
$$

elevator and rudder terms in wing-body loads should only be used if tail loads are not present
$C_{M}, C_{D}$, and $C_{L}$ function of $\alpha_{W B}$ and Mach number
$C_{\ell}, C_{n}$, and $C_{y}$ function of $\beta_{W B}$ and Mach number coefficients based on wing-body area $S$, chord $c$, span $b$

FOR HORIZONTAL TAIL:

$$
\binom{D / q}{L / q}=\binom{S C_{D}}{S C_{L}}+\binom{D_{\delta_{e}} / q}{L_{\delta_{e}} / q} \delta_{e}
$$

FOR VERTICAL TAIL:

$$
\binom{D / q}{L / q}=\binom{S C_{D}}{S C_{L}}+\binom{D_{\delta_{r}} / q}{L_{\delta_{r}} / q} \delta_{r}
$$

$C_{D}$ and $C_{L}$ function of $\alpha_{H T}$ or $\alpha_{V T}$, and Mach number
coefficients based horizontal or vertical tail area $S$

## THREE-DIMENSIONAL OR FOUR-DIMENSIONAL TABLES

coefficients function of $\alpha, \beta, M, \delta_{f}, \delta_{e}, \delta_{a}, \delta_{r}, \delta_{F}$ (any combination of three or four independent variables)
equations still needed for rate terms, and perhaps for the control and flap terms
control effects may be in equations or tables, but should not be in both
loads are calculated using same equations as for twodimensional table

FOR WING-BODY:
elevator and rudder terms should only be used if tail loads are not present

FOR HORIZONTAL AND VERTICAL TAILS:
sideslip angle, flaperon deflection, aileron deflection, and flap angle ignored
two-dimensional table: coefficients must all be functions of same independent variables (angle and Mach number)
three-dimensional or four-dimensional tables: different independent variables can be defined for each coefficient
if only one value in table for third independent variable, 3D table is effectively 2D

## 5-18 Airframe Flow Field Component

## AIRFRAME FLOW FIELD COMPONENT CALCULATES PERTURBATION AERODYNAMIC INTERFERENCE VELOCITY PRODUCED BY AIRFRAME (OR OTHER OBJECT)

figure 27 illustrates functionality

## AERODYNAMIC MODEL

QUASISTATIC MODEL
designed to be used with mean (filtered) values of component input
shed wake and noncirculatory loads needed for high frequency aerodynamics are not included
by using appropriate frame for component, location and velocity of wings and bodies can be filtered to eliminate high frequency oscillations
then flow field is quasistatic relative to component frame location of collocation point relative wings and bodies might still be time varying, so should not be filtered

INTERFERENCE SOURCES:
velocity calculated
velocity from table file


Figure 5-27 Functionality of airframe flow field component.

## CALCULATED INTERFERENCE VELOCITY

## PRODUCED BY SIMPLE REPRESENTATION OF AIRFRAME: SET OF WINGS AND BODIES

wing modelled as constant strength horseshoe vortex (lift) and constant strength doublet line (thickness)
body modelled by potential flow (no separation) about nonlifting body-of-revolution at arbitrary angle of attack
velocities calculated at actual locations of collocation points relative airframe
approximate model, requiring calibration for reasonable results near the wings and bodies
figure 28 shows the geometry

## WINGS

wing defined by quarter-chord, in two straight segments input = locations of left tip, middle, and right tip (at quarter chord)
wing not necessarily horizontal or symmetric
right-hand rule for vector from "left" tip to "right" tip defines positive direction of bound circulation (hence wing upper surface)
circulation and thickness lines placed distance $x c_{w}$ behind leading edge (to allow calibration of velocity field near wing)
for far field velocity, circulation line should be at $1 / 4$ chord, and thickness line should be at $3 / 8$ chord


Figure 5-28 Geometry of wings and bodies.
wing circulation strength is:

$$
\frac{\Gamma}{V}=\frac{L / q}{2 b_{w}}=\frac{c_{w} C_{L}}{2}
$$

evaluated from combination of input value, and the wing-body, horizontal tail, and vertical tail lifts:

$$
\frac{\Gamma}{V}=\frac{\Gamma_{0}}{V}+\frac{1}{2 b_{w}}\left[f_{W}(L / q)_{W B}+f_{H}(L / q)_{H T}+f_{V}(L / q)_{V T}\right]
$$

with factors $f_{W}, f_{H}, f_{V}$ to calibrate model
dipole strength obtained from wing airfoil cross section area $A_{X S}$ (approximately $0.68 \tau_{w} c_{w}^{2}$ )

## BODIES

body defined by axis, length, and thickness ratio input = locations center and nose (on axis)
figure 28 shows the geometry
shapes:
ovary (thin) ellipsoid, created by revolving ellipse about its major axis (exact solution)
planetary (flat) ellipsoid, created by revolving ellipse about its minor axis (exact solution)
sphere (exact solution)
airfoil-shaped body, created by revolving NACA 4-digit airfoil thickness distribution about chord line (modified slender-body theory solution)

## VELOCITY TABLE FILE

## PROVIDES ACCESS TO EXTERNALLY CALCULATED INTERFERENCE

typically obtained using panel method with detailed representation of airframe geometry
velocities calculated for specified locations of collocation points relative airframe

## TWO-DIMENSIONAL TABLE

dependent variables are three components of interference velocity $\Delta v^{F} / V$, in component frame F
first independent variable $=$ span station value interpolated to span station of collocation points
second independent variable $=$ time or azimuth interpolated (cyclically for azimuth) to current time value
with only one time value, table is function of span station only

## 5-19 Lifting Line Wing Component

## LIFTING LINE WING COMPONENT CALCULATES AERODYNAMIC FORCES ON A WING

AERODYNAMIC MODEL IS LIFTING-LINE THEORY, USING STEADY TWO-DIMENSIONAL AIRFOIL CHARACTERISTICS AND A VORTEX WAKE

WAKE MODEL IMPLEMENTED IN SEPARATE COMPONENT
wing component has as input the wake-induced interference velocity (and any other interference terms in velocity of air)
figure 29 illustrates functionality

## AERODYNAMIC MODEL

LIFTING-LINE THEORY ASSUMES THAT WING HAS HIGH ASPECT RATIO
more generally, assume small spanwise variations of aerodynamic environment
assumption allows problem to be split into separate wing and wake models
solved individually and combined
assumptions of lifting-line theory generally well satisfied for rotor blades


Figure 5-29 Functionality of lifting line wing component.
this component solves wing problem of lifting-line theory viscous and compressibility effects included by using experimental data for two-dimensional airfoil characteristics
other components solve wake problem of lifting-line theory vortex wake model used to calculate nonuniform induced velocity on wing
or approximate wake model based on ideal-wing theory can be used ("uniform inflow")
with uniform inflow, three-dimensional flow effects at wing tips are lost, requiring additional corrections in wing model (tip loss factor)

## MODEL FEATURES:

stall models (static, empirical dynamic stall model, none) unsteady lift and moment for attached flow yawed and swept flow corrections, and spanwise drag drag and moment increments, zero lift angle increment static stall delay

Reynolds number corrections
trailing edge flaps
prescribed lift, drag, and moment increments from an external aeroacoustic analysis

## WING IS SURFACE, DESCRIBED BY SET OF QUARTER CHORD AND THREE-QUARTER CHORD COLLOCATION POINTS ALONG SPAN

lifting-line theory deals with section aerodynamic environment
required section geometry (bent-chord axes) obtained from geometry of collocation points arbitrary geometry, including droop and sweep of quarter-chord line, and twist
geometry actually calculated on structural dynamic side of interfaces with wing component
component interfaces are velocity and force normal to wing surface (discretized)
component analyzes the entire wing
figure 30 shows relation between geometry of wing and geometry of structure


Figure 5-30 Relation between wing geometry and structure geometry.

## BASIC LIFTING-LINE THEORY

## THREE-DIMENSIONAL WING AERODYNAMIC PROBLEM IS UNSTEADY, COMPRESSIBLE, VISCOUS <br> ASSUMPTION OF HIGH ASPECT-RATIO SPLITS IT INTO INNER AND OUTER PROBLEMS - INTO WING AND WAKE MODELS

figure 31 illustrates approach

## OUTER PROBLEM IS WAKE

trailed and shed vorticity behind lifting-line (bound vortex)

## INNER PROBLEM IS TWO-DIMENSIONAL AIRFOIL

infinite wing in uniform, yawed free stream

## CONNECTED THROUGH THE WAKE-INDUCED VELOCITY AND THE BOUND CIRCULATION

outer problem calculates induced velocity at wing, from wake with strength determined by bound circulation
induced velocity not needed at arbitrary point, just at lifting-line
inner problem calculates bound circulation from aerodynamic environment, with wake-induced velocity included in free stream
pressure on wing not needed, just bound circulation (and section lift, drag, and moment in order to calculate performance and couple with structural dynamics)

UNIFORM INFLOW FROM IDEAL-WING (MOMENTUM OR VORTEX) THEORY IS APPROXIMATION FOR SOLUTION OF OUTER PROBLEM (WAKE)


Figure 5-31 Lifting line theory.

## PERTURBATION THEORY

FORMAL LIFTING-LINE THEORY IS SOLUTION FOR THREEDIMENSIONAL WING LOADING USING METHOD OF MATCHED ASYMPTOTIC EXPANSIONS
lowest-order fixed wing solution is Prandtl's theory (steady, no sweep)
developments found in literature include higher orders, unsteady, transonic, and swept flow
higher order theories from Weissinger (intuitive) and van Dyke (perturbation theory)
these theories generally obtain analytical solutions for both inner and outer problems, and are in quadrature rather than integral form
often inner solution is inviscid, or even a thin airfoil

## PERTURBATION THEORY PROVIDES GUIDE TO

 DEVELOPMENT OF PRACTICAL APPROACHfor rotary wing, must include stall (high angles of attack) in inner solution, and include distorted and rolled-up wake geometry in outer solution

## WANT LIFTING-LINE THEORY WITH

numerical solutions for inner and outer problems, iterative method based on matching procedure
must retain two-dimensional airfoil tables in inner solution, for viscous effects
accept whatever approximations are required to retain airfoil tables

## SECOND-ORDER LIFTING-LINE THEORY

second-order lifting-line theory gives nearly same results as lifting-surface theory, including lift produced in close wing-vortex interactions
second-order theory also improves loads calculations for swept tips, yawed flow, and low aspect-ratio wings
second-order outer solution
wing is dipole line + quadrupole line, equivalent to dipole at quarter chord
dipole solution is wake of vortex sheets
second-order inner solution
boundary condition is wake-induced velocity varying linearly in space
get same lift with uniform induced velocity, by using value at three-quarter chord
moment error, since linear induced-velocity variation over chord produces moment about quarter chord, but uniform induced-velocity does not
moment still first order
RETAINING AIRFOIL TABLES, SECOND-ORDER THEORY ONLY MEANS PLACING LIFTING-LINE AT QUARTER CHORD AND COLLOCATION POINT AT THREE-QUARTER CHORD
first-order theory has collocation point at quarter chord

## PRACTICAL IMPLEMENTATION OF LIFTING-LINE THEORY OUTER PROBLEM IS INCOMPRESSIBLE VORTEX WAKE BEHIND LIFTING-LINE, WITH DISTORTED GEOMETRY AND ROLLUP

lifting-line (bound vortex) at quarter chord
approximation for quadrupole line of second-order loading
trailed wake begins at bound vortex
shed wake created quarter chord aft of collocation point on the wing
lifting-line approximation for unsteady loading
three components of wake-induced velocity evaluated at collocation points, excluding contributions of bound vortex collocation points at three-quarter chord (in direction of local flow)
approximation for linearly varying induced velocity introduced by second-order wake
induced velocity at three-quarter chord used only to calculate angle of attack for loading solution
local section aerodynamic environment, including orientation of lift and drag and hence magnitude of induced power, still obtained from induced velocity at quarter chord

INNER PROBLEM IS UNSTEADY, COMPRESSIBLE, VISCOUS FLOW ABOUT INFINITE ASPECT-RATIO WING, IN UNIFORM FLOW CONSISTING OF YAWED FREE STREAM AND THREE COMPONENTS OF WAKE-INDUCED VELOCITY
split into parts
two-dimensional, steady, compressible, viscous flow = airfoil tables
plus corrections
corrections account for
swept and yawed flow
tip flow
Reynolds number
unsteady flow: small angle-of-attack noncirculatory loads, without any shed wake
dynamic stall: empirical model

## GENERALLY SECOND-ORDER ACCURATE FOR LIFT

including effects of sweep and yaw
with typical blade-vortex separations, as accurate as lifting-surface theory for vortex-induced lift calculations

BUT LESS ACCURATE FOR SECTION MOMENTS

## YAWED AND SWEPT FLOW CORRECTIONS

INNER PROBLEM IS INFINITE WING WITH YAW AND SWEEP
planform defined relative to wing reference line, so spanwise velocity produces yawed flow, while sweep obtained from locus of quarter chord relative to wing reference line
figure 32 shows geometry
SECTION LOADS OBTAINED FROM AIRFOIL TABLES (SOLUTION FOR WING WITH NO YAW OR SWEEP), USING EQUIVALENCE ASSUMPTION FOR SWEPT WINGS
must derive equivalent angle of attack and Mach number for evaluating coefficients, and corrections for coefficients from tables
and correctly account for chord and wing area when multiplying coefficients by chord, dynamic pressure, and panel width to obtain section loads


Figure 5-32 Swept and yawed wing aerodynamics.

## PRINCIPAL ASSUMPTIONS

lift-curve slope of normal sections not affected by spanwise flow

Mach number normal to (swept) quarter chord defines compressibility effects
angle of attack and chord in local flow direction define drag and stall
total viscous drag force (vector addition of spanwise and chordwise components) has same direction as local yawed flow, so spanwise drag component can be obtained from section drag coefficient
skin friction drag not pressure drag that has spanwise component; really in direction of velocity at bottom not top of boundary layer
analysis can calculate spanwise drag from chordwise drag coefficient, or from drag coefficient at zero angle of attack

## NO CORRECTIONS IMPLEMENTED FOR EFFECTIVE SHAPE AND THICKNESS OF AIRFOIL IN YAWED FLOW

all information about shape and thickness is in airfoil tables
probably airfoil tables should correspond to shape of cross-section in local flow direction (or mean direction for time-varying flow)
so for rotor blade, shape perpendicular to wing reference line (straight or drooped) through center of rotation is appropriate
which may or may not correspond to how the section geometry was defined

## DISCUSSING HERE HOW TO ANALYZE WING IN SWEPT/YAWED FLOW

how to manufacture wing is separate issue
can define airfoil sections relative straight reference axis, or relative swept quarter chord
how to describe wing geometry is separate issue
here describe planform by chord measured perpendicular to reference line (usually straight reference line, so wing area is easily calculated)

## STATIC STALL DELAY

THERE IS EVIDENCE THAT ROTATIONAL EFFECTS ON BOUNDARY LAYER PRODUCE A DELAY OF SEPARATION, PARTICULARLY FOR INBOARD SECTIONS OF ROTATING WINGS
three-dimensional aerodynamic phenomenon
modelled using input stall delay factors to modify the lift, drag, and moment coefficients obtained from tables

TIP FLOW CORRECTIONS
COMPRESSIBLE TIP RELIEF
small reduction in effective Mach number implemented using input factor for Mach number

## TIP LOSS FACTOR

needed when induced velocity is obtained from uniform inflow model (an approximation for wake solution)
wing has no lift outboard of span station $B$
typically tip loss factor $B$ represents about $3 \%$ of wing span

## TIP FLOW CORRECTIONS

## SPAN STATION OF ROLLED UP TIP VORTEX

tip loss factor not used for nonuniform inflow, but must consider span station of rolled-up tip vortex when it reaches trailing edge of wing, $r_{T V}$
wing has no circulation outboard of $r_{T V}$
figure 33 illustrates formation of vortex on wing tip implementation similar to tip loss factor typically tip vortex forms 1-2\% span inboard of wing tip (rectangular planform)
little effect on loading; should suppress effect to avoid convergence problems
for highly tapered tip, vortex can form 5-6\% span inboard of wing tip
significant effect on loading
if all or most of an aerodynamic panel is outboard of $r_{T V}$, wake induces large upwash at it, perhaps stalling wing section
for highly tapered tip this is representation of physical effect
for rectangular tip it is just source of convergence problems

TIP VORTEX INITIAL SPAN STATION IN UNDISTORTED WAKE GEOMETRY CAN BE INBOARD OF WING TIP
feature of wake geometry model, in addition to this affect on loading

## RECTANGULAR PLANFORM



HIGHLY TAPERED TIP


Figure 5-33 Formation of vortex on wing tip.



## REYNOLDS NUMBER CORRECTIONS

IF THE REYNOLDS NUMBER $R e$ OF ROTOR SECTION IS NOT EQUAL REYNOLDS NUMBER $R e_{t}$ OF AIRFOIL TABLE, DRAG AND LIFT COEFFICIENTS CAN BE CORRECTED for airfoil Reynolds number lower than table, drag increased and maximum lift decreased correction uses factor $K=\left(R e / R e_{t}\right)^{n}, n=0.125$ to 0.2

## TABLE REYNOLDS NUMBER CALCULATED FROM

 REYNOLDS NUMBER CORRESPONDING TO MACH NUMBER $M=1: R e_{t}=M_{e} R e_{1}$approximation for $R e$ vs $M$ of table this correction is the only use analysis makes of Reynolds number information in airfoil table

## ADJUSTMENTS TO AIRFOIL TABLE DATA (FUNCTION OF SPAN STATION)

LIFT, DRAG, MOMENT COEFFICIENT INCREMENTS; AERODYNAMIC CENTER INCREMENT provide means to modify aerodynamic characteristics without changing airfoil table
examples: drag increment for Reynolds number effect; aerodynamic center increment for three-dimensional effects at wing tip

TRAILING-EDGE FLAP DERIVATIVES;
MACH NUMBER, MAXIMUM LIFT, DRAG CORRECTIONS;
STATIC STALL DELAY;
AIRFOIL TABLE $\alpha$ AND MOMENT REFERENCES

## UNSTEADY LOADS IN ATTACHED FLOW

## NEED THE NONCIRCULATORY LOADS (INDEPENDENT OF SHED WAKE)

static loads obtained from airfoil tables, and usually shed wake effects accounted for through wake-induced velocity noncirculatory terms essential for aerodynamic pitch damping, and sometimes unsteady lift and even virtual mass terms are important

## MODELS (THIN-AIRFOIL THEORY):

INCOMPRESSIBLE
ONERA EDLIN LEISHMAN-BEDDOES
including effects of time-varying free stream and reverse flow, and trailing-edge flaps
options for lift-curve slope, aerodynamic center, and drag recovery factor

## SHED WAKE CAN BE IN WING COMPONENT OR IN WAKE COMPONENT

if shed wake is in wing component, omit near shed vorticity of wake component
with near shed vorticity in wake component, or with dynamic inflow, should not include shed wake in wing component
however, Leishman-Beddoes "circulatory" terms include more than shed wake effects

## DYNAMIC STALL

DYNAMIC STALL CHARACTERIZED BY
delay in occurrence of separated flow produced by wing motion
high transient loads induced by vortex shed from leading edge when stall does occur

## EMPIRICAL DYNAMIC STALL MODELS IMPLEMENTED: <br> JOHNSON <br> BOEING <br> LEISHMAN-BEDDOES <br> ONERA EDLIN <br> ONERA BH

still use airfoil table for steady characteristics, evaluated at effective angle of attack that includes the dynamic stall delay
require 2D table of dynamic stall parameters (function of Mach number and span station)
sample cases include table for NACA 0012 airfoil

## UNSTEADY LOADS AND DYNAMIC STALL MODELS MAY INTRODUCE STATES

usually implicit (solved internal to component)
or the component can have degrees of freedom and equations of motion

## GEOMETRY AND AXES

figure 34 shows geometry of wing model ANALYSIS IS PERFORMED IN AXES OF WING FRAME

## WING REFERENCE LINE USED TO DEFINE GEOMETRY

arbitrary curvature and torsion of reference line straight reference line simplest, and frequently sufficient rotorcraft shell uses straight line through center of rotation for wing reference line

## WING CONSISTS OF SET OF AERODYNAMIC PANELS

quarter chord and three-quarter chord collocation point at each panel
component input at each collocation point:
motion relative air velocity including interference terms dynamic pressure rate of change of velocity
location of wing, measured from the origin of the wing frame
position
rate of change of position
component output at each collocation point:
aerodynamic force


Figure 5-34 Geometry for definition and analysis of wing.

## WING DESCRIPTION

geometry defined relative to wing reference line
section properties measured in planes perpendicular to wing reference line chord, quarter chord position, pitch arbitrary sweep and droop (offsets of quarter chord from wing reference line)
arbitrary twist (from positions of three-quarter chord and quarter chord points in section plane)
span station $r$ identifies positions on wing reference line positive $r$ runs from root to tip of rotor blade for both counter-clockwise and clockwise rotation
span scale factor $R$ such that $R r$ is true measure of distance along reference line
for rotor blade with straight reference line and no droop, $r=$ dimensionless radial variable and $R=$ blade radius
with large droop, will need wing reference line that includes the droop
need not change definition of span station $r$ panel width (including $R$ ) times chord gives wing area

## INPUT DATA

aerodynamic panels defined by span stations at edges wing tips are first and last edges collocation points (QC and 3QC) are at panel midpoints
aerodynamic properties defined at arbitrary span stations, with linear variation between

## ANALYSIS

aerodynamic analysis in section axes
chord, normal, tangent vectors calculated from positions of collocation points
loads in sections axes are chord and normal forces conventional output available in wing plane axes (rotated from section axes by pitch angle $\theta$, so $x$-axis is in hub plane)
figure 35 shows section aerodynamic environment
calculate section forces and pitch moment at point $c x_{A F}$ aft of airfoil leading edge
$x_{A F}$ is position of moment reference axis of airfoil table moment data
$\theta_{A F}$ accounts for difference between twist definition and chord line of airfoil table data

wing plane axes W


Figure 5-35 Section aerodynamic environment (nose up pitch moment and spanwise drag not shown; lift and drag act a distance $c x_{\mathrm{AF}}$ aft of the leading edge).

## TRAILING-EDGE FLAPS

WING CAN HAVE ONE OR MORE TRAILING-EDGE FLAPS
each flap extends over one or more aerodynamic panels flap deflection angle $\phi$ and its derivatives identified as elements of some control vector (component input)

## AIRFOIL TABLE FILE INCLUDES EFFECTS OF FLAPS

total coefficients and flap coefficients, function of $\alpha, M, \phi$, span station
in addition, flap deflection can produce increments in the coefficients through input derivatives

UNSTEADY LOADS INCLUDE EFFECTS OF FLAPS
with aerodynamically balanced flap (flap leading edge forward of flap hinge), unsteady loads depend on whether gap at flap leading edge is sealed or open

## EXTERNAL AEROACOUSTIC ANALYSIS

## CAMRAD II CAN BE USED WITH EXTERNAL AEROACOUSTIC ANALYSIS

external analysis typically employs methods of computational fluid dynamics
possibly with limited computational domain that does not encompass entire wing wake
external analysis may not calculate effects of structural dynamic motion on aerodynamic boundary conditions
figure 36 illustrates communcation between CAMRAD II and external aeroacoustic analysis

PROCEDURE KNOWN AS LOOSE COUPLING

COMMUNICATION BASED ON WING MOTION (AERODYNAMIC OR STRUCTURAL DYNAMIC SENSORS), AND PRESCRIBED INCREMENTS IN SECTION COEFFICIENTS

## IF CFD ANALYSIS ACCOUNTS FOR ENTIRE FLOW FIELD, THEN IT REQUIRES JUST STRUCTURAL MOTION

otherwise, may use partial angle-of-attack $\alpha_{P}$ for boundary condition
so wake not used twice, wake-induced velocity used to evaluate $\alpha_{P}$ is calculated excluding wake vorticity in computational domain of external analysis
$\alpha_{P}$ includes structural motion


Figure 5-36 Communication with an external aeroacoustic analysis.

SETTING LIFT COEFFICIENT TO VALUE FROM EXTERNAL ANALYSIS DOES NOT ACCOUNT FOR CHANGES IN ANGLE-OF-ATTACK AS WING MOTION AND WAKE EFFECTS ARE UPDATED
approach:

$$
\begin{aligned}
c_{\ell \text { total }}(\alpha) & =c_{\text {ext }}\left(\alpha_{\text {old }}\right)+c_{\ell_{\alpha}}\left(\alpha-\alpha_{\text {old }}\right) \\
& =c_{\text {ext }}\left(\alpha_{\text {old }}\right)+c_{\ell \text { int }}(\alpha)-c_{\ell \text { old }}\left(\alpha_{\text {old }}\right) \\
& =c_{\ell \text { int }}(\alpha)+\left(c_{\ell \text { ext }}\left(\alpha_{\text {old }}\right)-c_{\ell \text { old }}\left(\alpha_{\text {old }}\right)\right) \\
& =c_{\ell \text { int }}+\Delta c_{\ell}
\end{aligned}
$$

so update of increment is:

$$
\begin{aligned}
\left(\Delta c_{\ell}\right)_{n+1} & =\left(c_{\ell \text { ext }}\right)_{n}-\left(c_{\ell \text { old }}\right)_{n} \\
& =\left(\Delta c_{\ell}\right)_{n}+\left(c_{\ell \text { ext }}-c_{\ell \mathrm{total}}\right)_{n}
\end{aligned}
$$

aerodynamic or structural dynamic sensors also provide quantities required for boundary conditions in the external analysis

## PRESCRIBED COEFFICIENT INCREMENTS OBTAINED FROM TWO-DIMENSIONAL TABLE

coefficient $\Delta c_{\ell}, \Delta c_{d}, \Delta c_{n}, \Delta c_{x}, \Delta c_{m}, \Delta c_{r}, \Delta c_{g}$
$\Delta c_{\ell f}, \Delta c_{d f}, \Delta c_{n f}, \Delta c_{x f}, \Delta c_{m f}, \Delta c_{r f}$
or $\Delta\left(M^{2} c\right)$ form
or load $\Delta F_{Q C}, \Delta F_{3 Q C}, \Delta M_{Q C}, \Delta F_{f}, \Delta M_{f}$
function of span station and time, or span station and azimuth

## PARTIAL ANGLE-OF-ATTACK

external analysis may account for - and wake component must exclude - when calculating induced velocity:
wake vorticity directly behind wing
and perhaps tip vortices from other wings
or all vorticity inside domain
wing component calculates $\alpha_{P}$ from this induced velocity, including structural motion
computational domain approximated by box domain boundary must be correct where it intersects wake; $\alpha_{P}$ probably not sensitive to small errors in that intersection

OFTEN EXTERNAL ANALYSIS REQUIRES HIGH RESOLUTION IN SPAN, TIME, AND WAKE AGE
so calculate $\alpha_{P}$ and other quantities in separate loop in trim task
using fixed wake geometry and structural motion, after converged solution solution for structural dynamic response obtained

## 5-20 Rotor Inflow Component

## ROTOR INFLOW COMPONENT CALCULATES WAKE INDUCED

 VELOCITY OF A ROTOR"UNIFORM INFLOW" COMPONENT (ALTHOUGH VELOCITY CAN VARY OVER ROTOR DISK)

THERE MAY BE CORRESPONDING NONUNIFORM INFLOW COMPONENT
which component gives nonzero velocity depends on specified model, and perhaps on wake loop level
figure 37 illustrates functionality

INDUCED VELOCITY MODEL
GLOBAL, QUASISTATIC MODEL FOR ROTOR
designed to be used with mean (filtered) values of component input

## EMPIRICAL MODEL BASED ON IDEAL-WING THEORY

momentum theory
induced velocity at rotor = mean term + linear variation over rotor disk

OPTIONAL DUCTED FAN MODEL
OPTIONAL DIFFERENTIAL MOMENTUM THEORY


Figure 5-37 Functionality of rotor inflow component.

## MOMENTUM THEORY

$$
\lambda_{i}=\kappa \frac{C_{T}}{2 \sqrt{\lambda^{2}+\mu^{2}}}
$$

with corrections for
nonideal induced losses in hover and forward flight (factors $\kappa_{h}$ for axial flow and $\kappa_{f}$ for edgewise flow) singularity of momentum theory at ideal autorotation of axial flow
vortex ring state
circulation loop divergence for hovering rotor near zero thrust
ground effect
linear inflow variation over rotor disk, from edgewise flow and hub moments and angular velocity
assume that induced velocity is in direction of $z$-axis of rotor axes (shaft axes or tip-path plane axes)
can calculate interference velocity at points off this rotor induced velocity $=$ (input interference factor) times (mean inflow at rotor) input interference factor accounts for fraction of system at collocation point affected by wake, and for fraction of fully developed wake achieved at collocation point

## DYNAMIC INFLOW

FOR INDUCED VELOCITY IN TRANSIENT OR FLUTTER TASK (PERTURBATIONS FROM TRIM INFLOW)

GLOBAL, LOW FREQUENCY MODEL
MEAN AND LINEAR INDUCED VELOCITY COMPONENTS RELATED TO NET AERODYNAMIC THRUST AND HUB MOMENTS ON ROTOR
inflow degrees of freedom

$$
\delta \lambda=\left(\begin{array}{l}
\delta \lambda_{u} \\
\delta \lambda_{x} \\
\delta \lambda_{y}
\end{array}\right)
$$

and loading

$$
\delta L=\left(\begin{array}{c}
\delta C_{T} \\
\delta C_{M y} \\
\delta C_{M x}
\end{array}\right)
$$

first order differential equations (linearized model, with time lags)

$$
\tau \delta \dot{\lambda}+\delta \lambda=\left(\frac{\partial \lambda}{\partial L}\right) \delta L
$$

must also include induced velocity perturbations produced by wing speed changes

## STATIC DERIVATIVE MATRIX $(\partial \lambda / \partial L)$ FROM

differential momentum theory (perturbation of uniform inflow model), which gives good results hover or from unsteady actuator disk theory (Pitt and Peters), which is needed for good results in forward flight

## DIFFERENTIAL MOMENTUM THEORY

INDUCED VELOCITY CALCULATED FROM BOUND CIRCULATION AT SPAN STATION OF COLLOCATION POINT
hence $\lambda_{i}$ varies over disk
intended for axial flow, but can be used in forward flight as well

INCLUDING TIP LOSS CORRECTION

## DUCTED FAN

COMPONENT INPUT = ROTOR FORCE AND DUCT FORCE duct force calculated by some aerodynamic component

## MOMENTUM THEORY FOR DUCTED FAN

depends on ratio rotor thrust to total thrust which determines wake velocity ratio and area ratio (ratio of far wake value and value at rotor disk)

## SIMPLE DUCT FORCE MODEL: SPECIFY RATIO ROTOR LOAD TO TOTAL LOAD <br> apply duct force to structure, at rotor hub node

## HOVERING ROTOR NEAR ZERO THRUST

## CIRCULATION ITERATION

consider inflow solution depending on rotor lift $L$; relaxation can be applied to $L$ :

$$
\bar{L}_{n}=\lambda L_{n}+(1-\lambda) \bar{L}_{n-1}
$$

circulation iteration can be represented as operation

$$
L_{n}=A\left(\bar{L}_{n-1}\right)
$$

successive substitution iteration will converge if

$$
\left|1-\lambda+\lambda A^{\prime}\right|<1
$$

for any finite $A^{\prime}$, relaxation factor can be found that produces convergence
but method fails if $A^{\prime}=\infty$
to analyze system in such case, necessary to change definition of problem: either change order that parts are solved in loop; or change physical model that is source of sensitivity of $A$ to $L$

## CIRCULATION CONVERGENCE IS PROBLEM FOR HOVERING ROTOR AT LOW THRUST

uniform inflow gives for hover

$$
\begin{aligned}
& C_{T}=c_{1} \theta-c_{2} \lambda=c_{1} \theta-c_{3} C_{T}^{1 / 2}=A\left(C_{T}\right) \\
& A^{\prime}\left(C_{T}\right)=-\frac{c_{3}}{2} C_{T}^{-1 / 2}
\end{aligned}
$$

so $A^{\prime}=\infty$ at zero thrust

## AVOID PROBLEM BY FIXING WAKE GEOMETRY

 uniform inflowfor hover, calculate induced velocity from

$$
\lambda_{i}=\frac{\kappa_{h}^{2} C_{T}}{2 \lambda_{0}}
$$

where $\lambda_{0}=\kappa_{h} \sqrt{C_{T 0} / 2}$, and $C_{T 0}$ is fixed, nominal value of rotor thrust
then $A^{\prime}=-c_{4} / \lambda_{0}$ is finite (and independent of $C_{T}$ ), and circulation iteration will converge for small enough nonzero relaxation factor
nonuniform inflow
fix wake geometry directly by using input values of constants $K_{1}, K_{2}, K_{3}$, and $K_{4}$ (instead of calculating them from rotor thrust)

## RESULTING INDUCED VELOCITY IS NOT ACCURATE

but acceptable for cases of small thrust, as long as reasonable value of $C_{T 0}$ is used

## 5-21 Rotor Dynamic Wake Component

## ROTOR DYNAMIC WAKE COMPONENT CALCULATES WAKE INDUCED VELOCITY OF A ROTOR

figure 38 illustrates functionality

INDUCED VELOCITY CALCULATED USING GENERALIZED DYNAMIC WAKE THEORY DEVELOPED BY PETERS AND HE

UNSTEADY WAKE THEORY FOR LIFTING ROTORS
based on acceleration potential for actuator disk
induced inflow at rotor disk is expressed as Fourier series azimuthally and polynomials radially

## RESULT IS SYSTEM OF FIRST-ORDER, ORDINARY DIFFERENTIAL EQUATIONS FOR INFLOW STATES

in trim task, obtain static solution for inflow variables (no degrees of freedom)
in transient and flutter tasks, solve ordinary differential equations

FINITE-STATE INDUCED FLOW MODEL PARTICULARLY SUITABLE FOR ROTORCRAFT AEROELASTIC ANALYSIS


Figure 5-38 Functionality of rotor dynamic wake component.

## DYNAMIC WAKE THEORY

INFLOW VELOCITY DEFINED BY DEGREE OF FREEDOM VECTOR $\alpha$

ROTOR LOADING REPRESENTED BY GENERALIZED FORCE VECTOR $\tau$

RELATED BY FIRST ORDER DIFFERENTIAL EQUATION

$$
L M \dot{\alpha}+\alpha=L \tau
$$

DERIVATIVE MATRIX $L=\partial \alpha / \partial \tau$ DEPENDS ON WAKE SKEW ANGLE $\chi=\tan ^{-1}|\mu / \lambda|$

## WAKE-INDUCED VELOCITY CALCULATED FROM $\alpha$

assume that induced velocity is in direction of $z$-axis of rotor axes (shaft axes or tip-path plane axes)
can calculate interference velocity at points off this rotor
induced velocity $=$ (input interference factor) times (mean inflow at rotor)
input interference factor accounts for fraction of system at collocation point affected by wake, and for fraction of fully developed wake achieved at collocation point

## NONLINEAR FORMULATION USED

uniform induced velocity term produced by the rotor thrust is the usual momentum theory result
including corrections for
nonideal induced losses in hover and forward flight (factors $\kappa_{h}$ for axial flow and $\kappa_{f}$ for edgewise flow) singularity of momentum theory at ideal autorotation of axial flow (correction covers vortex ring state and turbulent wake state)
wake skew angle $\chi=\tan ^{-1}|\mu / \lambda|$ evaluated using total normal velocity $\lambda$, not just $\mu_{z}$
rotor velocity replaced by $V=1 / 2\left(\partial C_{T} / \partial \lambda\right)$ to account for influence of induced flow on total steady flow through disk

## 5-22 Wing Inflow Component

WING INFLOW COMPONENT CALCULATES WAKE INDUCED VELOCITY OF ONE NON-ROTATING WING
figure 39 illustrates functionality

## INDUCED VELOCITY MODEL

GLOBAL, QUASISTATIC MODEL FOR WING
EMPIRICAL MODEL BASED ON IDEAL-WING THEORY
induced velocity at wing = mean term
from solution for minimum induced-drag of wing ( $A R=$ $b^{2} / S$ is wing aspect-ratio)

$$
\lambda_{i}=\frac{v_{i}}{\left|V_{x}\right|}=\frac{C_{L}}{\pi e A R}
$$

with correction for non-ideal losses (factor $e$ )


Figure 5-39 Functionality of wing inflow component.

## 5-23 Wing Wake Component

## WING WAKE COMPONENT CALCULATES WAKE INDUCED VELOCITY OF A WING SET

ONE OR MORE WINGS IN SET; ONE WAKE PER WING

## "NONUNIFORM INFLOW" COMPONENT <br> THERE MAY BE CORRESPONDING UNIFORM INFLOW COMPONENT

which component gives nonzero velocity depends on specified model, and perhaps on wake loop level
figure 40 illustrates functionality

## VORTEX WAKE OF ROTOR IS FACTOR IN MOST PROBLEMS OF HELICOPTER

PERFORMANCE, BLADE LOADS, VIBRATION, NOISE accurate calculation of wake-induced nonuniform inflow and resulting airloads needed in order to predict rotor behavior need free wake geometry below advance ratio of about $\mu=0.20$

## AIRLOADS BEHAVIOR OF ROTARY WINGS

## LOW SPEED

impulsive loading on advancing and retreating blades, caused by encounters of blade with tip vortices

## HIGH SPEED

blade-vortex interaction loading still evident; negative loading on advancing tip common, consequence of flap moment balance, with stall-limited loads on retreating side


Figure 5-40 Functionality of wing wake component.



## AERODYNAMIC MODEL

## VELOCITY CALCULATED AT COLLOCATION POINTS

"induced velocity" at collocation points on this wing set (on wing generating wake, or on other wings)
"interference velocity" at other points (on other aerodynamic components, or arbitrary point in flow field)

## VORTEX WAKE MODEL, WITH DISCRETIZED VORTEX ELEMENTS

incompressible flow in wake
integral equation (inflow depends on past circulation), which requires implicit solution
usually vortex lattice (straight-line segments) approximation
small viscous core radius for tip vortices
vortex sheet elements can be used for inboard wake, but usually sufficient (and more efficient) to approximate sheets by line segments, with large core radius to eliminate large velocities
model of wake rollup process
eventually tip vortex has strength of wing peak bound circulation at time that the wake elements were created
tip vortex core radius is input parameter

## MODEL FEATURES

FOR LIFTING-LINE THEORY, COLLOCATION POINTS ON THIS WING SET CAN BE AT QUARTER CHORD OR THREEQUARTER CHORD
lifting line can be at wing quarter chord, or approximated by straight line

## LINE AND SHEET WAKE ELEMENTS

constant or linear strength lines
nonplanar-quadrilateral sheets (or planar-rectangular; or approximate using lines)

## MODEL OF WAKE ROLLUP PROCESS

tip vortex rollup at one or both wing tips, or neither possibly two bound circulation peaks (inboard and outboard peaks of opposite sign): single-peak or dualpeak model
entrainment and stretching in rollup process can be defined
far wake trailed vorticity can be divided into several spanwise panels, to provide more detailed structure for inboard vorticity

> perhaps with consolidation of trailed lines in the wake geometry

## GROUND EFFECT

influence of ground can be included in wake-induced velocity calculation, through use of image elements in wake model
wake geometry components can also include ground plane influence

## EXTERNAL AEROACOUSTIC ANALYSIS

induced velocity can be calculated excluding vortex elements inside computational domain of external analysis

## BLADE-VORTEX INTERACTION LOADING

calculated using second-order lifting-line theory (threequarter chord collocation point)
or first-order theory (quarter chord collocation point) with a larger vortex core size
vortex core radius can be constant, or grow with wake age; and can include term that scales with trailed vorticity moment
can suppress blade-vortex interaction on inboard part of blade (as observed in measurements), by using large core radius when calculate velocity induced there

## 5-23.1 Wake Structure and Rollup

THREE-DIMENSIONAL WING TRAILS BOUND CIRCULATION $\Gamma$ INTO A WAKE
figure 41 illustrates wake of rotor blade
SPANWISE VARIATION OF Г PRODUCES TRAILED VORTICITY (ORIENTED IN LOCAL FREE STREAM DIRECTION)

TIME VARIATION OF Г PRODUCES SHED VORTICITY (ORIENTED SPANWISE)
strength of trailed and shed vorticity determined by span and time derivatives of $\Gamma$ at time wake element created

BOUND CIRCULATION TYPICALLY DROPS QUICKLY TO ZERO AT WING TIPS
so trailed sheet has high strength at wake outer edges quickly rolls up to form concentrated tip vortices

ROLLUP PROCESS (ALSO INFLUENCED BY TIP GEOMETRY) PRODUCES LINE VORTEX WITH SMALL CORE RADIUS, HENCE LARGE VELOCITIES

## ROTOR WAKE ROLLUP

BOUND CIRCULATION PEAK OCCURS NEAR BLADE TIP, GRADIENT OF BOUND CIRCULATION LOW AT BLADE ROOT
so blade tip vortex is strong and concentrated, while root vortex weaker and more diffuse

BASIS FOR DEVELOPING MODEL OF ROTOR WAKE, ALTHOUGH ROTOR WAKES ARE NOT ALWAYS SO SIMPLE


Figure 5-41 Trailed and shed vorticity in rotor wake.

## WAKE GEOMETRY

## WAKE GEOMETRY DESCRIBES POSITION OF WAKE VORTICITY IN SPACE

UNDISTORTED GEOMETRY OBTAINED FROM MOTION OF WING
wake element is convected by wind, from position in air at which it was created

THIS GEOMETRY IS DISTORTED BY SELF-INDUCED VELOCITY OF ALL WAKES IN SYSTEM, AND BY WINGS AND BODIES

## ROTOR WAKE GEOMETRY

WAKE CONVECTED DOWNWARD RELATIVE ROTOR BY MEAN INDUCED VELOCITY AND FREE STREAM, AFT IN FORWARD FLIGHT BY FREE STREAM
in hover, tip vortices trailed in helix
in forward flight, convected rearward as well as downward with substantial distortion from self-induced velocities

ROTOR WAKE = CONCENTRATED TIP VORTICES, TRAILED IN DISTORTED, INTERLOCKING HELICES; ONE BEHIND EACH BLADE, SKEWED AFT IN FORWARD FLIGHT
figure 42 shows typical wake geometry


Figure 5-42 Calculated rotor free wake geometry $\left(C_{T} / \sigma=0.087\right.$ and $\left.\mu=0.18\right)$.

TIP VORTICES
VORTICITY IN TIP VORTEX DISTRIBUTED OVER SMALL BUT FINITE REGION, BECAUSE OF VISCOSITY

VORTEX CORE IS IMPORTANT FACTOR IN INDUCED VELOCITY CHARACTER SINCE IT DEFINES AND LIMITS MAXIMUM VELOCITIES NEAR TIP VORTICES

STRONG, CONCENTRATED TIP VORTICES ARE DOMINANT FEATURE OF HELICOPTER ROTOR WAKE
produce highly nonuniform flow field
so any aerodynamic component encountering this flow field will see large vortex-induced loadings because of rotation, rotor blade encounters tip vortices from preceding blades
vortex-induced loading is principal source of higher harmonic loads on blades

## 5-23.2 Blade-Vortex Interaction

## AIRLOADS PRODUCED BY BLADE-VORTEX INTERACTION DEPEND ON NUMEROUS PHYSICAL EFFECTS

extent of tip vortex rollup
tip vortex strength
size of viscous core
distorted wake geometry
lifting-surface effects on induced loading and possibly even vortex bursting, or vortex-induced stall on the wing, or wing-induced geometry changes

## VORTICITY IN TIP VORTEX DISTRIBUTED OVER SMALL BUT FINITE REGION, BECAUSE OF VISCOSITY

## VISCOUS CORE DETERMINES VELOCITY NEAR LINE VORTEX CONSIDER TANGENTIAL OR CIRCUMFERENTIAL VELOCITY $v$ ABOUT LINE VORTEX, AT DISTANCE $r$ FROM LINE

core radius $r_{c}=$ distance $r$ where have maximum $v$
figure 43 shows influence of core radius on peak velocity


Figure 5-43 Tip vortex core radius and peak velocity.

## FOR POTENTIAL LINE VORTEX (NO CORE):

$v=\Gamma / 2 \pi r$, where strength $\Gamma$ is some fraction of maximum bound circulation (determined by rollup process)
at small $r$, viscosity reduces magnitude of $v$, by spreading vorticity over nonzero domain

## FOR RANKINE VORTEX CORE

solid body rotation of fluid inside $r_{c}$
uniform vorticity distribution, concentrated entirely within core radius
maximum tangential velocity is $v_{\max }=\Gamma / 2 \pi r_{c}$

## FOR DISTRIBUTED VORTICITY CORE

Scully model (a simple possibility) has circulation proportional to $r^{2} /\left(r^{2}+r_{c}^{2}\right)$, so half of vorticity is outside core radius
maximum tangential velocity is $v_{\max }=\Gamma / 4 \pi r_{c}$
figure 44 shows influence of core type on velocity
DISTRIBUTED CORE MODEL IS MORE REALISTIC
measurements show that maximum tangential velocity is much less than $\Gamma / 2 \pi r_{c}$, so substantial fraction of vorticity is outside core radius


Figure 5-44 Tip vortex core types.

## CORE RADIUS

ONLY LIMITED EXPERIMENTAL DATA ON CORE SIZE, PARTICULARLY FOR FULL SCALE ROTOR BLADES ANALYSIS REQUIRES CORE RADIUS $r_{c}$ OF ABOUT 20-25\% CHORD
based on correlation with measured blade-vortex interaction loads on rotors in low speed flight using distributed core model, and second-order lifting-line theory (three-quarter chord collocation point)

LIMITED DATA SUGGEST THAT THE ACTUAL VISCOUS CORE RADIUS IS ABOUT 10\% CHORD (FULL SCALE) AIRLOADS CORRELATION SUGGESTS THAT CORE SIZE VARIES WITH AZIMUTH
or rollup varies with azimuth, so less strength (compared to peak bound circulation) on retreating side

## CORE SIZE $r_{c}$ IS CONVENIENT PARAMETER TO CONTROL AMPLITUDE OF CALCULATED BLADE-VORTEX INTERACTION LOADS

determines maximum tangential velocity about vortex (inversely proportional to $r_{c}$ )
use value of $r_{c}$ that accounts for actual viscous core radius, and also for all phenomena of interaction that are not otherwise modelled (or are inaccurately modelled)

## BLADE-VORTEX INTERACTION ON HELICOPTER ROTORS IN LOWSPEED FORWARD FLIGHT

when vortex-induced loads are calculated using core size that gives good correlation at blade tip, find that strength of bladevortex interaction is significantly overpredicted for inboard radial stations (compared to flight test measurements)
measured vortex-induced loading is high when blade first encounters vortex, on advancing side;
decreases inboard as blade sweeps over vortex, on front of disk;
and then recovers again on retreating side

## EVIDENTLY THERE IS SOME PHENOMENON LIMITING THE LOADS

MECHANISM REMAINS SPECULATIVE; POSSIBILITIES:
local distortion of the vortex geometry
bursting of vortex core, induced by blade
interaction of vortex with trailed wake it induces behind blade, with effect of diffusing and reducing circulation in vortex
local flow separation produced by high radial pressure gradients on blade
viscous effects on interaction, because blade-vortex separation is much smaller on front of disk than on sides

## CORE SIZE CAN BE INCREASED FOR COLLOCATION POINTS ON INBOARD PART OF BLADE IN ORDER INCLUDE THIS EFFECT IN CALCULATIONS

simple way to model effects on airloads, but no explanation of the physics is intended

## 5-23.3 Nonuniform Inflow Calculation

## NONUNIFORM INFLOW CALCULATED BY INTEGRATING BIOTSAVART LAW OVER WAKE

induced velocity produced by one wing at a collocation point:

$$
v(t)=\iint \mathbf{C}(t, \tau ; r) \Gamma(t-\tau ; r) d r d \tau
$$

$\Gamma=$ wing bound circulation
$\mathbf{C}=$ influence coefficients (differential operator, calculated from wake geometry and modelling assumptions)
at time $t$, wake element and its position in wake identified by wake age $\tau$
$t-\tau=$ time wake element was created

SO INDUCED VELOCITIES DEPEND ON STRENGTH AND GEOMETRY OF WAKE VORTICITY
strength defined by span and time variation of bound circulation
geometry can be obtained from rigid, prescribed, or free models

## INTEGRATION OVER WAKE

can not evaluate integrals analytically, even with no distortion
because of helical wake geometry of rotor direct numerical integration not satisfactory either
large variations in integrand (produced by bladevortex interactions) require small step sizes for accuracy
so wake modelled by set of discrete elements
Biot-Savart law integrated analytically for each element
total velocity obtained by summing contributions from all elements
accurate and efficient

## DISCRETIZED WAKE

bound circulation and hence wake discretized spanwise wake age discretized, using geometry and strength only at fixed wake age increment $\Delta \tau$
integral equation for the wake-induced velocity becomes:

$$
\begin{array}{r}
v_{A}^{I}(t)=\sum_{\text {wings }}\left[\sum_{\text {panels }} \sum_{k=0}^{K_{F}}\left(C_{L}\left(t, \tau_{k}\right) \Gamma_{L}\left(t-\tau_{k}\right)+C_{R}\left(t, \tau_{k}\right) \Gamma_{R}\left(t-\tau_{k}\right)\right)\right. \\
\left.+\sum_{k=0}^{K_{N}} \sum_{i=1}^{M} C_{N}\left(t, \tau_{k} ; r_{A i}\right) \Gamma\left(t-\tau_{k} ; r_{A i}\right)\right]
\end{array}
$$

## DISCRETIZED WAKE MODEL

## TIP VORTICES MODELLED AS CONNECTED SERIES OF STRAIGHT LINE SEGMENTS, WITH SOME KIND OF CORE REPRESENTATION

good model for most important part of wake

## INBOARD TRAILED AND SHED VORTICITY TYPICALLY MODELLED AS VORTEX MESH OR LATTICE

with lattice (line segment) model of inboard wake, a large core size is needed
not representation of physical effect, but produces approximation for sheet element, by eliminating singularities in velocity for close passage of following wings
find line segments give about same results as sheet elements
model for inboard wake is not as good as tip vortex representation, but inboard wake is not as important

## APPROXIMATIONS INVOLVED USING SET OF DISCRETE VORTEX ELEMENTS

replacing curvilinear geometry by series of straight line or planar elements; simplified distribution of strength over individual element (constant or linear); sheets replaced by lines, or planar-rectangle approximation for nonplanarquadrilateral

PRACTICAL MODEL BALANCES ACCURACY AND EFFICIENCY OF SUCH APPROXIMATIONS

## DEVELOPMENT OF DISCRETIZED MODEL

ASSUME WING BOUND CIRCULATION KNOWN AT DISCRETE POINTS ALONG SPAN AND IN PAST
bound circulation $\Gamma(r, t)$ calculated at aerodynamic span stations $r_{A i}$, and at wake ages $\tau_{k}=k \Delta \tau, k=0$ to $K$
linear variation of $\Gamma$ between these points means wing generates wake of sheet elements (figure 45)

## STRENGTH OF TRAILED AND SHED VORTICITY IN AN ELEMENT OBTAINED FROM Г AT TIME THAT VORTICITY WAS CREATED

spanwise change in $\Gamma$ produces trailed vorticity time change in $\Gamma$ produces shed vorticity wake geometry provides locations at four corners of element

## ECONOMICAL APPROXIMATION FOR SHEET ELEMENT IS LINE SEGMENTS, WITH LARGE CORE TO AVOID VELOCITY SINGULARITIES NEAR LINE

so vortex lattice model obtained by collapsing all sheet elements to finite strength line segments (figure 46)
line segments are in center of sheet element, and points at which induced velocity are calculated (collocation points) are at midpoints of vortex lattice grid
locating collocation points midway between trailers is standard practice, to avoid singularities at lines
locating collocation points midway in time is required to correctly obtain shed wake effects


Figure 5-45 Wake model constructed from sheet panels (right tip vortex rollup).


Figure $5-46 \mathrm{a}$ Wake model with line segments replacing sheet panels (right tip vortex rollup).


Figure 5-46b Wake model constructed from line segments (right tip vortex rollup).

## WAKE MODEL FOR COLLOCATION POINTS ON WING THAT GENERATED THE WAKE

## WAKE DIRECTLY BEHIND THIS WING IS CALLED THE "NEAR WAKE"

## IMPLEMENTATION OF LIFTING-LINE THEORY INVOLVES PRIMARILY THIS REGION OF WAKE

most important requirement is to model detailed variation of wake strength, both spanwise and in time
to accurately get classical three-dimensional (Prandtl) and unsteady (Theodorsen) effects of wake on wing loading
rollup process is less important for near wake VORTEX LATTICE IS ALWAYS USED FOR NEAR WAKE (NOT SHEET ELEMENTS)
discretization of wake is better behaved with line segments than with sheet elements

## ROLLUP PROCESS NOT CONSIDERED IN MODELLING THE NEAR WAKE

except for effect of rollup on tip loading of generating wing modelled by prescribing spanwise location of tip vortex, forcing wing to trail remaining bound circulation into wake at that span station implemented in wing component $\left(r_{T V}\right)$

## MODEL OF ROLLUP PROCESS

CONSIDER WAKE WHEN IT REACHES COLLOCATION POINTS ON A FOLLOWING WING OR OTHER AERODYNAMIC COMPONENT

THIS WAKE REGION CALLED THE "FAR WAKE"
most important to model rollup process

## WAKE QUICKLY ROLLS UP AT OUTER EDGES TO FORM CONCENTRATED TIP VORTICES

so tip vortices dominate flow field, and must be modelled well
inboard vorticity less important, so more approximations are acceptable

BOUND CIRCULATION CALCULATED AT MANY SPANWISE POINTS ON WING
but because of entrainment and stretching during rollup process, such detailed knowledge of vorticity strength not available in far wake
so far wake model constructed using bound circulation at small number of points - typically just circulation peaks

## 5-23.4 Wake Rollup Process

## WAKE ROLLUP PROCESS

## CALCULATION OF WAKE ROLLUP FROM FIRST PRINCIPLES

 IS NEEDED, BUT IT IS A DIFFICULT PROBLEMvortex core largely formed at wing trailing edge, so problem is not inviscid rollup of vortex sheet discretization of wake for rollup calculation is difficult (may not be well-posed even for inviscid, two-dimensional problem)
so rollup problem involves three-dimensional, unsteady, viscous fluid dynamics; compressible for rotors
beginning to be attacked using computational fluid dynamics methods

## HERE WAKE ROLLUP PROCESS IS MODELLED NOT CALCULATED

structure and properties of rolled up wake are determined from assumptions and input parameters
and from spanwise distribution of bound circulation where wake was created

## MODEL MUST ACCOUNT FOR INFLUENCE OF

 strength of tip vortex when it encounters following wing core radius when vortex is fully rolled up
## ASSUMED ROLLUP PROCESS FOR WAKE MODEL

 LIFT CONCENTRATED AT TIP OF WINGso trailed vorticity strength high at edge of wake, and vortex sheet quickly rolls up into concentrated tip vortex tip vortex may form at both tips (for nonrotating wing), or at just one tip (for rotor blade)

FORMATION OF VORTEX INFLUENCED BY TIP GEOMETRY with core largely formed by time vortex leaves trailing edge

ROLLED UP TIP VORTEX QUICKLY ATTAINS STRENGTH NEARLY EQUAL TO MAXIMUM BOUND CIRCULATION

TIP VORTEX HAS SMALL CORE RADIUS, DEPENDING ON WING GEOMETRY AND LOADING

SUCH CONCEPTS ARE BASIS FOR DEVELOPING MODEL OF ROTOR WAKE

## HELICOPTER BLADE LOADING

wing loading at given time need not be all positive or all negative helicopter rotor in high speed forward flight:
typically has negative lift on advancing tip, in second quadrant
for flapping rotor, net pitch and roll moment on hub must be small (zero with no flap hinge offset)
in forward flight, lift capability on retreating side limited by combination of low dynamic pressure and stall of airfoil
so lift on advancing side must also be small, to maintain roll balance
at high speed, lift on advancing tip can become negative
large twist of blade (built-in or elastic) increases negative loading vortex-induced loading on following blades shows that this negative loading produces substantial negative trailed vorticity in wake
over much of rotor disk, $\Gamma$ still positive all along span
range of azimuth on advancing side with negative peak near tip and positive peak inboard
figure 47 illustrates wake structure


Figure 5-47 Rotor wake rollup structure.

## SINGLE-PEAK AND DUAL-PEAK ROLLUP

## SINGLE-PEAK CASE

often bound circulation has same sign along entire wing span at a given time then rolled up wake is constructed solely from magnitude and position of peak bound circulation
spanwise maximum of bound circulation is $\Gamma_{\max }$
in far wake (where rollup process is complete) have rolledup tip vortices with strength equal to value of $\Gamma_{\max }$ at time that wake element was created
corresponding negative trailed vorticity with total strength $-\Gamma_{\text {max }}$ in inboard sheet
tip vortex modelled as line segment with this strength, and small core radius (input parameter)
any error in assumed strength (because vortex is partially or over rolled-up), is compensated for by value of core radius
detailed structure of inboard vortex sheet not known (no calculation of stretch and rollup of vorticity)
so use single sheet element with trailed and shed vorticity models of tip vortex entrainment (partial rollup) and stretching of inboard wake implemented
but seldom have enough information to select input parameters

## DUAL-PEAK CASE

bound circulation can be positive over part of wing span, and negative over remainder
then rolled up wake constructed from magnitude and position of left and right bound circulation peaks
two peaks of opposite sign (left peak $\Gamma_{L}$ and right peak $\Gamma_{R}$ ) in spanwise distribution of bound circulation
for rotor blade, with only right tip vortex rolled up:
tip vortex consists of trailed vorticity outboard of right peak, hence strength $\Gamma_{R}$
inboard sheet has both positive and negative trailed vorticity, divided at span station corresponding to left peak
trailed vorticity between two peaks may partially roll up as well modelled by using line segment with physically-meaningful core radius

## IF SINGLE-PEAK MODEL APPLIED TO DUAL-PEAK CIRCULATION DISTRIBUTION, CAN RESULT IN WRONG SIGN OF TIP VORTEX STRENGTH

wake structure (single-peak or dual-peak) used when influence coefficients are calculated may not correspond to wing loading distribution
either because solution has not converged or because wrong wake model was specified
figure 48 illustrates possible rotor wake models with dual-peak loading
single-peak model based on $\Gamma_{R}$ rather than $\Gamma_{\max }$ at least gets correct sign and magnitude of tip vortex strength

## BLADE BOUND CIRCULATION



FAR WAKE STRUCTURE dual-peak model

single-peak model, using outboard peak

single-peak model, using peak with largest magnitude


Figure 5-48 Wake models with dual-peak loading.

## MULTIPLE-TRAILER WAKE MODEL FAR WAKE TRAILED VORTICITY

far wake can be divided into several spanwise panels, to provide more detailed structure of inboard vorticity rollup can occur at boundaries between panels

## THERE MAY BE FEATURES OF INBOARD WAKE STRUCTURE THAT MUST BE MODELLED

for example, trailing-edge flap or rapid change in chord can produce step in bound circulation
which might produce significant rollup of inboard vorticity
figure 49 illustrates the wake configuration

## CAN SPECIFY SPAN STATIONS OF SUCH INBOARD ROLLUP, BASED ON GEOMETRY OF WING

far wake trailed vorticity then divided into several spanwise panels
single-peak or dual-peak model applied to each panel


Figure 5-49 Rotor wake with multiple rolled-up trailed vorticity.

## ALTERNATIVELY, CAN HAVE TRAILED VORTEX LINE EMANATING FROM EACH AERODYNAMIC PANEL EDGE

rollup is not well calculated even with many trailed vortex lines, because of the coarse discretization and neglect of viscosity
trailed lines can be consolidated into single rolled-up line, using trailed vorticity moment to scale the rate of rollup
figure 50 illustrates the wake model options

## CONSOLIDATION IS SIMULATION OF TIP VORTEX FORMATION PROCESS

implemented in wake geometry calculation, requires use of wing wake geometry component
trailed vorticity partitioned into sets of adjacent lines that have same sign (bound circulation increasing or decreasing)
assume that all vorticity in a set eventually rolls up into a single vortex, located at centroid of original vorticity distribution
using the total strength and trailed vorticity moment of the set $\left(\Gamma / r_{G}^{2}\right)$ to scale the rate of rollup

a) rolled-up wake model

b) multiple-trailer wake model

c) multiple-trailer wake model, with consolidation (entrainment form)

d) multiple-trailer wake model, with consolidation (compression form)

Figure 5-50 Illustration of wake models.

## 5-23.5 Summary of Wake Model

## WAKE REGIONS

wake behind a wing divided into near wake and far wake regions near wake only used for collocation points on wing that generated the wake elements in near wake always modelled using line segments
extent of near wake should be at least one-half span, or 30 deg azimuth for rotor
far wake used for rest of wake behind each wing

## FAR WAKE

tip vortices modelled using line segments inboard wake modelled using line segments or sheet elements
extent of far wake can be different for collocation points on and off the wing set
for rotor, collocation points on wing set need at least two revolutions of wake, more at low speed
roll up of both tip vortices, or just left or right tip vortex partial entrainment and partial stretching can be included in rollup process
consists of one or more spanwise panels single-peak or dual-peak model for each panel inboard rolled-up trailed vortex structures can be defined based on wing geometry
can use multiple-trailer model with consolidation

## ONLY RIGHT TIP VORTEX ROLLED UP

model for rotor, with counter-clockwise rotation for clockwise rotation, span stations still ordered from root to tip

## DUAL-PEAK MODEL AND ONLY RIGHT TIP VORTEX ROLLED UP

inboard (left) peak breaks sheet into two elements at span station $r_{L}$ (calculated location of left circulation peak)
trailed vorticity between two peaks may partially roll up
modelled by using line segment with physically-meaningful core radius
wake geometry iteration must produce converged $r_{L}$
inboard peak position $r_{G I}$ is part of wake geometry description, needed to calculate influence coefficients these influence coefficients used to evaluate wing loading, hence new value of location of left peak, $r_{L}$ next wake geometry iteration uses $\rho_{G I}=r_{L}$ to calculate influence coefficients

## 5-24 Wing Wake Geometry Component

## WING WAKE GEOMETRY COMPONENT CALCULATES WAKE GEOMETRY FOR A WING SET

figure 51 illustrates functionality

## WAKE GEOMETRY MODEL

## RIGID, PRESCRIBED, AND FREE WAKE GEOMETRY MODELS

rigid geometry
calculates distortion from mean interference velocity at wing set
prescribed geometry
obtained from empirical model for hovering rotor (same as rotor wake geometry component)
both rigid and prescribed models considered "rigid wake geometry" for trim wake loop (LEVEL 2)
free geometry
obtained by calculating distortion simultaneously for all wings in the wing set
using wake model of the wing wake component
free wake geometry available for trim and transient tasks


Figure 5-51 Functionality of wing wake geometry component.

## WAKE GEOMETRY DESCRIBES POSITION OF WAKE VORTICITY IN SPACE

undistorted geometry obtained from motion of wing
wake element convected by wind, from position in air at which it was created
this geometry is distorted by wake self-induced velocity

## NONUNIFORM INDUCED FLOW AT WAKE SURFACE PRODUCES SIGNIFICANT DISTORTION OF GEOMETRY

distortion may be unimportant, or adequately represented by convection with mean induced velocity
wake geometry (including distortion) important when wing or body (perhaps wing that generated wake) passes close to wake
wake-induced loading then very sensitive to separation between wings and tip vortices

## CHARACTER OF ROTOR WAKE GEOMETRY

ROTOR WAKE GEOMETRY CONSISTS OF DISTORTED, INTERLOCKING HELICES, ONE BEHIND EACH BLADE, SKEWED AFT IN FORWARD FLIGHT
typically only vortex from blade tip rolls up significantly
relative rotor disk, wake is convected downward (normal to disk plane) by mean induced velocity and free stream and convected aft in forward flight by inplane component of free stream
self-induced velocity produces substantial distortion of vortex filaments as they are convected

## DISTORTED WAKE GEOMETRY EXHIBITS AN OVERALL PATTERN IN WHICH EDGES OF WAKE ARISING FROM ROTOR DISK ROLL UP TO FORM VORTICES, AS BEHIND CIRCULAR WING

actually this wake consists of helical tip vortices from individual blades
consequence of this pattern near rotor disk:
tip vortices tend to move upward on sides of disk
tend to move downward in middle of disk
so self-induced distortion moves tip vortices closer to blades on advancing and retreating sides (compared to rigid geometry)
increasing blade-vortex interaction loads

## FREE WAKE GEOMETRY HAS LARGE INFLUENCE ON

 BLADE AIRLOADING AT LOW SPEEDneed free wake geometry in analysis for advance ratios below about $\mu=0.20$ to 0.25
at higher speeds, wake convected quickly downstream and helicopter rotor has large tip-path-plane angle of attack to provide propulsive force
so wake is convected away from disk by normal component of free stream, and distorted geometry is less important
nonuniform inflow at rotor disk still important
hover wake geometry, $\mathrm{C}_{\mathrm{T}} / \sigma=0.099$
(view: azimuth $=20$, elevation $=10$ )


$$
\mu=0.1, \mathrm{C}_{\mathrm{T}} / \sigma=0.08
$$



$$
\mu=0.1, \mathrm{C}_{\mathrm{T}} / \sigma=0.08
$$



SA-349 Flight Test, $\mathrm{C}_{\mathrm{T}} / \sigma=.065, \mu=.14$


H-34 Flight Test, $\mathrm{C}_{\mathrm{T}} / \sigma=.091, \mu=.29$


SA-349 Flight Test, $\mathrm{C}_{\mathrm{T}} / \sigma=.071, \mu=.36$



H-34 Flight Test, $\mathrm{C}_{\mathrm{T}} / \sigma=.097, \mu=.29$



SA-349 Flight Test, $\mathrm{C}_{\mathrm{T}} / \sigma=.065, \mu=.14$






TIP VORTEX FORMATION
EFFECTS ON WAKE GEOMETRY ARE MODELLED
important for hover free wake geometry
INITIAL SPAN STATION
tip vortex can form at span station $r_{T V}$ inboard of wing tip input parameter
or Betz rollup (calculated assuming centroid of rolled up trailed vorticity is conserved)

## INITIAL CONVECTION

with three-dimensional wing, Kutta condition requires that wake leave trailing-edge tangent to wing surface extent (in wake age) of this initial convection is input parameter

## WAKE GEOMETRY DISTORTION <br> RIGID GEOMETRY <br> wake elements all convected by average interference velocity <br> with empirical corrections factors for normal and inplane convection

## PRESCRIBED GEOMETRY

based on empirical model for hovering rotor effects of turn rate or transient motion of rotor included in undistorted geometry, but not in prescribed distortion
interference from other aerodynamic components is not considered
distortion is described by two-stage vertical convection $D_{z}$ and exponential spanwise contraction $D_{r}$ :

$$
\left.\begin{array}{rl}
D_{z} & = \begin{cases}K_{1} \phi & \phi<\phi_{1} \\
K_{1} \phi_{1}+K_{2}\left(\phi-\phi_{1}\right) & \phi>\phi_{1}\end{cases} \\
D_{r} & =\left(1-e^{-K_{3} \phi}\right)\left(1-K_{4}\right)
\end{array} \quad \phi=\Omega \tau \text { is dimensionless wake age }\right\} \text {. } \begin{aligned}
& =2 \pi / N \text { is age at encounter with following blade } \\
& \quad(N=\text { number of blades })
\end{aligned}
$$

## PRESCRIBED MODELS IMPLEMENTED

RIGID: convection by normal interference velocity
$K_{1}=f_{1} \lambda$ and $K_{2}=f_{2} \lambda$, with input values of contraction constants $K_{3}$ and $K_{4}$

EMPIRICAL MODEL FOR HOVERING ROTOR: equations for $K_{1}, K_{2}, K_{3}, K_{4}$

Landgrebe model, or Kocurek and Tangler model from thrust coefficient $C_{T}$, or from peak bound circulation
equations depend on $N, \sigma, \theta_{\mathrm{tw}}$
INPUT $K_{1}, K_{2}, K_{3}, K_{4}$

## FREE GEOMETRY

distortion calculated simultaneously for all wings in the wing set
wing set can include wings of several rotors, and nonrotating wings
using wake model of the wing wake component including models for distributed vorticity in vortex core, and core radius growth
only far wake region of wing wake component considered for wake geometry calculation
far wake trailed vorticity can be divided into several spanwise panels
wake can roll up at boundaries between panels
wake geometry can be calculated optionally for tip vortices; for all rolled up trailed vortices; and for inboard sheet edges as well
rigid distortion used where free distortion not calculated
with multiple-trailer wake model, trailed lines can be consolidated into a single rolled-up line using the trailed vorticity moment $\left(\Gamma / r_{G}^{2}\right)$ to scale the rate of rollup
free wake geometry available for trim and transient tasks free distortion calculated for entire period at first time step of solution for wake geometry in trim task
or at next time step in transient task
distortion calculated using time and wake age increment of wake geometry component
distortion is interpolated or extrapolated as required to times of solution procedure, and wake ages of wake component

## WIND AND GUST

variation of wind velocity in ground boundary layer, and gust velocities, can be included in convection term of distortion

## GROUND EFFECT

influence of ground can be included in free wake geometry calculation, through use of image elements in wake model

## AIRFRAME FLOW FIELD INFLUENCE

calculated using simple model of airframe flow field component
set of wings and bodies, producing velocity perturbations that distort the wake geometry
mutual interference between wake elements and airframe flow field elements not included

## 5-25 Rotor Wake Geometry Component

## ROTOR WAKE GEOMETRY COMPONENT CALCULATES WAKE GEOMETRY FOR A ROTOR

figure 52 illustrates functionality

## WAKE GEOMETRY MODEL <br> RIGID, PRESCRIBED, AND FREE WAKE GEOMETRY MODELS

rigid geometry
calculates distortion from mean interference velocity at wing set
prescribed geometry
obtained from empirical model for hovering rotor both rigid and prescribed models considered "rigid wake geometry" for trim wake loop (LEVEL 2)
free geometry
obtained by calculation for helicopter rotor
using Scully's method or Johnson's method
free wake geometry generated only for tip vortex of single rotor
simplified wake model
free wake geometry only available for trim task JOHNSON METHOD IS SPECIALIZED VERSION OF WING WAKE GEOMETRY COMPONENT


Figure 5-52 Functionality of rotor wake geometry component.

## WAKE GEOMETRY DISTORTION

## FREE GEOMETRY

Scully method or Johnson method
distortion calculated only for tip vortex of single rotor
rigid or prescribed geometry model used for distortion of inboard sheet
constant tip vortex core radius, with Scully vorticity distribution
effects of turn rate included in undistorted geometry, but not in free distortion
interference from other aerodynamic components is not considered
distortion calculation assumes periodicity of wake geometry, hence model can only be used in trim task
free distortion calculated for entire rotor period at first time step of solution for wake geometry
distortion calculated using internally generated time and wake age increment (typically 15 degrees in azimuth) distortion is interpolated or extrapolated as required to times of solution procedure, and wake ages of component

## FREE WAKE GEOMETRY MODELS AT VERY LOW SPEED

Scully method applicable only in forward flight
good performance and airloads results at $\mu=0.125$ and above
results unrealistic at lower $\mu$

Johnson method can be used in both forward flight and hover
simplified version of wing wake geometry
component
baseline parameters give good performance and airloads results at $\mu=0.125$ and above
good performance results at lower advance ratios using appropriate parameters
can include influence of ground

## 5-26 Performance Components

# WING PERFORMANCE COMPONENT CALCULATES <br> PERFORMANCE QUANTITIES FOR ONE NONROTATING WING 

 figure 53 illustrates functionality
## ROTOR PERFORMANCE COMPONENT CALCULATES PERFORMANCE QUANTITIES FOR A ROTOR

figure 54 illustrates functionality

## ROTORCRAFT PERFORMANCE COMPONENT CALCULATES PERFORMANCE QUANTITIES FOR A ROTORCRAFT

figure 55 illustrates functionality
wing set force and moment wing set velocity total wng set power average ind and int velocity

|  | velocity <br> force and moment <br> power <br> performance indices |
| :--- | :--- |
|  | sensors |

Figure 5-53 Functionality of wing performance component.


Figure 5-54 Functionality of rotor performance component.


Figure 5-55 Functionality of rotorcraft performance component.

## 5-27 Rigid Wing Component

RIGID WING COMPONENT COMBINES RIGID BODY COMPONENT AND LIFTING LINE WING COMPONENT

STRUCTURAL DYNAMIC COMPONENT, WITH INTERNAL AERODYNAMIC MODEL
no elastic motion
internal calculation of velocity, position, and force on wing (not implemented as interfaces)

## COMPONENT TRADES CONFIGURATION FLEXIBILITY FOR COMPUTATIONAL EFFICIENCY

figure 56 illustrates the combination


Figure 5-56 Rigid wing as combination of rigid body and lifting line wing.

## 5-28 Helicopter Tail Boom Component

## HELICOPTER TAIL BOOM COMPONENT CALCULATES

 AERODYNAMIC FORCES ACTING ON CIRCULATION-CONTROL BOOM AND REACTION JETCIRCULATION-CONTROL BOOM OPERATES IN WAKE OF MAIN ROTOR
intended to provide torque reaction and yaw control for single main-rotor helicopter
figure 57 illustrates functionality

COMPONENT IMPLEMENTS USER-DEFINED CALCULATIONS
basic version is simple model of tail boom
other versions constructed by modifying subroutines called by helicopter tail boom component


Figure 5-57 Functionality of helicopter tail boom component.

## MODEL

## AERODYNAMIC MODEL OF BOOM IS LIFTING-LINE THEORY

 includes approximate induced angle-of-attack calculation based on ideal-wing theory for boomcomponent typically obtains aerodynamic coefficient data from tables
simple model of the basic version uses a two dimensional table: lift and drag coefficient as function of angle of attack and blowing coefficient user-defined calculations can make use of any table class and type

REACTION JET MODELLED AS FORCE WITH SPECIFIED DIRECTION AND MAGNITUDE

COMPONENT DEGREES OF FREEDOM MAY BE NEEDED TO ACCOUNT FOR BEHAVIOR OF INTERNAL AERODYNAMICS
component equations can be static, or first-order differential equations

## 5-29 Computational Fluid Dynamics Component

## COMPUTATIONAL FLUID DYNAMICS COMPONENT CALCULATES

 AERODYNAMICS OF A SET OF OBJECTSUSER PROVIDES CFD ANALYSIS, BY MODIFYING SUBROUTINES CALLED BY COMPONENT

CFD analysis can be viscous or inviscid
general interface with CFD analysis implemented
figure 58 illustrates functionality
figure 59 shows the operation of the interface routines

## AERODYNAMIC SYSTEM CONSISTS OF WINGS AND OTHER OBJECTS

## BASIC VERSION OF COMPONENT

can be used to test component, and used if component is being perturbed
for basic version, wings modelled as in lifting line wing component (without states, dynamic stall, or prescribed coefficient increments)


Figure 5-58 Functionality of computational fluid dynamics component.

Computational Fluid Dynamics Component


Figure 5-59 Operation of interface routines.

## 5-30 Plugin Component

THE PLUGIN COMPONENT ALLOWS SOFTWARE DEVELOPERS AND USERS TO ADD CONTENT TO CAMRAD II

## PLUGINS CAN ADD TO FUNCTIONALITY OF BOTH SHELL AND COMPONENTS

## SHELL PLUGINS CAN BE USED INDEPENDENTLY OF

 PLUGIN COMPONENTSextend capability of shell to construct core input

PLUGIN COMPONENTS CAN BE USED WITH CORE INPUT ENTIRELY
but corresponding shell plugin is recommended, to make it easy to use the new component technology

## PLUGIN COMPONENT CAN NOT BE A STRUCTURAL DYNAMICS COMPONENT

Chapter 6

## ROTORCRAFT THEORY

GENERAL REFERENCE FOR CHAPTER: CAMRAD II Documentation, Volume III, Rotorcraft Theory

## 6-1 Rotorcraft Shell

## CONFIGURATION CONSTRUCTED BY SHELL

ROTORCRAFT: AIRCRAFT WITH ONE OR TWO OR MORE ROTORS
configurations: one rotor, single main-rotor and tail-rotor helicopter, tandem helicopter, coaxial helicopter, tilting proprotor aircraft, multirotor aircraft
in free flight or in wind tunnel
figure 1 is simplified description of model ROTOR: ARBITRARY $N$-BLADED ROTOR
hub and blade root configurations: articulated, hingeless, teetering, gimballed, and bearingless
control configurations: swashplate mechanism (with pitch horns and pitch links), or blade root pitch
optionally with higher harmonic control
aerodynamic model includes a sophisticated wake analysis to calculate the rotor nonuniform inducedvelocities; using rigid, prescribed, or free wake geometry


Figure 6-1 Rotorcraft model (simplified).

## MODEL CONSTRUCTED BY SHELL

ROTOR STRUCTURAL DYNAMICS
beam theory, with large pitch and twist; isotropic material with elastic axis, or anisotropic material; arbitrary beam axis (straight within beam segments); arbitrary hinge order; exact kinematics of hinges and bearings; sweep and droop of tip; point masses on blade

## ROTOR AERODYNAMICS

lifting line theory (steady two-dimensional airfoil characteristics plus vortex wake); empirical dynamic stall models; unsteady aerodynamic forces from thin airfoil theory; yawed-flow and swept-blade corrections; sweep and droop of tip
induced velocity from momentum theory or vortex wake model
momentum theory includes mean and linear variation over rotor disk (produced by forward flight and hub moment); rotor-on-rotor and rotor-on-airframe interference models; and dynamic inflow model (perturbation momentum theory or actuator disk theory) or dynamic wake model

## ROTOR WAKE

vortex lattice, with small viscous core for tip vortices; nonplanar, quadrilateral sheet elements available for inboard wake; second-order lifting line theory (threequarter chord collocation point)
wake rollup process modelled, including possibility of two bound circulation peaks (inboard and outboard peaks of opposite sign); and simulation of rollup process in wake geometry calculation (consolidation)
rotor-on-rotor interference; mean induced-velocity at airframe; induced-velocity at arbitrary point in space wake geometry models: undistorted (rigid); hover prescribed; calculated free wake

## AIRFRAME

elastic airframe modes; drive train model; auxiliary forces (constant and higher harmonic terms)
airframe aerodynamic forces from nonlinear or linearized (stability derivative) model, from equations or tables; airframe-to-rotor interference velocity, from simple model or table

## SOLUTION PROCEDURE CONSTRUCTED BY SHELL

## ITERATIVE TRIM SOLUTION

wake loop, with levels: uniform inflow; nonuniform inflow with prescribed wake geometry; nonuniform inflow with free wake geometry
trim loop (modified Newton-Raphson solution); circulation and motion loops (successive substitution solution)
part solutions: arbitrary time step; harmonic solution method for rotor, airframe, and drive train parts (differential equations), optionally with modal transform

## CONSTRUCT IDENTICAL, EQUALLY-SPACED BLADES

for efficiency, trim analysis can assume that all blades have same motion, so only necessary to analyze one blade
but all blades are constructed, and assumption that motion is same need not be made
rotorcraft shell can also construct a rotor with unequal blade spacing
more general problems, including dissimilar blades, can be implemented using core input

## USUALLY ASSUME SYSTEM MOTION IS PERIODIC FOR TRIM ANALYSIS

so necessary to exclude vibratory dynamic and aerodynamic interaction between rotors operating at different rotational speeds (as for main rotor and tail rotor configuration)
can also analyze rotors and airframe using common period, with full interaction




## 6-2 Rotorcraft System

## FIGURES 2 to 28 SHOW SYSTEM CONSTRUCTED BY

 ROTORCRAFT SHELLconventions in figures:
components $=$ rectangle; name in upper case, type in lower case
input/output interfaces = triangle
structural dynamic interfaces $=$ diamond
input and output $=$ semicircle
input/output interfaces and structural dynamic components (shaded) connect the figures

FOR NAMES OF SYSTEM PIECES AND VARIABLES APPEARING IN FIGURES 2 to 28:

CAMRAD II Documentation, Volume III, Rotorcraft Theory; Chapter "Rotorcraft System"

PART AND LOOP NAMES ARE LISTED IN OUTLINES OF TRIM, TRANSIENT, AND FLUTTER SOLUTION PROCEDURES


Figure 6-2 Rotorcraft control.


Figure 6-3 Gust and auxiliary force control.

rotor n control to swashplate or blade, not both (dashed line)


Figure 6-4 Rotor swashplate control.


Figure 6-5 Rotor blade control.


Figure 6-6 Airframe.


Figure 6-7 Drive train.


Figure 6-8 Rotor hub and control system.


Figure 6-9 Rotor blade structure (single load path).


Figure 6-10 Rotor blade structure (bearingless).


Figure 6-11 Rotor blade aerodynamics.


Figure 6-12 Rotor blade structure and aerodynamics (rigid wing).


Figure 6-13 Rotor uniform inflow.


Figure 6-14 Rotor nonuniform inflow.


Figure 6-15a Induced and interference velocities.


BLADE
AVERAGE

Figure 6-15b Induced and interference velocities.


Figure 6-15c Induced and interference velocities.


Figure 6-16 Rotor tip-path-plane tilt, coning, power, and frame (performance).


Figure 6-17 Rotor frame (inflow).


Figure 6-18 Rotor and rotorcraft performance.


Figure 6-19 Tail boom.


Figure 6-20 Rotor duct force.


Figure 6-21 Rotor blade trailing edge flap control.


Figure 6-22 Slung load.


Figure 6-23a Post-trim rotor blade aerodynamics.


Figure 6-23b Post-trim rotor blade aerodynamics.


Figure 6-23c Post-trim rotor blade aerodynamics.


Figure 6-24 Computational fluid dynamics analysis.


Figure 6-25 Autopilot.


Figure 6-26 Rotor blade reaction jet control.


Figure 6-27 Rotor control system with nonrotating actuators.


Figure 6-28 Rotor pylon.

## 6-3 Frames, Periods, and Weights

FRAMES
FRAMES ARE ESTABLISHED FOR AIRFRAME (F), ROTORS
NONROTATING (S) AND ROTATING (R), AND BLADES (B)
frame degrees of freedom used for each
corresponding frame components are airframe, rotor hub, rotor rotating hub, and rotor blade root
figure 29 illustrates the conventions

## INERTIAL FRAME

gravity acts in $+z$ direction; so $z$-axis positive downward

FRAME = AIRFRAME

## F FRAME IS BASE FRAME FOR SYSTEM

parent is inertial frame
angular motion of F measured from orientation of inertial axes

FREE FLIGHT
airframe is frame component airframe constructed so normally F axes are center-of-mass, mean axes of system, with origin at rotorcraft center of gravity
$x$-axis forward, with aircraft velocity positive in $x$ direction
$y$-axis to right; $z$-axis down

ROTOR n BLADE m (B frame)


ROTOR n ROTATING (R frame)


ROTOR n
(S frame)

right

AIRFRAME (F frame)


Figure 6-29 Rotorcraft frames.
degrees of freedom are typically body axis velocity and Euler angles (aircraft convention)
yaw about $z$-axis, then pitch about $y$-axis, then roll about $x$-axis
body axis velocity representation does not produce complete calculation of frame displacement can use inertial axis displacement for linear degrees of freedom, if rotorcraft position relative inertial frame is important (such as for ground effect, or for wake geometry in transients)

## WIND TUNNEL

airframe is constrained component
$F$ axes are axes of wind tunnel test module, origin at center of test module
$x$-axis forward, with wind positive from $x$ direction $y$-axis to right; $z$-axis down
fixed orientation of $F$ frame relative inertial frame, specified by Euler angles
yaw about $z$-axis, then pitch about $y$-axis, then roll about $x$-axis
velocity of air given by wind speed (flight speed not used)

FRAME $=$ ROTOR $n$

## S FRAME IS NONROTATING HUB FRAME (SHAFT AXES) OF ROTOR

parent is airframe frame
$z$-axis is shaft axis, positive in positive thrust direction rotor rotates about $z$-axis, clockwise or counterclockwise
$x$-axis positive downstream
blade azimuth angle measured from $x$-axis, in direction of rotor rotation
$y$-axis is on advancing side for counter-clockwise rotation, on retreating side for clockwise rotation
$x-y$ plane is rotor hub plane; origin of frame is at center of rotation

## S AXES ARE WING SET AXES (REFERENCE AXES OF ROTOR AERODYNAMICS)

## MOTION REPRESENTATION

if airframe is rigid body, then $S$ frame has constant position and orientation relative F frame
if airframe has elastic motion, then $S$ frame motion includes effects of airframe dynamics

FRAME = ROTOR n ROTATING R FRAME IS ROTATING HUB FRAME (SHAFT AXES) OF ROTOR
parent is rotor frame
$z$-axis is shaft axis, positive in positive thrust direction origin of frame is at center of rotation MOTION REPRESENTATION
nonrotating hub component has no elastic motion, and joint at origin to produce rotor rotation
origin of rotating hub component attached to joint
so $R$ frame has same origin as $S$ frame, and is rotated from S frame by angle $\psi$ about shaft

$$
\psi=\text { azimuth of reference blade }(m=N)
$$

without drive train model, $\psi$ is prescribed rotation of joint, at constant angular speed $\Omega$
with drive train model, $\psi$ is degree of freedom

FRAME $=$ ROTOR $n$ BLADE $m$ B FRAME IS ROTATING FRAME OF $m$-th BLADE OF ROTOR ( $m=1$ TO $N$ )
separate frame for each blade, parent is rotor rotating frame
$z$-axis is shaft axis, positive in positive thrust direction $y$-axis is spanwise, positive radially outward $x$-axis is chordwise
$x$-axis positive towards blade trailing edge for counter-clockwise rotation
$x$-axis positive towards blade leading edge for clockwise rotation
origin of frame is at center of rotation

## B AXES ARE WING AXES (REFERENCE AXES OF BLADE AERODYNAMICS)

## MOTION REPRESENTATION

rotating hub component has no elastic motion
origin of blade root component attached to rotating hub component, possibly through gimbal joint
so $B$ frame has same origin as $R$ frame, and is rotated from $\mathbf{R}$ frame by angle ( $m \Delta \psi-90$ ) about shaft
$\Delta \psi=2 \pi / N=$ interblade spacing
blade azimuth angle is $\psi_{m}=\psi+m \Delta \psi$
for rotor with unequal blade spacing, specified blade azimuth angle $\Delta \psi_{m}$ replaces $m \Delta \psi$

## AXES OF BLADE ELEMENT

axes of rigid body motion (origin at inboard end of element), following conventions of beam component $x$-axis is beam axis of element, approximately spanwise, positive radially outward
$z$-axis is approximately parallel to shaft axis, positive in positive thrust direction
$y$-axis is in $x-y$ plane of B frame, approximately chordwise $y$-axis positive towards blade leading edge for counterclockwise rotation
$y$-axis positive towards blade trailing edge for clockwise rotation

## PERIODS

## ROTATIONAL SPEED OF FIRST ROTOR $\left(\Omega_{1}\right)$ IS BASE PERIOD FOR ROTORCRAFT

all other periods are children of $\Omega_{1}$

## PERIODS REQUIRED

| period name | frequency |
| :--- | :--- |
| ROTOR 1 | $\Omega_{1}$ |
| ROTOR n | $\Omega_{n}=r_{n} \Omega_{1}$ |
| ROTOR n BLADE PASSAGE | $N_{n} \Omega_{n}$ |
| DRIVE TRAIN INTERCONNECT SHAFT | $r_{I} \Omega_{1}$ |
| ROTORCRAFT FLUTTER | $r_{f} \Omega_{1}$ |
| ROTOR n FLUTTER | $r_{f} \Omega_{n}$ |
| ROTOR n TRIM | $\Omega_{n} / K_{n}=\left(r_{n} / K_{n}\right) \Omega_{1}$ |
| AIRFRAME TRIM | $\left(1 / K_{A}\right) \Omega_{1}$ |
| DRIVE TRAIN TRIM | $\left(1 / K_{D}\right) \Omega_{1}$ |

$r_{n}=$ gear ratio for $n$-th rotor shaft
$r_{I}=$ gear ratio for interconnect shaft
$N_{n}=$ number of blades for $n$-th rotor
$r_{f}=$ flutter gear ratio (used in flutter analysis)
$K=$ number of periods (used in trim part solution)

## WEIGHTS

WEIGHTS SYSTEM PIECE DEFINED FOR EACH ROTOR
name = ROTOR n
required to provide standard weights for perturbation and convergence of variables

## 6-4 Control System

## ROTORCRAFT CONTROL SYSTEM <br> SET OF CONTROL VARIABLES $c$ <br> CALCULATED FROM SYSTEM INPUT VECTORS $u$ AND PRESCRIBED CONTROL VARIABLES $p$

control $c$ (interface variables) used by rest of system as required
generally units $=$ degrees for input vectors, and radians for control vectors
except for tail boom and throttle variables, and lb or N for forces

GUST VELOCITIES CALCULATED FROM INPUT VECTOR $g$ gust generally not used for trim task, but available in trim to simulate aerodynamic interference
figures 2 to $5,21,25$ to 28 describe configuration constructed

## ROTOR MODEL CONSTRUCTED CAN HAVE SWASHPLATE MECHANISM

WITHOUT SWASHPLATE:
rotor collective and cyclic control produce rotating frame pitch control (at blade pitch joint)

IF SWASHPLATE EXISTS:
swashplate used to generate rotor control, perhaps with individual-blade-control

## ROTOR BLADE AERODYNAMIC MODEL CAN HAVE ONE OR MORE TRAILING-EDGE FLAPS

perhaps connected to rotor primary control
ROTOR BLADE CAN HAVE ONE OR MORE REACTION JETS FOR CONTROL
modelled as applied forces acting on beam elements

## PRESCRIBED CONTROL ONLY USED FOR TRANSIENT TASK <br> CONSTRUCT PRESCRIBED CONTROL $p$ CORRESPONDING TO ALL INPUT VECTORS $u$ AND $g$ <br> except for higher harmonic pitch and force

## SEPARATE PRESCRIBED CONTROL COMPONENT CONSTRUCTED FOR EACH ELEMENT OF $u$ AND $g$

shell defines different amplitude but same time history shape for these prescribed control components
core input can be used to define different time history shapes for individual elements

## AUTOPILOT CAN BE CONSTRUCTED FOR TRANSIENT TASK

using designated airframe sensors
core input required to complete definition: time constants and gains; perhaps more states (poles and zeros in transfer function), different sensors, nonlinear output

## INPUT VECTORS

## VALUES OF INPUT VARIABLES REQUIRED FOR TRIM, TRANSIENT, AND FLUTTER TASKS

## INPUT VARIABLES CONSTRUCTED (VECTORS)

pilot stick: $u_{P}=\left(\delta_{0}, \delta_{c}, \delta_{s}, \delta_{p}, \delta_{t}\right)^{T}$
airframe aerodynamic controls: $u_{A}=\left(\delta_{f}, \delta_{e}, \delta_{a}, \delta_{r}\right)^{T}$
pylon controls: $u_{Y n}=\left(\delta_{Y c}, \delta_{Y p}, \delta_{Y y}\right)^{T}$
tail boom controls: $u_{T}=\left(\delta_{b}, \delta_{j}\right)^{T}$
engine throttle: $u_{E}=\left(\theta_{t n}\right)$
auxiliary force: $u_{F}=\left(F_{m}\right)$
collective governor: $u_{G}=\left(\delta_{g}\right)$
rotor primary controls: $u_{R n}=\left(\theta_{0}, \theta_{1 c}, \theta_{1 s}\right)^{T}$
blade pitch: $u_{B n}=\left(\theta_{m}\right)$
blade trailing-edge flap: $u_{D n j}=\left(\phi_{m}\right)$
blade reaction jet: $u_{J n}=(J)$
rotating frame HHC pitch:

$$
u_{B H n}=\left(0,0,0, \theta_{2 c}, \theta_{2 s}, \ldots, \theta_{k c}, \theta_{k s}\right)^{T}
$$

rotating frame higher harmonic trailing-edge flap:

$$
u_{D H n j}=\left(0, \phi_{1 c}, \phi_{1 s}, \phi_{2 c}, \phi_{2 s}, \ldots, \phi_{k c}, \phi_{k s}\right)^{T}
$$

nonrotating frame HHC swashplate control:

$$
\begin{aligned}
& u_{R H 0 n}=\left(0,\left(\theta_{p N c}\right)_{0},\left(\theta_{p N s}\right)_{0}\right)^{T} \\
& u_{R H c n}=\left(0,\left(\theta_{p N c}\right)_{1 c},\left(\theta_{p N s}\right)_{1 c}\right)^{T} \\
& u_{R H s n}=\left(0,\left(\theta_{p N c}\right)_{1 s},\left(\theta_{p N s}\right)_{1 s}\right)^{T}
\end{aligned}
$$

higher harmonic auxiliary forces:

$$
u_{F H n m}=\left(0,\left(F_{p N c}\right)_{m},\left(F_{p N s}\right)_{m}\right)^{T}
$$

gust velocities: $g=\left(u_{G}, v_{G}, w_{G}\right)^{T}$

## SIGN CONVENTIONS

## PILOT'S CONTROLS

collective stick, $\delta_{0}$ lateral cyclic stick, $\delta_{c}$ longitudinal cyclic stick, $\delta_{s}$ pedal, $\delta_{p}$ throttle, $\delta_{t}$
positive up positive right positive forward positive yaw right positive increase power
typically then positive pilot's longitudinal and lateral cyclic stick produce respectively $(-\sin \psi)$ and $(-\cos \psi)$ variation of blade pitch

## GUST VARIABLES ARE UNIFORM VELOCITY PERTURBATIONS, IN NEGATIVE WIND/GUST AXES

longitudinal gust velocity, $u_{G}$ lateral gust velocity, $v_{G}$ vertical gust velocity, $w_{G}$
positive from forward
positive from right
positive from below

## CALCULATION OF CONTROL VECTORS

CONTROL VECTORS $c$ (INPUT/OUTPUT INTERFACE VARIABLES) CALCULATED FROM INPUT VECTORS $u$ AND $g$, AND PRESCRIBED CONTROL VARIABLES $p$

## OPERATIONS INCLUDE:

connection of pilot's controls to airframe, pylon, tail boom, engine, auxiliary force, and rotor controls
connection defined in terms of control matrix $T_{C}$ conversion of units from degrees to radians evaluation of control in time domain from harmonics incorporation of feedback variables
collective governor $=$ feedback from drive train mean governor (throttle governor is part of drive train model)

## CONTROL MATRICES

CONTROL SYSTEM MODEL CONNECTS PILOT'S CONTROLS TO AIRFRAME, PYLON, TAIL BOOM, ENGINE, AUXILIARY FORCE, AND ROTOR CONTROLS
connection defined in terms of control matrices $T_{C}$
linear relation, typically of form:

$$
c_{X}=\frac{1}{57.3}\left(T_{C X} u_{P}+u_{X}\right)
$$

$u_{X}=$ input vector specifying control position with all sticks centered ( $u_{P}=0$ )

$$
T_{C X}=\text { control matrix }
$$

## FOR MODERN ROTORCRAFT, COUPLING BETWEEN PILOT'S CONTROLS AND ROTOR/AIRCRAFT CONTROLS CAN BE QUITE COMPLEX <br> involving nonlinear functions and gain scheduling with flight condition <br> FOR TRIM TASK, ONLY NECESSARY TO CORRECTLY ACCOUNT FOR COUPLING BETWEEN REDUNDANT ROTOR AND AIRCRAFT CONTROLS

then trim solution will produce correct positions of individual controls
and true position of pilot's sticks (if required) can be determined from the actual control system geometry

CONTROL MATRICES $T_{C}$
INPUT DIRECTLY
OR USE STANDARD FORMS, CALCULATED FROM GAIN FACTORS AND SWASHPLATE PHASE ANGLES

## STANDARD FORMS

CAMRAD II Documentation, Volume III, Rotorcraft Theory; Chapter "Control System"
for airframe, tail boom, and engine
for auxiliary forces
first auxiliary force for propulsion (connected to throttle)
second auxiliary force for anti-torque (connected to pedal)
for rotor controls, depending on configuration:

| configuration | first rotor |
| :--- | :--- |
| one rotor |  |
| single main-rotor and tail-rotor | main rotor |
| tandem main-rotor | front rotor |
| coaxial main-rotor | lower rotor |
| tilting proprotor aircraft | right rotor |

## 6-5 Airframe

## ROTORCRAFT AIRFRAME CONSISTS OF AN ELASTIC BODY WITH ONE OR MORE ROTORS ATTACHED

## FREE FLIGHT OR WIND TUNNEL

for free flight: rigid and elastic airframe motion
in wind tunnel: rotor support can be flexible; no rigid body motion of system

## ELASTIC MOTION DESCRIBED BY LINEAR MODES

linear normal modes component
modes often obtained from large finite-element analysis of structure (free-vibration modes for free flight; constrained modes for wind tunnel)

## QUASISTATIC MODEL FOR AERODYNAMIC FORCES AND MOMENTS

rigid airframe aerodynamics component
nonlinear model for trim task
nonlinear or stability derivatives model for flutter and transient tasks
typically based on wind tunnel measurements of aerodynamic characteristics

## AIRFRAME-INDUCED INTERFERENCE VELOCITY AT ROTORS

calculated using simplified representation of airframe
or interference velocities can be obtained from large panel-method analysis

## TAIL BOOM

optional circulation-control boom and reaction jet
torque reaction and yaw control for single main-rotor helicopter configuration
programmable component, so model can be user-defined; interfaces constructed should still be appropriate

## ROTOR PYLON

airframe model can include pylon for each rotor attached to airframe at a pivot with spring, damper, and control
hub, swashplate, and swashplate actuators attached to pylon

## SLUNG LOADS

one or more slung loads can be attached to airframe load suspension is one or more cables from each load to airframe
figures 6, 19, 22, and 28 describe configuration constructed

## FRAMES AND INERTIA

FIGURES 30 AND 31 SUMMARIZE FRAMES, AND CONVENTIONS FOR DESCRIBING INERTIA
orientation of blade and element axes depends on rotor direction of rotation

FOR FREE FLIGHT, AIRFRAME IS CONSTRUCTED SO F AXES ARE NORMALLY CENTER-OF-MASS, MEAN AXES OF SYSTEM, WITH ORIGIN AT ROTORCRAFT CENTER OF GRAVITY
by using the linear normal modes component, the rigid motion describes center-of-mass, mean axes motion of airframe component
inertial properties of components defined so this property can hold for entire system

AIRFRAME ELASTIC MODES DESCRIBE ENTIRE SYSTEM, EXCEPT FOR EFFECTS OF ROTATING BLADES
obtained by calculation using finite-element analysis, or by shake test of actual airframe
various conventions in use by industry to simulate effects of rotor inertia in these modes
rotorcraft shell designed to accommodate these conventions

ROTOR n BLADE m ELEMENT k beam axes, B frame

$$
\sum \begin{aligned}
& M=M_{b} \\
& \sum I=I_{b}
\end{aligned}
$$

ROTOR n BLADE m ROOT
$B$ frame component massless


AIRFRAME
F frame component
$M=\frac{1}{g} W-\sum N M_{b}+\sum M_{R}$
$I$ and modal mass $=$ input
counter-clockwise rotation


B frame


S frame


F frame

gross weight $W$ and CG position input for entire system
inertia $I$ and modal mass input without rotor inertia ( $M_{b}$ and $I_{b}$ ), plus $M_{R}$ and $I_{R}$ at hub node (canceled by terms in AIRFRAME ROTOR $\mathbf{n}$ ) solution procedure requires rotor inertia terms ( $M_{b}$ and $\left.I_{b}\right)$ in ROTOR n HUB and AIRFRAME ROTOR n

Figure 6-30 Rotorcraft frames and inertia (counter-clockwise rotation).

ROTOR n BLADE m ELEMENT k beam axes, B frame $\sum \begin{aligned} M & =M_{b} \\ \sum I & =I_{b}\end{aligned}$


ROTOR n BLADE m ROOT $B$ frame component

ROTOR n HUB
S frame component

$$
\begin{aligned}
& M=-N M_{b} \\
& I_{z z}=-N I_{b}
\end{aligned}
$$

AIRFRAME ROTOR n $S$ axes, $F$ frame
$M=N M_{b}-M_{R}$
$I=N I_{b} \quad-I_{R}$
$\begin{array}{ll}I_{z z}=N I_{b} & -I_{R z z} \\ I= & -I_{R}\end{array}$
$I=\quad-I_{R}$

AIRFRAME
F frame component
$M=\frac{1}{g} W-\sum N M_{b}+\sum M_{R}$
$I$ and modal mass $=$ input
clockwise rotation


B frame


S frame


F frame

gross weight $W$ and CG position input for entire system
inertia $I$ and modal mass input without rotor inertia ( $M_{b}$ and $I_{b}$ ), plus $M_{R}$ and $I_{R}$ at hub node (canceled by terms in AIRFRAME ROTOR $\mathbf{n}$ )
solution procedure requires rotor inertia terms ( $M_{b}$ and $I_{b}$ ) in ROTOR n HUB and AIRFRAME ROTOR n

Figure 6-31 Rotorcraft frames and inertia (clockwise rotation).

## INPUT DESCRIPTION OF SYSTEM INERTIAL PROPERTIES

definition of rotor structure (blades and flexbeams, perhaps with point masses and flaps) includes inertia
all in rotating frame, outboard of rotor blade root component
$M_{b}=$ total mass of a blade
$I_{b}=$ corresponding rotational moment of inertia
input of rotorcraft gross weight $W$ and center of gravity:
for entire system, including rotors
input of rigid body moments of inertia $I$ and elastic mode generalized masses:
for the system without rotating frame inertia (same inertia that gives $M_{b}$ and $I_{b}$ )
plus equivalent hub inertia consisting of mass $M_{R}$ and moments of inertia $I_{R}$
equivalent hub inertia corresponds to configuration analyzed or tested to obtain the modal properties
typically $M_{R}=N M_{b}$
in practice many approaches are used to represent rotational inertia of rotor, and sometimes total system mass is not matched
figure 32 illustrates determination of equivalent hub mass $M_{R}$ (similar for moments of inertia)

MASS OF ACTUAL ROTORCRAFT CONFIGURATION


MASS OF AIRFRAME COMPONENT CREATĖD BY CAMRAD II remove mass accounted for by rotor input (blade and flexbeam mass, trailing-edge flap mass, point masses)


MASS OF SHAKE TEST OR FINITE-ELEMENT CONFIGURATION input inertia $I$ and modal mass are for this configuration

test or analysis configuration

$$
M_{R}>0
$$

test or analysis configuration

$$
M_{R}=0
$$

test or analysis configuration

$$
M_{R}<0
$$

$M_{R}$ is the mass that must be subtracted from the test or analysis configuration, to match the required airframe component

Figure 6-32 Determination of equivalent hub inertia.

## INERTIAL PROPERTIES OF COMPONENTS ARE CALCULATED FROM INPUT DATA

rigid body component is placed at rotor hub node, to subtract equivalent hub inertia ( $M_{R}$ and $I_{R}$ )
mass and center-of-gravity position of airframe component calculated
if $M_{R}=N M_{b}$, then F axes are motion of system center-ofgravity
for convergence of motion loop (iteration between rotor and airframe solutions), inertial reactions of rotor mass to hub motion should be on airframe side
so rotor mass $N M_{b}$ and rotational inertia $N I_{b}$ added to rigid body component at hub node (on airframe side)
and subtracted from rotor hub component (on rotor side) as shown in figures 30 and 31

## GEOMETRY

## AIRFRAME GEOMETRY DESCRIBED IN TERMS OF LOCATIONS OF STRUCTURAL DYNAMIC COMPONENT

## LOCATIONS USED (IN ORDER DEFINED):

rotor hub: for each rotor
rotor swashplate: for each rotor
general locations: for swashplate nonrotating actuators, auxiliary forces, tail boom, sensors, airframe wings and bodies, slung loads, and rotor-induced velocity calculation airframe aerodynamic collocation points: wing-body, horizontal tail, vertical tail, and stability derivative

## POSITION OF LOCATION SPECIFIED IN TERMS OF REFERENCE AXES

reference axes are parallel to F axes, but have arbitrary origin and different directions (figure 33)

| fuselage station (FS) | positive aft |
| :--- | :--- |
| buttline (BL) | positive right |
| waterline (WL) | positive up |

## GENERAL LOCATIONS

ORIENTATION AT GENERAL LOCATION SPECIFIED IN TERMS OF AZIMUTH ANGLE AND ELEVATION ANGLE, RELATIVE F AXES
azimuth angle $\psi=$ rotation about positive $z$-axis
elevation angle $\theta=$ rotation about resulting $y$-axis, measured from $x-y$ plane of F axes


Figure 6-33 Definition of aircraft geometry.

## ROTOR LOCATIONS

ORIENTATION OF LOCATION AXES AT ROTOR HUB = ORIENTATION OF UNDEFLECTED NONROTATING HUB FRAME (S AXES) RELATIVE AIRFRAME FRAME (F AXES)
position is center of rotation
$z$-axis is shaft axis, positive in positive thrust direction $x$-axis is positive downstream, azimuth measured from $x$ axis

## SPECIFICATION (RELATIVE BASIC ORIENTATION WITH S AXES PARALLEL F AXES)

for main rotor
first shaft roll $\phi_{R}$ (positive right)
then shaft angle of attack $\theta_{R}$ (positive rearward)
then azimuth reference angle $\psi_{R}$ about final $z$-axis (thrust axis)
for tail rotor
first shaft cant $\phi_{R}$ (positive up)
then shaft angle of attack $\theta_{R}$ (positive rearward)
then azimuth reference angle $\psi_{R}$ about final $z$-axis (thrust axis)

## SWASHPLATE LOCATIONS

axes at swashplate location are parallel to rotor $S$ axes on shaft ( $z$ axis), distance $h_{S P}$ below hub

## TILTING PROPROTOR AIRCRAFT GEOMETRY DEFINED IN HELICOPTER MODE, FOR RIGHT ROTOR

nacelle tilt angle $\alpha$ is zero for airplane mode, and 90 degrees for helicopter mode
specification of rotor orientation (at $\alpha=90$ )
first shaft roll $\phi_{R}$ (positive right)
then shaft angle of attack $\theta_{R}$ (positive rearward)
then azimuth reference angle $\psi_{R}$ about final $z$-axis (thrust axis)
nacelle pivots about axis with sweep $\psi_{P}$ (positive aft) and dihedral $\phi_{P}$ (positive up) relative the $y$-axis of F frame position of pivot and right rotor hub are specified for helicopter mode ( $\alpha=90$ )
usually geometry at left rotor calculated assuming that aircraft is symmetric about $\mathrm{BL}=0$
or geometry can be separately specified for left rotor, to model configuration that is not symmetric
with more than two rotors, first two rotors always tilt, other rotors may or may not tilt
if all rotors tilt, aircraft may or may not be symmetric

## MODE SHAPES OF ELASTIC MOTION

## MODES SHAPES DESCRIBED BY LINEAR COMPONENTS $\xi$ AND ANGULAR COMPONENTS $\gamma$

at each airframe location
input in F axes, or in undeflected location axes
ASSUMING THAT GENERALIZED COORDINATE $q_{k}$ HAS UNITS OF FEET OR METERS, MODAL PROPERTIES HAVE FOLLOWING UNITS:
generalized mass $M_{k}$ : slug or kg
linear mode shape $\xi_{k}$ : $\mathrm{ft} / \mathrm{ft}$ or $\mathrm{m} / \mathrm{m}$ (dimensionless) angular mode shape $\gamma_{k}$ : rad/ft or rad/m

## ELASTIC MODES CAN BE ARBITRARILY SCALED

if $\xi_{k}$ and $\gamma_{k}$ multiplied by factor $S$ (at all locations) then $M_{k}$ should be multiplied by $S^{2}$
and solution for $q_{k}$ will be divided by $S$

## AIRFRAME FINITE-ELEMENT ANALYSIS MIGHT USE INCHES FOR LENGTH UNITS

to convert to feet: angular modes shapes $\gamma_{k}$ and generalized mass $M_{k}$ multiplied by 12

COORDINATE SYSTEM OF FINITE-ELEMENT ANALYSIS CAN HAVE $x$-AXIS POSITIVE AFT, $y$-AXIS POSITIVE TO RIGHT, AND $z$-AXIS POSITIVE UPWARD
to convert to F axes, change signs of $x$ and $z$-components of linear and angular mode shapes ( $\xi_{k}$ and $\gamma_{k}$ )

## EFFECTS OF ELASTIC MOTION AND FORCES ARE SUPPRESSED AT AIRFRAME AERODYNAMIC COLLOCATION POINTS <br> so mode shapes not required at these locations

## OPTIONALLY, CAN CALCULATE MODE SHAPES AT SWASHPLATE LOCATIONS FROM MODE SHAPES AT ROTOR HUB LOCATIONS

assuming rigid motion of shaft
so there will be no pitch/mast-bending coupling

## TRANSMISSION COMPONENTS USED TO MODEL DRIVE TRAIN

so actual load paths between rotor, airframe, and drive train are not modelled
and rotor torque always reacted at hub node of airframe (and perhaps at swashplate node), with or without drive train model
because true load path through transmission is not modelled, may be appropriate to suppress effect of rotor torque on airframe elastic modes
accomplish by setting to zero the shaft-axis component of angular mode shape at hub (and swashplate)

## AUXILIARY FORCES

AUXILIARY FORCES CAN BE APPLIED TO AIRFRAME, USING AN APPLIED LOAD INTERFACE
location and axis at that location are identified for each force
forces are connected to control vectors
CONVENTIONS FOR CONTROL MATRIX
first auxiliary force for propulsion (connected to throttle) second auxiliary force for anti-torque (connected to pedal)

FOLLOWING SOURCES CONSTRUCTED:
constant forces, from input vector $u_{F}$
can represent fixed auxiliary lift or propulsive force
forces proportional to the pilot control, from the input vector $u_{P}$
can be used to trim rotorcraft
both $u_{F}$ and $u_{P}$ terms can be used to perturb system in transient or flutter
higher-harmonic forces, from the input vector $u_{F H n m}$
can represent effects such as fuselage loads produced by rotor wake impingement
prescribed forces $p_{F m}$, generated for the transient task

## 6-6 Drive Train

# ROTORCRAFT DRIVE TRAIN CONSISTS OF ONE OR MORE ROTORS, AND ONE OR TWO ENGINES, CONNECTED THROUGH FLEXIBLE TRANSMISSION 

DRIVE TRAIN LOAD PATHS CONSIDER TORQUE BALANCE ONLY
rotor torque acting on airframe is reacted at the hub component, with or without a drive train model

ROTOR SPEED GOVERNOR IS INCLUDED, USING COLLECTIVE OR THROTTLE FEEDBACK OF ROTOR SPEED ERROR
governor dynamics represented by second-order transfer function

DRIVE TRAIN CONNECTED TO JOINT ON HUB COMPONENT, AND THUS TO THE ROTOR
when swashplate is constructed, there is also a transmission component connecting hub joint and rotating swashplate
so swashplate azimuth and rotor azimuth remain consistent

## ROTORCRAFT CAN BE CONSTRUCTED WITHOUT DRIVE TRAIN SUBSYSTEM <br> then rotor and swashplate rotate at constant speed, determined by prescribed motion of hub and swashplate joints

## ROTOR CAN HAVE FREE-ROTATION HUB

hub joint is degree of freedom
connected to swashplate (if it exists) but not connected to drive train

## HELICOPTER HAS OVERRUNNING CLUTCH THAT

 DISCONNECTS ROTORS FROM ENGINE (BUT NOT FROM EACH OTHER) AT ZERO OR NEGATIVE TORQUEso for autorotation operation, input parameters should be set to eliminate engine and engine shaft, throttle input, and governor
for operation with engines out (but still connected to rotors), input parameters should be set to eliminate engine damping, throttle input, and governor
figure 7 describes configuration constructed

## DRIVE TRAIN INERTIAL PROPERTIES

FIGURE 34 SUMMARIZES DRIVE TRAIN CONNECTIONS, AND CONVENTIONS FOR DESCRIBING INERTIA
rotational inertia is specified for rotor and engine shafts
$I_{b}=$ total rotational inertia of a blade

## INERTIAL PROPERTIES OF COMPONENTS ARE

 CALCULATED FROM INPUT DATAfor convergence of motion loop (iteration between rotor and drive train solutions), inertial reactions of rotor mass to hub motion should be on drive train side
so rotor rotational inertia $N I_{b}$ added to rotor shaft (on drive train side)
and subtracted from rotor azimuth component (on rotor side)

solution procedure requires rotor inertia terms ( $I_{b}$ ) in ROTOR n AZIMUTH and DRIVE TRAIN ROTOR n SHAFT

Figure 6-34 Drive train connections and inertia.

## TRANSMISSION CONFIGURATIONS AND MOTION

 CONFIGURATIONS ILLUSTRATED IN FIGURE 35 conventionssolid dots define structural dynamic interfaces (torque kind) between components
circles indicate gear ratios for branches of transmission
squares indicate rotational inertia of root or branch control torque $Q_{E}$ is applied load on engine branch also a damping term for engine branch
zig-zag structures indicate rotational springs of branches also a damper for each branch
there is an inertia corresponding to each degree of freedom (solution procedure requires that there be no massless elastic degrees of freedom)
elastic motion of any branch can be suppressed

TRANSMISSION CONFIGURATION MUST BE CONSISTENT WITH ROTORCRAFT CONFIGURATION
one rotor

asymmetric

asymmetric


Figure 6-35a Drive train configurations.
symmetric, two engines

symmetric, center engine
hub 1

hub 2
number of masses $=$ number of degrees of freedom

$$
\text { = } 1 \text { rigid + number of springs }
$$

solution procedure requires $N I_{b 1}$ and $N I_{b 2}$

Figure 6-35b Drive train configurations.

## CONFIGURATIONS AND ASSOCIATED SYSTEM DEGREES OF FREEDOM

one rotor: $\psi, \psi_{R 1}, \psi_{E 1}$
asymmetric, engine by first rotor: $\psi, \psi_{R 1}, \psi_{R 2}, \psi_{E 1}, \psi_{I 1}$ asymmetric, engine by second rotor: $\psi, \psi_{R 1}, \psi_{R 2}, \psi_{E 1}$, $\psi_{I 1}$
symmetric, two engines: $\psi, \psi_{R 1}, \psi_{R 2}, \psi_{E 1}, \psi_{E 2}, \psi_{I 1}, \psi_{I 2}$ symmetric, center engine: $\psi, \psi_{R 1}, \psi_{R 2}, \psi_{E 1}, \psi_{I 1}, \psi_{I 2}$
$\psi=$ azimuth
$\psi_{R n}=$ rotor shaft
$\psi_{E n}=$ engine shaft
$\psi_{\text {In }}=$ interconnect shaft
each transmission component has rigid degree of freedom, all but one of which is eliminated by structural dynamic interfaces
other degrees of freedom are elastic motion of transmission shaft (can be suppressed)

WITH MORE THAN TWO ROTORS, ADD BRANCH FOR $n$-th ROTOR ( $n \geq 3$ ) AS FOR SECOND ROTOR

RIGID DEGREE OF FREEDOM OF DRIVE TRAIN SYSTEM (INCLUDING THE ROTORS) IS AZIMUTH $\psi$
$\psi=$ rigid motion of appropriate transmission component (first rotor shaft, except for symmetric case)
response nominal for $\psi$ is $\psi_{\text {nom }}=\Omega_{1} t+\psi_{01}$
$\Omega_{1}=$ rotational speed of first rotor
$\psi_{01}=$ reference azimuth of first rotor

## AZIMUTH $\psi$ IS SYSTEM RIGID DEGREE OF FREEDOM

steady state motion is specified rotational speed of rotor corresponding system rigid force is net torque on drive train
trim loop can solve for throttle (or rotor speed or other variable) that makes this net torque zero

## GOVERNOR AND ENGINE CONTROL

DRIVE TRAIN AZIMUTH/SPEED ERROR $y_{\text {gov }}$ OBTAINED FROM TRANSMISSION COMPONENT WITH AZIMUTH $\psi$
component calculates errors $\dot{\psi}_{e}$ and $\psi_{e}$ relative reference rotation at prescribed rotor speed $\Omega$
sensor has proportional (rotational speed error) and integral (rotation angle error) terms, with constant gains:

$$
y_{\mathrm{gov}}=\theta_{g \text { static }}=K_{P} \dot{\psi}_{e}+K_{I} \psi_{e}
$$

## GOVERNOR DYNAMICS DESCRIBED BY SECOND-ORDER

 DIFFERENTIAL EQUATIONin terms of transfer function for governor degree of freedom $\theta_{g}$

$$
\frac{\theta_{g}}{\theta_{g \text { static }}}=\frac{\omega_{n}^{2}}{s^{2}+2 \omega_{n} \zeta s+\omega_{n}^{2}}
$$

## GOVERNOR FEEDBACK

control to rotor collective and engine throttle

$$
\begin{aligned}
\theta_{\mathrm{gov} R} & =K_{R \mathrm{gov}} \theta_{g} \\
\theta_{\mathrm{gov}} E & =K_{E \operatorname{gov}} \theta_{g}
\end{aligned}
$$

feedback to engine throttle is part of drive train subsystem (directly coupled to transmission dynamics)
feedback to rotor collective implemented through filter, so mean value can be included in steady collective pitch for trim

## ENGINE CONTROL MODELLED BY APPLIED LOAD (TORQUE) ON TRANSMISSION BRANCH REPRESENTING THE ENGINE

control torque produced by engine control $c_{E}$, and governor feedback term
gain $G_{E}$ converts total control from throttle units to torque

## 6-7 Rotor Structure

ROTOR STRUCTURE CONSISTS OF ONE OR MORE IDENTICAL, EQUALLY-SPACED BLADES

HUB AND BLADE ROOT CONFIGURATIONS INCLUDE ARTICULATED, HINGELESS, TEETERING, GIMBALLED, AND BEARINGLESS

## STRUCTURAL MODEL BASED ON BEAM THEORY FOR COUPLED FLAP-LAG-TORSION MOTION OF BLADE, INCLUDING LARGE PITCH AND LARGE TWIST

assumption that structural elements have high aspect-ratio is normally well satisfied for rotor blades
although beam theory may be less accurate modelling root of hingeless or bearingless blade
structural model: isotropic with elastic axis; or anisotropic/ composite
geometric model for elastic motion: second-order, almostexact, or exact

CONTROL CONFIGURATIONS INCLUDE SWASHPLATE MODEL (WITH PITCH HORNS AND PITCH LINKS), OR BLADE ROOT PITCH

ROTOR WITH UNEQUAL BLADE SPACING CAN BE CONSTRUCTED

ROTOR MODEL CAN BE USED TO CONSTRUCT A NONROTATING WING

## ROTOR BLADE AND FLEXBEAM ARE DIVIDED INTO ELEMENTS

modelled by beam components, with joints for required hinges and bearings
beam component has straight beam axis and only use blade properties at Gaussian integration points
so blade having properties that vary rapidly along its length must be modelled by breaking it into many elements, with major jumps in properties at the nodes
optionally, elements can have only rigid motion, or be modelled by rigid body components
rigid wing component can be used for last element of blade
combines rigid body component and lifting line wing component, for computational efficiency

## ANALYSIS CAN USE MODAL TRANSFORMATION AND TRUNCATION IN SOLVING FOR ROTOR MOTION

in trim, transient, or flutter tasks
modes encompass any control system components, but not rotating hub component or swashplate component (which couple structural dynamics of blades)
optionally modes can exclude joints, all elements and point masses inboard of joints, pitch horn and pitch link, and flexbeam
figures 8 to 10 describe rotor hub, blade, and control system configuration constructed

## ROTOR INERTIAL PROPERTIES

## DEFINITION OF ROTOR STRUCTURE (BLADES AND FLEXBEAMS, PERHAPS WITH POINT MASSES AND FLAPS) INCLUDES INERTIA

this inertia is all in rotating frame, outboard of rotor blade root component
used to calculate blade mass $M_{b}$ and moment of inertia $I_{b}$ (for system inertia)

INPUT VALUE OF BLADE AND FLEXBEAM MASS IS PER UNIT SPAN STATION (slug/ft or kg/m)
so integral over span station gives the correct blade mass when elements representing blade are constructed, input is converted to mass per length along beam axis

## MAJOR CONFIGURATION OPTIONS

ROOT CONFIGURATION IS SINGLE LOAD PATH, OR BEARINGLESS
for articulated and hingeless rotors, blade structure has single load path from root to tip
for bearingless rotors, structure has dual load path, consisting of blade and flexbeam

## GIMBAL OR TEETER JOINT MAY EXIST BETWEEN ROTATING

 HUB AND ROOT COMPONENTSteeter hinge for $N \leq 2$, gimbal for $N \geq 3$

## DRIVE TRAIN MODEL MAY EXIST

including connection between rotating hub and rotating swashplate

## SWASHPLATE MECHANISM CAN BE CONSTRUCTED

including pitch link and pitch horn components, perhaps with individual-blade-control
optionally can use nonrotating actuators
without swashplate mechanism, rotor control introduced at blade pitch joint

## POINT MASSES MAY BE ATTACHED TO BLADE AND FLEXBEAM

BLADE CAN HAVE ONE OR MORE TRAILING-EDGE FLAPS
figure 36 shows components, connections, and joints used to implement these options


Figure 6-36a Construction of rotor hub, blade, and control system.


Figure 6-36b Construction of rotor hub, blade, and control system.

## HUB AND BLADES

ROTOR HUB COMPONENT CONNECTED TO AIRFRAME
through a rigid body component, in order to handle description of rotor inertia
hub has joint for rotor rotation
rotating hub component attached to that joint rotor blade root components of all blades attached to connections on rotating hub component rotating hub component can have gimbal or teeter joint gimbal joint is Rodrigues hinge, not universal joint teeter joint can include pitch-flap coupling ( $\delta_{3}$ ) can have rotating shaft component between hub and rotating hub components, with rotational spring and inertia
optionally the gimbal or teeter hinge can be simulated by a flap hinge at the center of rotation on each blade
trim solution filtered so only $p N \pm 1$ harmonics nonzero
flutter degrees of freedom just tip-path plane tilt ( $\beta_{1 c}, \beta_{1 s}$ ) or teeter ( $\beta_{1}$ )
approximate model, but simpler
can solve for just reference blade, can even use pitch bearing control instead of swashplate
hub, rotating hub, rotating shaft, and root components are compact:
all locations, joints, and connections are at hub node (except the snubber connection)
so hub node is origin of $S, R$, and $B$ frames, center of rotation, and location of gimbal or teeter hinges
hub node also point at which hub and blade root loads are measured

## FIRST BLADE ELEMENT CONNECTED TO ROTOR BLADE ROOT COMPONENT

blade elements form single load path from root to tip (except for bearingless configuration)
each element can have joint at its outboard end
flap, lag, or pitch hinge, or damper
control can be introduced at pitch bearing
outboard of all joints is droop-sweep node
where sweep and droop of blade reference line introduced
with a swashplate mechanism, pitch horn component connected to blade at specified radial station
point mass component can be connected to any element

# FOR BEARINGLESS CONFIGURATION, ELEMENTS FORM TWO LOAD PATHS: BLADE AND FLEXBEAM 

flexbeam: no joints
first element connected to rotor blade root component
last element connected to blade at flexbeam node (inboard of droop-sweep node)
point mass component can be connected to any element
blade: constructed as for single load-path configuration with point masses, but no joints
no pitch bearing, so need swashplate for control all aerodynamic interfaces on blade, not flexbeam inboard end of blade can be free, or connected to snubber joint on hub or flexbeam
snubber: connects blade to hub (structure rigid from hub node to snubber attachment) or to flexbeam
for snubber axes fixed to hub or flexbeam:
snubber joint has three linear degrees of freedom, with pinned interface to blade
if snubber axes rotate with blade:
snubber joint has three angular and then three linear degrees of freedom, then cantilever interface to blade

## CONTROL SYSTEM

## SWASHPLATE MECHANISM

if swashplate mechanism exists, rotor swashplate component connected to airframe, at swashplate node on shaft axis, below the hub
swashplate component has joint for swashplate control
one linear motion for collective, then two angular motions for cyclic
cyclic joint is Rodrigues gimbal (not universal joint)
joint axes defined so longitudinal and lateral cyclic control properly phased
swashplate gains calculated from nominal geometry, so control can still be interpreted as blade pitch angle
joint motion = degrees of freedom with spring and offset actuator
rotor swashplate plane component attached to connection at that joint
optionally swashplate motion can be produced by 3 nonrotating actuators
each actuator connected to swashplate plane component and to actuator plane component
each actuator has linear joint with spring and offset actuator
swashplate plane component has joint for rotation of rotating swashplate
rotor blade pitch link components of all blades attached to connections at that joint
these connections are at appropriate location in swashplate plane, not at center of rotation
except for collective joint motion and pitch link connections, swashplate and swashplate plane components are compact
with all locations, joints, and connections at swashplate node
rotor blade pitch horn component connected to blade, at specified radial station (inboard of node or joint)
rotor blade pitch link component connects pitch horn and swashplate plane
at connection to pitch horn, pitch link component has linear joint, with spring for control system flexibility
optionally with actuator for individual-blade-control at connection to swashplate plane, pitch link component has gimbal joint (and cantilever interface)
pinned interface (and no joint) here would leave pitch link axial rotation unconstrained; a cantilever interface is also needed so that the interface force can be kept in rotating frame

## CONTROL SYSTEM MODELS

can construct:
swashplate mechanism with nonrotating actuators swashplate mechanism with swashplate control swashplate mechanism
no swashplate mechanism
swashplate mechanism introduces additional load paths and additional rotating-to-nonrotating frame interfaces for system
not restricted to small motions of linkages
with swashplate control, there must be pitch flexibility inboard of pitch horn
pitch bearing for single load-path configuration
flexbeam torsion flexibility for bearingless configuration
swashplate collective and cyclic joints have degrees of freedom, with offset actuators to introduce control
swashplate control =
rotor primary control (collective and cyclic)

+ nonrotating frame higher-harmonic control
control system flexibility represented by springs in pitch link and swashplate joints
locked swashplate obtained using quasistatic solution, with a large spring


## SINGLE LOAD-PATH CONFIGURATION CAN INCLUDE PITCH BEARING

can construct:
pitch bearing with pitch control pitch bearing no pitch bearing
pitch bearing with control not available if swashplate mechanism used
pitch joint has degree of freedom, with offset actuator to introduce control
control =
rotor primary control

+ blade pitch control
+ rotating frame higher-harmonic control
with a swashplate and individual-blade-control actuators, this control goes to pitch link joint, not to root pitch joint
control system flexibility can be represented by spring in pitch bearing (in absence of swashplate mechanism, this is only source of flexibility) locked pitch bearing obtained using quasistatic solution, with a large spring


## DRIVE TRAIN

## WITHOUT DRIVE TRAIN MODEL

rotation of rotor and swashplate prescribed, at constant angular velocity

## WITH DRIVE TRAIN MODEL

 this rotation is a degree of freedom, with nominal motion equal to the prescribed motion
## COMPONENTS

hub component has two additional rotational joints, with same joint variable as rotor rotation
swashplate plane component has one additional rotational joint, with same joint variable as rotating swashplate transmission component (rotor azimuth) connects rotating swashplate joint to hub rotation joint
so control system has same azimuth as rotor
drive train rotor shaft component connected to remaining joint on hub

## ROTOR CAN HAVE FREE-ROTATION HUB

hub joint is degree of freedom; connected to swashplate (if it exists) but not connected to drive train

## TRANSMISSION COMPONENTS USED TO MODEL DRIVE TRAIN

so actual load paths between rotor, airframe, and drive train are not modelled
and rotor torque always reacted at hub node of airframe (and perhaps at swashplate node), with or without a drive train model

## GENERAL CHARACTERISTICS

## ROTOR COMPONENTS

finite element beam components: elements of blade and flexbeam, and pitch horns
elastic motion can be suppressed; or can be replaced by rigid body component
pitch horn constructed without elastic degrees of freedom
rigid body components: all other
last element of blade can be replaced by rigid wing component

## INERTIA

only elements of blade and flexbeam, point masses, and flaps have inertia
all other components of rotor model are massless

## GENERALLY BLADES HAVE INDEPENDENT STRUCTURAL DYNAMICS

coupled through airframe, drive train, and aerodynamics so may be possible to solve for motion of just reference blade
gimbal or teeter hinge, or flexibility in swashplate joints, will couple blade structural dynamics

## SENSORS

## NONROTATING FRAME HUB FORCE AND MOMENT MEASURED ON ROTOR HUB COMPONENT

sum of loads applied at location of rotor rotation joint
loads at this location include inertial reactions and weight of rotors

## SHAFT TORQUE AND POWER, ROTATIONAL SPEED MEASURED ON ROTOR HUB COMPONENT

torque is shaft-axis component of hub moment power sensor is power of rotor rotation joint rotational speed is angular velocity of rotor rotation joint

SWASHPLATE FORCE AND MOMENT MEASURED ON SWASHPLATE COMPONENT

ROTATING FRAME BLADE ROOT FORCE AND MOMENT MEASURED ON ROTOR BLADE ROOT COMPONENT
sum of loads acting on hub

PITCH LINK LOAD
MEASURED ON ROTOR BLADE PITCH LINK COMPONENT
axial force, at location at top of pitch link

## BLADE SECTION FORCE AND MOMENT EVALUATED AT NODE, OR ON FINITE ELEMENT BEAM COMPONENT

node reaction measured as reaction on connection for structural dynamic interface at the node
in structural principal axes relative the tension center
on either side of node (since there might be jump in tension center or structural axes at node) section load evaluated from deflection, or by force balance

## BLADE POSITION SENSORS MEASURE MOTION OF BLADE

 flap, lag, pitch, and axial motion at span stations on blade and flexbeamdisplacement measured relative blade frame, rotating hub frame, nonrotating hub frame, airframe frame, or inertial frame
pitch rotation always measured relative rotating blade frame
and motion of all joints
flap, lag, pitch, damper joints
pitch link joint
snubber joint
gimbal or teeter joint
swashplate joint
CFD POSITION SENSORS MEASURE DISPLACEMENT AND EULER ANGLES OF BLADE
typically quarter chord position relative blade frame

## TIP-PATH PLANE SENSOR

## AERODYNAMIC AND PERFORMANCE CALCULATIONS, AND TRIM OPTIONS, REQUIRE ROTOR TIP-PATH PLANE ORIENTATION

calculated by reference plane component from sensor
$z_{T P P n m}$

## SENSOR CAN BE THE MOTION OF THE BLADE TIP (OR SOME OTHER SPAN STATION)

structural dynamic sensor that measures $z$-axis displacement of blade tip relative rotating hub frame (not blade frame, since blade frame is outboard of gimbal or teeter hinge)
displacement divided by rotor radius $R$, so sensor gives tilt of line from hub to blade tip, relative the hub plane
in general, can put sensor at span station $e_{T P P}$, on blade beam axis; scaled to tip value:

$$
z_{T P P n m}=\frac{z}{e_{T P P} R}
$$

need inboard span station for blade with swept tip, to avoid pitch motion contributing to vertical displacement
for gimballed or teetering rotor, sensor span station near hub will give just gimbal or teeter hinge motion

OR SENSOR CAN BE ROTATION OF FLAP HINGE (IF JOINT WITH FLAP MOTION EXISTS)

OR ROTATION OF GIMBAL OR TEETER HINGE

## BLADE GEOMETRY

FIGURE 37 SHOWS GEOMETRY OF BLADE AND CONTROL SYSTEM
specified in blade axes (B coordinates)
radial $y$-axis $=$ span station variable

## INPUT CONVENTIONS

shell input data dimensionless, scaled with rotor radius $R$ directions of $B$ axes relative blade are different for counter-clockwise and clockwise rotation
but shell input data same for both directions of rotation
input $x$-quantities always are positive towards trailing edge
blade properties are input as piecewise linear functions of span station

## BLADE TIP AND BLADE RADIUS

blade tip radial station is $e_{\text {tip }}$ (dimensionless)
usually $R$ is actual rotor disk radius, and $e_{\text {tip }}=1$
if $e_{\text {tip }} \neq 1$, then $R$ is not the physical radius, rather $R$ is only used for normalization of the input parameters and the output


Figure 6-37 Geometry of blade and control system.

## REFERENCE LINE

define reference line, extending from root to tip:
torque offset $x_{T O}$ and gimbal/teeter undersling $z_{U S}$ at root (probably negative)
precone angle $\beta_{p}$ at root (relative $y$-axis, positive up)
droop angle $\beta_{d}$ and sweep angle $\zeta_{s}$ (relative preconed line, positive down and aft) at droopsweep node at span station $e_{D S}$
so reference line consists of two straight segments, divided at droop-sweep node (which may be absent)
all joints are on first segment of reference line, at span stations inboard of droop-sweep node
so pitch bearing is aligned with preconed reference line
this reference line is used to describe blade geometry wing reference line is $y$-axis of rotating blade frame (B axes)

## GEOMETRY RELATIVE REFERENCE LINE

blade geometry specified in B coordinates, measured from the reference line:
locus of beam axis
locus of quarter chord
damper attachments
locations of point masses
aerodynamic twist measured about wing reference line ( $y$-axis), from $x-y$ plane of B frame

## BEAM AXIS

beam axis is elastic axis for isotropic structural model
beam axis must be straight within each element (between nodes)
such a beam axis constructed from input data, as straight line between specified beam axis positions at $10 \%$ and $90 \%$ element length (so jumps at the nodes are possible)
all structural dynamic interfaces between elements of blade and flexbeam are at mean of beam axis positions of ends of elements being connected
axes of interface are B frame axes

## BLADE PROPERTIES

blade properties are specified relative beam axis
positions of center of gravity and tension center: measured from straight beam axis of element, in section principal axes
pitch angle of principle axes: measured about beam axis, from element $x-y$ plane
section mass specified per unit length along span axis, not along element axis (so integration over span station gives correct total blade mass)

## CONTROL SYSTEM

geometry of control system:
position of swashplate node, on shaft axis $h_{S P}$ below hub (positive)
position of connection between swashplate and pitch link, in B axes (with $z \cong-h_{S P}$ )
position of connection between pitch link and pitch horn, in B axes (with $x$ positive for trailing pitch horn)
span station $e_{P H}$ of pitch horn connection to blade
geometry defined for zero collective and cyclic control and reference blade pitch (typically corresponding to zero pitch at $75 \%$ radius, depending on definition of aerodynamic twist)

## BEARINGLESS CONFIGURATION

inboard end of blade is span station $e_{\text {blade }}$
flexbeam connected to hub at span station $e=0$
snubber joint can connect blade root component and inboard end of blade
location of snubber on blade root component specified in B axes (as for pitch link ends)
connection of snubber to blade specified
in B coordinates, measured from the reference line (as for beam axis), at radial station $e_{\text {root }}$
or snubber joint can connect flexbeam element and inboard end of blade
location of snubber on flexbeam element specified in B coordinates, measured from the reference line (as for beam axis), at radial station $y_{\text {snub }}$
connection of snubber to blade specified
in $B$ coordinates, measured from the reference line (as for beam axis), at radial station $e_{\text {root }}$
snubber joint has three linear degrees of freedom, each with spring and damper
snubber joint axes rotated by precone angle $\beta_{p}$, and then by snubber pitch angle $\theta_{s}$

ELEMENTS, NODES, AND JOINTS

## BLADE STRUCTURE DIVIDED INTO SET OF ELEMENTS (COMPONENTS) BY NODES (STRUCTURAL DYNAMIC INTERFACES) POSITIONED ON REFERENCE LINE

 nodes identified by span station $e$
## NODES

automatic nodes
connection of first element to rotor blade root component is first node, at $e=0$
blade tip is last node, at $e=e_{\text {tip }}$
droop-sweep node required at $e_{D S}$ if reference line has nonzero droop or sweep angle
node at each hinge or bearing
additional nodes can be specified as required
to account for large changes in beam axis definition, since beam axis is straight within each element
to increase number of elastic degrees of freedom
figure 38 illustrates connection between specification of nodes and construction of beam axis


BEAM AXIS OF CONSTRUCTED ELEMENTS
straight within element

- = specified nodes
-     -         -             - = input beam axis
_ = constructed beam axis

minimum number of nodes

more nodes


Figure 6-38 Blade beam axis constructed by rotorcraft shell.

## ELEMENTS

blade element constructed between each pair of nodes options for component type:
finite element beam component
without elastic motion if length is less than a specified minimum, or if element entirely within a specified range
finite element beam component, with no elastic motion
rigid body component (with inertial properties evaluated as by beam component)
analysis may create zero-length elements to accommodate specified joints element with zero length is massless and rigid

## JOINTS

outboard end of an element can have a joint; all joints occur before droop-sweep node possible joints: flap and lag hinges, pitch bearing, damper

FLAP AND LAG HINGES, AT $e_{F}$ AND $e_{L}$
options:
none
flap hinge only
lag hinge only
both hinges
flap then lag universal joint
lag then flap universal joint
flap-lag gimbal (Rodrigues hinges)
each hinge can have spring and damper
hinges placed on reference line, with joint axes parallel B axes
flap hinge can have pitch-flap coupling ( $\delta_{3}$ )
lag hinge can have pitch-lag coupling ( $\alpha_{1}$ )
hinge skew frequently used on tail rotor
but for main rotor, coupling with pitch more often produced by kinematics of control system
flap and lag hinges can be rotated by fixed pitch angles relative the $B$ axes
optionally joints can be locked: joint exists, but motion is prescribed (to zero) rather than degree of freedom

## PITCH BEARING AT $e_{P}$

options:
none
pitch bearing
pitch bearing with control
joint can have spring and damper
with swashplate mechanism, pitch bearing typically has zero spring and damping
without swashplate mechanism, spring at pitch bearing represents control system flexibility

PITCH BEARING PLACED ON REFERENCE LINE, ALIGNED WITH PRECONED DIRECTION
for controlled pitch, joint has offset actuator
locked pitch bearing obtained by using quasistatic solution for joint motion, with large spring

## BEARINGLESS CONFIGURATION

two load paths constructed: blade and flexbeam blade extends from $e_{\text {blade }}$ to tip blade may be attached to root component or flexbeam through snubber joint
flexbeam extends from hub $(e=0)$ to connection with blade at $e_{F B}$
flexbeam attached to rotor blade root component by cantilever structural dynamic interface
$e_{F B} \leq e_{D S}$, so reference line for flexbeam has only one segment, at precone angle

## 6-8 Rotor Aerodynamics

ROTOR CONSISTS OF ONE OR MORE IDENTICAL, EQUALLYSPACED WINGS (BLADES)

## BLADE CONFIGURATION INCLUDES SWEPT AND DROOPED TIPS (ARBITRARY QUARTER CHORD LOCUS) <br> ROTOR WITH UNEQUAL BLADE SPACING CAN BE CONSTRUCTED

## WING MODEL IS BASED ON LIFTING-LINE THEORY

using steady two-dimensional airfoil characteristics and vortex wake; plus corrections for unsteady and yawed-flow effects
wing component includes: second order lifting line theory; empirical dynamic stall models; unsteady loads from thinairfoil theory; yawed and swept flow corrections; spanwise drag
blade can have one or more trailing-edge flaps

## INDUCED VELOCITY OBTAINED FROM: <br> UNIFORM INFLOW MODEL, BASED ON MOMENTUM THEORY

mean inflow plus linear variation over rotor disk (from edgewise flow and hub moments)
dynamic inflow can be used in transient and flutter tasks (low frequency, global model of wake influence in unsteady aerodynamics of rotor)
or dynamic wake model (unsteady wake theory based on acceleration potential for actuator disk)

## OR NONUNIFORM MODEL, USING VORTEX WAKE REPRESENTATION

wake geometry can be rigid, prescribed (for hover), or free

## UNIFORM INFLOW MODEL CAN BE FOR DUCTED FAN

 duct aerodynamic loads calculated from rotor loads, and specified ratio of rotor load to total loadfigures 11 to 18, 20, 23, and 24 describe configuration constructed

## INPUT CONVENTIONS

chord input is dimensional (ft or m)
wing properties are input as piecewise linear functions of span station $r$

## WING GEOMETRY

## WING REFERENCE LINE IS $y$-AXIS OF ROTATING BLADE FRAME (B AXES)

straight line, with origin at center of rotation
span station variable is dimensionless radial distance $r=y / R$
span scale factor $R=$ blade radius
with large droop, will need wing reference line that includes the droop
for moderate droop, just use core input to define span scale factor $R$ as a function of $r$, such that $(R \Delta r c)$ is wing panel area

## WING PLANFORM DEFINED BY QUARTER-CHORD LINE, AERODYNAMIC TWIST, AND CHORD

rotor structure input gives quarter-chord line and twist wing sections are defined in plane perpendicular to wing reference line
input blade chord measured in this section, so blade area properly calculated
pitch measured in this section, from $x$-axis of B frame
quarter chord locus defined in this section, but measured from reference line of blade structural definition

## 6-9 Trim Task

TRIM TASK OBTAINS EQUILIBRIUM SOLUTION OF SYSTEM EQUATIONS, FOR STEADY STATE OPERATING CONDITION

ASSUME:
system environment and input are constant or periodic, equilibrium solution exists,
solution is constant or periodic
assumption that system is periodic requires exclusion of any vibratory dynamic and aerodynamic interaction between rotors operating different rotational speeds (such as for main rotor and tail rotor configuration)
rotors and airframe can be also analyzed using a common period, with full interaction

USUALLY IDENTIFY PARAMETERS REQUIRED TO ACHIEVE A SPECIFIED OPERATING CONDITION (AN INVERSE PROBLEM)

TRIM TASK OBTAINS SOLUTION FOR PERIODIC MOTION AND FORCES, AND STEADY TRIM VARIABLES
then performance, structural loads, vibration, and other output quantities can be evaluated

FREE FLIGHT AND WIND TUNNEL OPERATING CONDITIONS CONSIDERED

## FREE FLIGHT

rotorcraft trimmed to force and moment equilibrium for specified flight condition
can also trim to specified rotor power
steady state implies unaccelerated flight path (or turn)
turns and climb or descent, as well as level flight

## IN WIND TUNNEL

rotor trimmed to specified values of designated parameters, such as rotor thrust and flapping

## CAN ALSO OMIT TRIM ITERATION

solve system equations for fixed controls for free flight case, rotorcraft will probably not be in equilibrium

## A REGULATOR CAN BE INTRODUCED, USING ANY CONTROLS NOT REQUIRED TO ACHIEVE EQUILIBRIUM SOLUTION

TRIM LOOPS
PARTITIONED, ITERATIVE PROCEDURE CONSTRUCTED
system is divided into parts, which solve subset of equations for periodic or constant response loops iterate between part solutions, until converged system solution is obtained
figure 39 describes trim loops and principal interface variables, and shows levels of wake loop


Figure 6-39a Basic loops of trim analysis (optional regulator loop not shown).


Figure 6-39b Levels of wake loop.

## POST-TRIM AERODYNAMIC CALCULATIONS

FOR HIGH AZIMUTHAL RESOLUTION, OR TO OBTAIN PARTIAL ANGLE-OF-ATTACK FOR EXTERNAL AEROACOUSTIC ANALYSIS

## POST-TRIM CALCULATIONS IMPLEMENTED IN SEPARATE LOOPS OF TRIM TASK

fixed wake geometry and structural motion
executed after converged solution for structural dynamic response obtained

DEFINE DUPLICATE WING, WAKE, AND WAKE GEOMETRY COMPONENTS
structural dynamic components not duplicated, but have additional aerodynamic interfaces

## CIRCULATION CAN BE CALCULATED, TO BE CONSISTENT WITH WAKE ANALYSIS

## OR CIRCULATION STRENGTH CAN BE FIXED DURING POSTTRIM SOLUTION

circulation obtained from coupled aerodynamic and structural solution
no iteration required in circulation loop
must be used when partial angle-of-attack evaluated, otherwise calculated circulation reflects the computational domain

## WAKE LOOP

ROTOR WAKE-INDUCED VELOCITY CAN BE OBTAINED FROM UNIFORM INFLOW OR NONUNIFORM INFLOW nonuniform inflow refers to calculation of wake-induced velocity using vortex model
or approximate wake model based on momentum theory can be used (called "uniform inflow", although induced velocity can have linear variation over rotor disk)

NONUNIFORM INFLOW REQUIRES WAKE GEOMETRY MODEL, WHICH CAN BE RIGID OR PRESCRIBED, OR FREE rigid model calculates wake distortion from mean induced and interference velocity at rotor
prescribed geometry is obtained from empirical model, based on measurements
free wake geometry obtained by calculation

## SUCCESSIVE SUBSTITUTION ITERATION, WITH UP TO

 THREE LEVELS:level 1 is uniform inflow
level 2 is nonuniform inflow with rigid or prescribed wake geometry
level 3 is nonuniform inflow with free wake geometry
for better convergence, uniform inflow used to initialize nonuniform inflow, and rigid wake geometry used to initialize free wake geometry
whether results of level 3 or level 2 are required depends on importance of free wake geometry and nonuniform inflow to the problem

## WAKE LOOP IS ITERATION ON WAKE GEOMETRY

wake geometry and influence coefficients calculated for given operating condition and loading
then trimmed rotorcraft solution obtained, with induced velocity evaluated from latest bound circulation and fixed influence coefficients
iteration is required if trimmed rotorcraft solution significantly changes any quantities that influence the wake geometry
with trim solution inside wake geometry loop, may not be necessary to iterate on wake geometry at each level
some problems need iteration in wake geometry loop (and relaxation factor on wake geometry)
if rotor is at fixed collective pitch (rather than trimmed to specified thrust), then overall geometry of wake is not known in advance, only after thrust calculated
or in hover and at low speeds, where rotor loading is very sensitive to wake geometry

## WAKE LOOP ITERATION EXECUTED SPECIFIED NUMBER OF TIMES, WITH NO TEST FOR CONVERGENCE

outer loop iteration considered too expensive to implement numerical test for convergence
experience shows that often only one or two iterations are required at last stage of loop

TRIM LOOP

## FREE FLIGHT

## STEADY STATE OPERATING CONDITION IMPLIES UNACCELERATED MOTION

so net force and moment on entire system must be zero
requirement that six components of mean force and moment be zero provides algebraic equations that must be satisfied in trim state specified rotor power can also be included

## UNACCELERATED MOTION ALSO IMPLIES

that system linear velocity is constant and angular velocity is zero (except for yaw rate in steady turn)

## SO OPERATING CONDITION DESCRIBED BY FOLLOWING TRIM VARIABLES:

magnitude and orientation of system velocity (flight path) and wind speed
orientation of system relative inertial frame
control settings
turn rate
rotor speed
some of these trim variables define operating condition, some can have arbitrary values
solution procedure must determine values of remaining variables

## IN WIND TUNNEL

ROTOR TRIMMED TO SPECIFIED VALUES OF DESIGNATED PARAMETERS
requirement that these parameters equal target values provides algebraic equations that must be satisfied in trim state
these trim quantities can be rotor forces, flapping, or power

## AVAILABLE TRIM VARIABLES

magnitude and orientation of wind speed orientation of system relative inertial frame control settings
rotor speed

TRIM LOOP IS ITERATION THAT ADJUSTS TRIM VARIABLES UNTIL TRIM CRITERIA ARE SATISFIED
values for trim variables are calculated by loop
then differential and integral equations of motion are solved for periodic response of rotor and airframe, for fixed values of trim variables
finally quantities required by trim criteria are evaluated

TRIM LOOP IS NEWTON-RAPHSON ITERATION

## CIRCULATION LOOP

CIRCULATION LOOP IS ITERATION ON WAKE-INDUCED VELOCITY AND ROTOR BOUND CIRCULATION induced velocity calculated from rotor loading, using uniform or nonuniform inflow model
then differential equations of motion are solved for periodic response of rotor and airframe, for fixed induced velocity
iteration is required, until induced velocity and loading are consistent

CIRCULATION LOOP IS SUCCESSIVE SUBSTITUTION ITERATION
relaxation factor of $\lambda=0.10$ or 0.05 is often required with nonuniform inflow

## MOTION LOOP

MOTION LOOP IS ITERATION ON ROTOR AND AIRFRAME MOTION
differential equations of motion are solved for periodic response of rotor, including effects of hub motion generated by airframe
then differential equations of motion are solved for periodic response of airframe, including effects of rotor hub forces
iteration is required, until hub motion and hub forces are consistent
if airframe does not produce hub motion, no iteration is required

MOTION LOOP IS SUCCESSIVE SUBSTITUTION ITERATION

ROTORCRAFT HAS ROTATING AND NONROTATING SUBSYSTEMS
equations of motion for rotor are in rotating frame equations of motion for airframe and drive train in nonrotating frame
so must have separate part solutions for rotor and airframe

FOR MAIN ROTOR AND TAIL ROTOR CONFIGURATION
must analyze airframe as responding separately to different frequencies of two rotors

## REGULATOR LOOP

## A SELF-TUNING REGULATOR CAN BE USED TO AUTOMATICALLY ADJUST SELECTED CONTROLS TO MINIMIZE A COST FUNCTION <br> COST FUNCTION = WEIGHTED SUM OF SQUARES OF SELECTED MEASUREMENTS

## A SELF-TUNING REGULATOR COMBINES RECURSIVE PARAMETER ESTIMATION WITH LINEAR FEEDBACK

control system characterized by
linear, quasistatic, frequency-domain model of rotorcraft response to control
identification of rotorcraft model by least-squarederror or Kalman-filter method
minimum variance or quadratic performancefunction controller
in trim task, regulator functions as an optimizer does not necessarily correspond to an actual control system

## TYPICAL APPLICATIONS FOR ROTORCRAFT:

multicyclic control to minimize vibration or loads, or improve performance
cyclic pitch feedback to control rotor flapping (for cyclic controls not used by trim loop)

## ROTORCRAFT SHELL CAN CONSTRUCT A REGULATOR LOOP, BETWEEN TRIM AND CIRCULATION LOOPS

measurements can include
hub, root, blade, or control loads
airframe vibration and other sensors
rotor flapping, rotor power
sensors created specifically for regulator
any controls not used by trim loop can be used by regulator
rotor and airframe primary controls
higher harmonic rotor pitch in the rotating or nonrotating frame
mean or higher harmonic auxiliary forces
should print standard output for measurements, in order to get descriptions of quantities

CORE INPUT IS REQUIRED TO COMPLETE LOOP DEFINITION
identify controls and measurements, and set loop parameters
in order to use means, harmonics, or other derived quantities as measurements, appropriate filter components must be defined and solved

## AS CREATED BY SHELL, REGULATOR LOOP DOES NOTHING

number of controls and number of measurements set to zero

## SHELL CONSTRUCTS INPUT/OUTPUT INTERFACES FOR ANY TRIM OUTPUT QUANTITIES REQUESTED

solved in implicit part "ROTOR n REGULATOR SENSOR", at end of circulation loop
airframe sensors
blade loads, pitch link loads blade root force and moment hub force and moment without regulator loop, only solved as output with regulator loop present, solved as interfaces as well, hence available to regulator as measurements also then available to rest of system, including trim loop (perhaps with regulator loop doing nothing)

## ROTOR AND AIRFRAME PARTS

ROTORCRAFT EQUATIONS OF MOTION AND GENERALIZED FORCES USUALLY PERIODIC
so steady state response is periodic
response of rotor subsystem has fundamental frequency $\Omega$ in rotating frame ( $\Omega=$ rotational speed)

## ROTORCRAFT SHELL USUALLY CONSTRUCTS ROTORS WITH $N$ IDENTICAL, EQUALLY SPACED BLADES

assuming that these blades have identical trim motion, follows that response of airframe subsystem has fundamental frequency $N \Omega$
analysis can filter solution for airframe, suppressing all harmonics not multiple of fundamental frequency
for airframe and drive train response, pass only $p N$ harmonics ( $p=$ integer)
for gimbal/teeter response, pass only $p N \pm 1$ harmonics ( $p=$ integer)
if rotor subsystem is not axisymmetric, then fundamental frequency of airframe will also be $\Omega$
axisymmetry can be lost if hub has universal-joint type gimbal
or if some change is introduced to one blade

## ROTOR AND AIRFRAME PARTS SOLVED USING HARMONIC METHOD OR TIME FINITE ELEMENT METHOD

using Fourier series representation of response, so time step can usually be kept large
time step determined by frequency content of generalized forces, not frequency spectrum of equations of motion

## EQUATIONS ARE GENERALLY NONLINEAR, SO ITERATIVE PROCEDURE REQUIRED

one iteration produces solution over period (one revolution)
convergence can be controlled using relaxation of generalized forces, and an estimate of aerodynamic damping

AIRFRAME AND DRIVE TRAIN PARTS IDENTIFIED AS TIMEINVARIANT
if two rotors have different rotational speeds, then part is solved for two periods:
primary $=$ first rotor period
secondary $=$ second rotor period
similarly with more than two rotors, part solved for each unique period
if all rotors have same period, then single solution is obtained

## ROTOR PARTS

## ROTORCRAFT SHELL CONSTRUCTS ROTORS WITH IDENTICAL, EQUALLY SPACED BLADES <br> SO TRIM SOLUTION WILL BE IDENTICAL MOTION OF BLADES

$N$-th blade (at azimuth $\psi$ ) is reference blade
$N=$ number of blades
$\Delta \psi=2 \pi / N=$ interblade spacing
for rotor with unequal blade spacing, specified blade azimuth angle $\Delta \psi_{m}$ replaces $m \Delta \psi$

WHEN ROTOR HAS IDENTICAL BLADES UNDERGOING IDENTICAL MOTION, MOST EFFICIENT TO SOLVE FOR ONLY RESPONSE OF ONE BLADE
methods implemented:
solve for response of only reference blade solve for response of each blade separately solve for response of all blades together
must solve for response of complete rotor:
if system changed so motion of blades no longer identical
or if blades are structurally coupled, even if blades still have identical motion
if rotor has a gimbal or teeter joint; perhaps if rotor has flexible swashplate

## SOLVE FOR RESPONSE OF REFERENCE BLADE

differential equation parts defined for rotor hub and $N$-th blade
structural components and interfaces still created for all blades
trim response of reference blade is parent for trim response of other blades (with phase shift)
wing created only for reference blade (unless flutter or transient task initialized or executed)

## SOLVE FOR RESPONSE OF EACH BLADE SEPARATELY

differential equation parts defined for rotor hub, and all $N$ blades

SOLVE FOR RESPONSE OF ALL BLADES TOGETHER
differential equation part defined for entire rotor, including hub and all $N$ blades
in cases such as gimballed or teetering hub, identical blades can still have identical motion, even though necessary to solve rotor system as a whole
enforced by replacing harmonic solution for blade motion (with phase shift) by its parent blade motion
parent motion is average (with phase shifts) of solution for all blades
for gimbal and teeter degrees of freedom, also only $p N \pm 1$ harmonics are nonzero

## TRIM SOLUTION PROCEDURE

## FIGURE 40 DEFINES SOLUTION PROCEDURE CONSTRUCTED FOR TRIM TASK

analysis prints actual solution procedure used

## LOOPS AND THEIR SOLUTION METHODS

| loop name | solution method | levels | test convergence |
| :--- | :--- | :--- | :--- |
| WAKE | successive substitution | 3 | no |
| TRIM | Newton-Raphson |  | yes |
| REGULATOR | regulator |  | yes |
| CIRCULATION | successive substitution | 1 | yes |
| MOTION | successive substitution | 1 | yes |
| AIRFRAME | no solution |  |  |
| POST TRIM WAKE | successive substitution | 1 | no |
| POST TRIM CIRCULATION | successive substitution | 1 | yes |
| POST TRIM MOTION | no solution |  |  |

wake loop has up to three levels (four with CFD component implemented), and no test for convergence; a specified number of iterations are executed at each level optionally, output can be obtained for each iteration of wake loop
post-trim circulation loop uses no-solution method for fixed circulation option

```
First Loop = WAKE
    Parts Solved in Loop
    Part = AIRFRAME VELOCITY
    Part = AIRFRAME MEAN VELOCITY
    Part = AIRFRAME LOCATION ROTOR n
    Part = ROTOR n LIFTING LINE POSITION
    Part = ROTOR n COLLOCATION POINTS
    Part = ROTOR n DISK POSITION
    Part = ROTOR n AIRFRAME POSITION
    Part = ROTOR n WING MEAN FRAME
    Part = ROTOR n DUCT FORCE
    Part = ROTOR n MEAN DUCT FORCE
    Part = ROTOR n FRAME
    Part = ROTOR n MEAN FRAME
    Part = ROTOR n WING LOAD
    Part = ROTOR n WING MEAN LOAD
    Part = ROTOR n WAKE GEOMETRY
    Part = ROTOR n WAKE GEOMETRY SENSOR
    Part = ROTOR n WAKE GEOMETRY DISPLAY
    Part = ROTOR n WAKE INFLUENCE COEFF
    Part = ROTOR n WAKE INFL COEFF SENSOR
        Child Loop = TRIM
            Parts Solved at End of Loop
            Part = AIRFRAME SENSOR ROTOR n
            Part = ROTOR n HUB LOAD SENSOR
            Part = ROTOR n CONTROL LOAD SENSOR
            Part = ROTOR n BLADE LOAD SENSOR
            Part = ROTOR n BLADE POSITION SENSOR
            Part = ROTOR n WING SENSOR
                Modules Written at End of Loop
                Module Convergence = WAKE
                    Module Shell = ROTORCRAFT 11
                    Module Shell = ROTOR n
                Module Shell = FRAMES 10
                Module Part = ROTOR n HUB
                Module Part = ROTOR n BLADE N
                Module Part = AIRFRAME
                Module Part = DRIVE TRAIN
                Module Output of Part = ROTOR n INTVEL trANSFORM
                    Module Output of Part = AIRFRAME SENSOR ROTOR n
                    Module Output of Part = ROTOR n HUB LOAD SENSOR
                    Module Output of Part = ROTOR n CONTROL LOAD SENSOR
                    Module Output of Part = ROTOR n BLADE LOAD SENSOR
                    Module Output of Part = ROTOR n BLADE POSITION SENSOR
                    Module Output of Part = ROTOR n WING SENSOR
                    Module Output of Part = ROTOR n WAKE GEOMETRY DISPLAY
                    Module Graphics, Part = ROTOR n GRAPHICS
```

Figure 6-40a Trim solution procedure.

```
Loop = TRIM
    Parts Solved in Loop
    Part = CONTROL
    Part = ROTOR n CONTROL
        Pass 1 = ROTOR n HARMONICS
        Pass 2 = ROTOR n CONTROL
            Child Loop = CIRCULATION
LOOp = CIRCULATION
    Parts Solved in Loop
    Part = ROTOR n UNIFORM INFLOW
    Part = ROTOR n UNIFORM INFLOW SENSOR
    Part = ROTOR n WAKE VORTICITY
    Part = ROTOR n NONUNIFORM INFLOW
    Part = AIRFRAME FLOW FIELD ROTOR n
    Part = ROTOR n INTVEL TRANSFORM
    Part = ROTOR n INTVEL ADDITION
    Part = ROTOR n INTVEL SPAN AVERAGE
    Part = ROTOR n INTVEL BLADE AVERAGE
    Part = ROTOR n INTVEL MEAN
    Part = ROTOR n INTVEL TOTAL
        Child Loop = MOTION
            Parts Solved at End of Loop
            Part = ROTOR n TORQUE
            Part = ROTOR n MEAN TORQUE
            Part = ROTOR n PERFORMANCE
            Pass 1 = ROTOR n POWER
                    Pass 2 = ROTOR n PERFORMANCE
                    Part = ROTORCRAFT PERFORMANCE
```

Figure 6-40b Trim solution procedure.

```
Loop = MOTION
    Parts Solved in Loop
    Part = ROTOR n HUB
    Part = ROTOR n BLADE N
        Pass 1 = ROTOR n AERODYNAMIC INTERFACE (V)
        Pass 2 = ROTOR n AERODYNAMIC INTERFACE (R)
        Pass 3 = ROTOR n WING AERODYNAMICS
        Pass 4 = ROTOR n AERODYNAMIC INTERFACE (F)
    Part = ROTOR n HUB
                Child Loop = AIRFRAME
                    Parts Solved at End of Loop
                    Part = ROTOR n DISK POSITION
                    Part = ROTOR n AIRFRAME POSITION
                    Part = ROTOR n TIP PATH PLANE
                    Part = ROTOR n MEAN TIP PATH PLANE
                    Part = ROTOR n WING LOAD
                    Part = ROTOR n WING MEAN LOAD
                    Part = ROTOR n WING FRAME
                    Part = ROTOR n WING MEAN FRAME
                    Part = ROTOR n DUCT FORCE
                    Part = ROTOR n MEAN DUCT FORCE
                    Part = ROTOR n FRAME
                    Part = ROTOR n MEAN FRAME
```

Last Loop = AIRFRAME
Parts Solved in Loop
Part = AIRFRAME AERODYNAMICS
Part = AIRFRAME
Part = DRIVE TRAIN
Parts Solved at End of Loop
Part = AIRFRAME VELOCITY
Part = AIRFRAME MEAN VELOCITY
Part $=$ AIRFRAME LOCATION ROTOR $n$
Part = DRIVE TRAIN MEAN GOVERNOR
Part = AIRFRAME LOAD
Part = AIRFRAME MEAN LOAD

Figure 6-40c Trim solution procedure.

```
First Loop = POST TRIM WAKE
    Parts Solved in Loop
    Part = RTREX n LIFTING LINE POSITION
    Part = RTREX n COLLOCATION POINTS
    Part = RTREX n MOTION
    Part = RTREX n WING LOAD
    Part = RTREX n WAKE GEOMETRY
    Part = RTREX n WAKE GEOMETRY SENSOR
    Part = RTREX n WAKE GEOMETRY DISPLAY
    Part = RTREX n WAKE INFLUENCE COEFF
    Part = RTREX n WAKE INFL COEFF SENSOR
            Child Loop = POST TRIM CIRCULATION
                Parts Solved at End of Loop
                        Part = RTREX n WING SENSOR
                    Modules Written at End of Loop
                    Module Convergence = POST TRIM WAKE
                    Module Output of Part = RTREX n INTVEL TRANSFORM
                    Module Output of Part = RTREX n WING SENSOR
                    Module Output of Part = RTReX n wake geometry display
                    Module Graphics, Part = RTREX n GRAPHICS
```

Loop = POST TRIM CIRCULATION
Parts Solved in Loop
Part $=$ RTREX $n$ WAKE VORTICITY
Part = RTREX n NONUNIFORM INFLOW
Part = RTREX n INTVEL TRANSFORM
Part = RTREX $n$ INTVEL ADDITION
Child Loop $=$ POST TRIM MOTION
Loop $=$ POST TRIM MOTION
Parts Solved in Loop
Part = RTREX n MOTION
Part $=$ RTREX n WING LOAD

Figure 6-40d Trim solution procedure.

## 6-10 Transient Task

## TRANSIENT TASK NUMERICALLY INTEGRATES SYSTEM EQUATIONS, FROM TRIM SOLUTION, FOR PRESCRIBED EXCITATION

ROTORCRAFT SHELL CONSTRUCTS TRANSIENT SOLUTION PROCEDURE WITH NO LOOPS (NO ITERATION):
one loop (no solution) to organize process, entire system solved in one part (integration method) relaxation applied to bound circulation, for convergence when nonuniform inflow model used

TRANSIENT TASK WITH VORTEX WAKE MODEL
NUMERICAL INTEGRATION SHOULD BE ITERATED AT EACH TIME STEP TO ACHIEVE CONSISTENT, CONVERGED SOLUTION
with nonuniform inflow wake model, relaxation of circulation required for this iteration to converge MORE CHAOTIC BEHAVIOR OF FREE WAKE GEOMETRY CAN BE EXPECTED IN TRANSIENT TASK
compared to trim task, where periodicity enforced in wake distortion solution
might use growth of core radius to control this chaotic behavior

## 6-11 Flutter Task

## FLUTTER TASK LINEARIZES SYSTEM EQUATIONS, ABOUT TRIM SOLUTION; THEN ANALYZES RESULTING DIFFERENTIAL EQUATIONS

## SHELL CONSTRUCT FLUTTER EQUATIONS

for entire system,
or for independent blade (of N -bladed rotor)

## FLUTTER EQUATIONS CAN BE TIME INVARIANT OR PERIODIC

time invariant equations obtained for truly time invariant system, or by averaging equations of periodic system (in first loop)

## FLUTTER ANALYSIS PERIOD $T$

for analysis of entire system: $2 \pi / T=N \Omega$ ( $N$ odd) or $N \Omega / 2$ ( $N$ even)
$N=$ number of blades
$\Omega=$ rotational speed of first rotor
for independent blade analysis, or without multiblade coordinates: $2 \pi / T=\Omega$
can specify $T=$ period of first rotor, for problems such as nonidentical blades
in more general cases, core input can be used to redefine flutter period
if second (or other) rotor has period not equal to system period, then its equations must be averaged by child loop

## FLUTTER LOOPS

## PARTITIONED PROCEDURE CONSTRUCTED TO GENERATE EQUATIONS

system is divided into parts, which obtain subset of differential equations
loops combine and operate on part solutions, to obtain system equations

## PRINCIPAL LOOPS ARE FOR ROTOR, AIRFRAME, AND DRIVE TRAIN SUBSYSTEMS

rotor loop has child loops for blade, hub, and inflow
each loop has corresponding differential equations part

## SEPARATE LOOP REQUIRED FOR EACH ROTOR IN ORDER TO IMPLEMENT DIFFERENT OPERATIONS

if rotors turn at different speeds, then always average equations for second (and other) rotor, airframe, and drive train
partitioning system also makes it possible to examine equations of each subsystem

## FLUTTER EQUATIONS DO NOT LINEARIZE FOLLOWING:

prescribed or higher-harmonic control collocation point positions rotor tip-path plane tilt or coning nonuniform inflow or wake geometry bound circulation or circulation peaks performance variables

Chapter 7

## USING CAMRAD II ANALYSIS

## 7-1 Output Format

## LISTING OF OUTPUT FOR TYPICAL CAMRAD II ANALYSIS

header and job parameters
case input, rotor elements constructed
shell header
shell input data
core and table input data
trim solution procedure, flutter solution procedure trim loop iteration and part solution trim convergence
trim solution: rotorcraft performance trim solution: rotor performance trim solution: analysis frames trim solution: output of part flutter solution procedure
flutter loop iteration and part solution
flutter solution: mode set
flutter solution: description of variables
flutter solution: analysis
computation times, data vector statistics
end of case


PLUGINS INSTALLED
NONE

READING NAMELIST NLJOB

JOB DESCRIPTION

NUMBER OF CASES, NCASES = 1
PLOT FILE WRITE ( 0 FOR NEVER, 1 FOR ONE FILE PER JOB), PLFILE = 1
INPUT CATEGORY ( 0 FOR CORE, 1 FOR SHELL), $\quad$ INFILE $=1$
SHELL INPUT FILE READ ( 0 FOR NEVER, 1 FOR FIRST CASE ONLY, 2 FOR EACH CASE),
EMPTY INPUT DATA VECTORS FOR NEXT CASE ( 0 NEVER, 1 SHELL/CORE, 2 TABLES, 3 ALL), OPSHLL $=1$
0
EMPTY INPUT DATA VECTORS FOR NEXT CASE ( 0 NEVER, 1 SHELL/CORE, 2 TABLES, 3 ALL), OPREIN $=$
DELETE TABLES NOT USED BY THIS CASE ( 0 FOR NEVER), 10 LOP 1 LOOP VARIABLES, 6 PART SOLUTION, 7 BOTH)
INITIALIZE TRIM SOLUTION FROM PREVIOUS JOB, READ FILE ( 0 NEVER, 1 ONLY FIRST CASE, 2 SAME, 3 SEPARATE),
INITIALIZE TRIM SOLUTION FROM PREVIOUS JOB, WRITE FILE ( 0 NEVER, 1 EACH CASE)
OPINIT =
OPITJR =
RESTART TRANSIENT SOLUTION FROM PREVIOUS JOB, READ FILE ( 0 NEVER)
ESTART TRANSIENT SOLUTION FROM PREVIOUS JOB, READ FILE ( 0 NEVER),
TRACE OF OPERATION ( 0 TO 4 ; 0 NONE, 4 HIGH LEVEL PRINT); FOR JOB START (DATA VECTOR UTILITIES), JOBTRC $=\quad 0$ RRACE OF OPERATION ( 0 TO 4; 0 NONE, 4 HIGH LEVEL PRINT); FOR CASE INPUT

CSITRC $=0$
TRACE OF OPERATION ( 0 TO 4; 0 NONE, 4 HIGH LEVEL PRINT); FOR TIMER AND DEBUG COUNTER,
TIMTRC $=0$
INPUT/OUTPUT UNIT NUMBERS

| NAMELIST INPUT, | NUIN $=$ | 5 |
| :--- | :--- | ---: |
| PRINTER OUTPUT, | NUOUT $=$ | 6 |
| SHELL INPUT DATA FILE, | NFSHLL $=$ | 41 |
| CORE INPUT DATA FILE, | NFCORE $=$ | 42 |
| PLOT DATA FILE, | NFPLOT $=$ | 43 |
| TRIM INITIALIZATION INPUT FILE, | NFITJR $=$ | 44 |
| TRIM INITIALIZATION OUTPUT FILE, | NFITJW $=$ | 45 |
| TRANSIENT RESTART INPUT FILE, | NFRNJR $=46$ |  |
| TRANSIENT RESTART OUTPUT FILE, | NFRNJW $=47$ |  |
| DATA VECTOR STATISTICS FILE, | NFSTAT $=48$ |  |
| TABLE FILES DEFAULT (NFTABL FOR ALL), | NFTABL $=49$ |  |

## SYSTEM PARAMETERS

| FILE OPEN UTILITY ( 0 FOR NO ACTION, 1 TO OPEN FILE), | OPNFIL $=$ |
| :--- | :--- | :--- |
| FILE CLOSE UTILITY ( 0 FOR NO ACTION, 1 TO CLOSE FILE, 2 TO REWIND FILE), | CLSFIL $=$ |
| MAXIMUM FILE UNIT NUMBER ( 0 FOR NO MAXIMUM, -1 FOR CONSTANT UNIT NUMBER), |  |

FILE DEFINITION FOR INTERACTIVE MODE ( 1 NAME, 2 UNIT NUMBER, 3 BOTH),
BLOCK COMPRESSION IN DATA VECTOR ( 0 FOR NONE, 1 TO COMPRESS),
DATA VECTOR STATISTICS FILE ( 0 FOR NONE, 1 TO APPEND TO OLD FILE, 2 TO CREATE NEW FILE) ACTION ON ERROR ( 0 FOR NONE, 1 TO EXIT, 2-3 FOR TRACEBACK),
ANALYSIS EXIT ( 0 FOR NO ACTION, 1 TO STOP ON ABNORMAL EXIT, 2 TO STOP), NUMERICAL PRECISION IN MATHEMATICS UTILITIES (1 SINGLE, 2 DOUBLE),
EIGENANALYSIS METHOD (1 FOR FIRST, 2 FOR SECOND),

DEFFIL = BLKCMP = DVSFIL = ERRACT = OPEXIT = OPMATH OPEIGN

EATING PLOT FILE FOR JOB
OPEN FILE 43, NAME: PLOTFILE = [CAMRADII]ROTOR.PLOT

SHELL INPUT, CASE NUMBER 1
OPEN FILE 41, NAME: SHELLINPUT = [CAMRADII.INPUT]ROTORINPUT.DAT
READING DATA VECTOR "SHLLCM" (TYPE = SHELL ) FROM FILE 41, FILE IDENTIFICATION = HH:MM:SS DD-MMM-YY CLOSE FILE 41
SHELL NAMELIST INPUT
READING NAMELIST NLDEF, SHELL INPUT
ACTION = CHANGE OLD SECTION (INITIALIZE IF NEW); CLASS = CASE
READING NAMELIST NLVAL
READING NAMELIST NLDEF, SHELL INPUT
ACTION = CHANGE OLD SECTION (INITIALIZE IF NEW); CLASS = TRIM
READING NAMELIST NLVAL
READING NAMELIST NLDEF, SHELL INPUT
ACTION = CHANGE OLD SECTION (INITIALIZE IF NEW); CLASS = TRIM ROTOR
NAME = ROTOR 1
READING NAMELIST NLVAL
READING NAMELIST NLDEF, SHELL INPUT
ACTION = CHANGE OLD SECTION (INITIALIZE IF NEW); CLASS = FLUTTER
READING NAMELIST NLVAL
READING NAMELIST NLDEF, SHELL INPUT
ACTION $=$ CHANGE OLD SECTION (INITIALIZE IF NEW); CLASS = FLUTTER ROTOR
NAME = ROTOR 1
READING NAMELIST NLVAL
READING NAMELIST NLDEF, SHELL INPUT
ACTION = END OF INPUT
GENERATING CORE INPUT FROM SHELL INPUT
GENERATING CORE INPUT FOR SHELL CASE DATA
GENERATING CORE INPUT FOR SHELL ROTORCRAFT DATA
GENERATING CORE INPUT FOR SHELL AIRFRAME DATA


GENERATING CORE INPUT FOR SHELL ROTOR 1 DATA


GENERATING CORE INPUT FOR SHELL SOLUTION DATA
CORE NAMELIST INPUT
READING NAMELIST NLDEF, CORE INPUT
ACTION = END OF INPUT
TABLE FILE INPUT, CASE NUMBER 1
REQUIRED TABLE NUMBER 1; CLASS = AIRFOIL, TYPE = CAMRAD, NAME = BLADEAIRFOIL1 READING TABLE FILE; CLASS = AIRFOIL, TYPE = CAMRAD
OPEN FILE 61, NAME: BLADEAIRFOIL1 = [CAMRADII.AIRFOIL]ROTORAIRFOIL.TAB
TITLE = ROTOR AIRFOIL (C81 TABLE)
FILE IDENTIFICATION $=\mathrm{HH}: \mathrm{MM}: S \mathrm{SS}$ DD-MMM-YY
CLOSE FILE 6
2; CLASS = AIRFOIL, TYPE = CAMRAD, NAME = BLADEAIRFOIL1 TABLE ALREADY PRESENT REQUIRED TABLE NUMBER TABLE AIPEADY PRESENT REQUIRED TABLE NUMBER TABLE ALREADY PRESENT

ROTOR 1 BLADE 1 ELEMENT 1
ROTOR 1 BLADE 1 ELEMENT 2
ROTOR 1 BLADE 1 ELEMENT 3
ROTOR 1 BLADE 1 ELEMENT 4 ROTOR 1 BLADE 1 ELEMENT 6

3; CLASS $=$ AIRFOIL, TYPE $=$ CAMRAD, NAME $=$ BLADEAIRFOIL1
4; CLASS = AIRFOIL, TYPE = CAMRAD, NAME = BLADEAIRFOIL1


AERODYNAMIC CONTROLS (DEG)

| FLAPERON | $=$ | 0.0 |
| :--- | :--- | :--- |
| ELEVATOR | $=$ | 0.0 |


$=0.00$

ENGINE CONTROL
THROTTLE $=0.0$

IGHER HARMONIC CONTROL (NUMBER OF HARMONICS) ROTATING FRAME $=0$ AUXILIARY FORCES = 0

TE FLAP $=0$

RIMARY CONTROLS (DEG)
COLLECTIVE
$\begin{array}{lll}\text { ATERAL CYCLIC } & = & 0.00\end{array}$
BLADE CONTROL (DEG)
PITCH = 0.00
0.00
$0.00000 \mathrm{E}+00$
JETS

IRFRAME AND ROTOR CONTROLS ARE VALUES WITH ZERO PILOT AND GOVERNOR CONTROLS

TRIM SOLUTION PROCEDURE
WAKE LOOP: NONUNIFORM INFLOW WITH PRESCRIBED WAKE GEOMETRY
NUMBER OF ITERATIONS $=1$ (UNIFORM), 1 (PRESCRIBED), 1 (FREE)
RELAXATION FACTOR $=1.00000$ (UNIFORM), 1.00000 (PRESCRIBED), 1.00000 (FREE)
TRIM LOOP: NUMBER OF VARIABLES $=3$, NUMBER OF ITERATIONS $=40$, RELAXATION FACTOR $=0.20000$, TOLERANCE $=5.00000$ TRIM VARIABLE

TRIM QUANTITY
ARGET
COLL CT/S
0.080000

LATGCYC
BETAS
BCTRIM $=0.000000$
CIRCULATION LOOP: NUMBER OF ITERATIONS $=200$, RELAXATION FACTOR $=0.05000$, TOLERANCE $=1.00000$
MOTION LOOP: NUMBER OF ITERATIONS $=40$, RELAXATION FACTOR $=1.00000$, TOLERANCE $=2.00000$
PART SOLUTION: NUMBER OF AZIMUTH STEPS PER REVOLUTION = 24 , AZIMUTH INCREMENT (DEG) = 15.00
ROTOR 1 PART SOLUTION
NONUNIFORM INFLOW WITH PRESCRIBED WAKE GEOMETRY
INGLE ROTOR PART FOR REFERENCE BLADE; HARMONIC SOLUTION METHOD
ROTOR: 10 HARMONICS, 40 ITERATIONS, RELAXATION FACTOR $=0.50000$, TOLERANCE $=2.00000$, DAMPING $=1.00000$
AIRFRAME: 10 HARMONICS, 40 ITERATIONS, RELAXATION FACTOR $=1.00000$, TOLERANCE $=2.00000$, DAMPING $=0.50000$

DEGREES OF FREEDOM IN TRIM SOLUTION (0 ZERO, 1 DYNAMIC, 2 QUASISTATIC)
AIRFRAME
ROTOR 1
BLADE MODES, $\quad$ DOFM $=1111111111 \quad 1122222222 \quad 222222222222222222$

FLUTTER EQUATIONS
TIME INVARIANT SYSTEM
CONSTANT COEFFICIENT APPROXIMATION; INDEPENDENT BLADE ANALYSIS
ROTOR 1: NO ROTOR AERODYNAMICS
ANALYSIS TASKS: EIGENANALYSIS; NO TIME HISTORY RESPONSE; NO FREQUENCY RESPONSE; NO RMS GUST RESPONSE

FLUTTER SOLUTION PROCEDURE
NUMBER OF STEPS PER REVOLUTION IN AVERAGE OF MATRICES = 12
REDUCTION OF EQUATIONS: ZERO IN LOOPS; QUASISTATIC IN LOOPS
TIME HISTORY RESPONSE: FROM MODES
FREQUENCY RESPONSE: CALCULATED FROM MODES
RMS GUST RESPONSE: STOCHASTIC (FROM MODES

VARIABLES IN FLUTTER SOLUTION
DEGREES OF FREEDOM ( 0 ZERO, 1 DYNAMIC, 2 QUASISTATIC)

```
    AIRFRAME
    AOTOR 1
        BLADE MODES, DOFM = 11111111111 2222222222 2222222222 2222222222
        DYNAMIC INFLOW,
        DOFL = 111
    DOFMBC = 1 111 11 1
SPECIFICATION OF ORDER (O DETERMINE FROM EQUATIONS, 1 FIRST ORDER, 2 SECOND ORDER
    AIRFRAME DEGREES OF FREEDOM,
        DOFORD = 0000 0
            RDR DOFA = UNSTEADY LOADS, DYNAMIC STALL
            ORDER DOFL = UNIFORM, LONGITUDINAL, LATERAI
            ORDER DOFMBC = (0, 1C,1S, NC,NS, N/2)
            ORDER DOFORD = AIRFRAME RIGID BODY (X,Y,Z,PSI) + DRIVE TRAIN RIGID (PSI)
CONTROLS (0 ZERO, 1 USED)
    ROTORCRAFT
        PILOT CONTROL, CONP = 00000
        GUST,
    AIRFRAME
        AERODYNAMIC CONTROLS, CONA = 0000
        ENGINE CONTROL,
    ROTOR 1
        PRIMARY CONTROLS,
        BLADE PITCH,
        BLADE TE FLAP
        BLADE REACTION JET,
        BLADE REACTION JET,
            ORDER CONP = COLLECTIVE, LATERAL CYCLIC, LONGITUDINAL CYCLIC, PEDAL, THROTTLE
            ORDER GUST = LONGITUDINAL, LATERAL, VERTICAL
            ORDER CONA = FLAPERON, ELEVATOR, AILERON, RUDDER
            ORDER CONR = COLLECTIVE, LATERAL CYCLIC, LONGITUDINAL CYCLIC
            ORDER CONMBC = (0, 1C,1S, NC,NS, N/2)
OUTPUT (0 ZERO, 1 USED)
    AIRFRAME
        AIRFRAME SENSORS, MSSEN = 0
    ROTOR 1
        HUB LOAD SENSORS, MHSEN = 0
        CONTROL LOAD SENSORS, MCSEN = 0
        BLADE LOAD SENSORS,
        BLADE POSITION SENSORS
        BLADE AERODYNAMICS
        BLADE AERODYNAMICS,
        MULTIBLADE COORDINATES, SENMBC = 1 11 11 1
            ORDER SENMBC = (0, 1C,1S, NC,NS, N/2)
```

***************
SHELL INPUT DATA
****************

| CLASS = CASE |  | NUMBER OF BLOCKS = | 1 |
| :---: | :---: | :---: | :---: |
| CLASS $=$ TRIM |  | NUMBER OF BLOCKS = | 1 |
| CLASS = TRIM ROTOR |  | NUMBER OF BLOCKS = | 1 |
| CLASS = FLUTTER |  | NUMBER OF BLOCKS = | 1 |
| CLASS = FLUTTER ROTOR |  | NUMBER OF BLOCKS = | 1 |
| CLASS = AIRFRAME | TYPE = STRUCTURE | NUMBER OF BLOCKS = | 1 |
| CLASS = AIRFRAME | TYPE = AERODYNAMICS | NUMBER OF BLOCKS | 1 |
| CLASS = AIRFRAME | TYPE = CONTROL | NUMBER OF BLOCKS | 1 |
| CLASS = AIRFRAME | TYPE = DRIVE TRAIN | NUMBER OF BLOCKS | 1 |
| CLASS $=$ ROTOR | TYPE = STRUCTURE | NUMBER OF BLOCKS = | 1 |
| CLASS $=$ ROTOR | TYPE = AERODYNAMICS | NUMBER OF BLOCKS | 1 |
| CLASS = ROTOR | TYPE = INFLOW | NUMBER OF BLOCKS | 1 |
| CLASS $=$ ROTOR | TYPE = WAKE | NUMBER OF BLOCKS | 1 |
| CLASS $=$ TABLES |  | NUMBER OF BLOCKS = | 1 |

UNITS FOR INPUT PARAMETERS
DIMENSIONAL INPUT PARAMETERS CAN BE IN ENGLISH OR METRIC (SI) UNITS, USING A CONSISTENT LENGTH-MASS-TIME SYSTEM

ENGLISH (OPUNIT = 1) FOOT-SLUG-SECOND
METRIC (OPUNIT $=2$ ) $\quad$ METER-KILOGRAM-SECOND
WITH THE FOLLOWING EXCEPTION
ROTORCRAFT GROSS WEIGHT (LB OR KG); AIRCRAFT SPEED AND WIND SPEED (KNOTS) ANGLES ARE INPUT IN DEGREES


[^1]B7 (Section 7-1
$* * * * * * * * * * * * * * *$
CORE INPUT DATA
***************

| CLASS $=$ CASE |  | NUMBER OF BLOCKS = | 1 |
| :---: | :---: | :---: | :---: |
| CLASS = COMPONENT | TYPE = RIGID BODY | NUMBER OF BLOCKS | 7 |
| CLASS = COMPONENT | TYPE = LINEAR NORMAL MODES | NUMBER OF BLOCKS | 1 |
| CLASS = COMPONENT | TYPE = FINITE ELEMENT BEAM | NUMBER OF BLOCKS | 24 |
| CLASS = COMPONENT | TYPE = REFERENCE FRAME | NUMBER OF BLOCKS | 245 |
| CLASS = COMPONENT | TYPE = FILTER | NUMBER OF BLOCKS | 29 |
| CLASS = COMPONENT | TYPE = REFERENCE PLANE | NUMBER OF BLOCKS | 1 |
| CLASS = COMPONENT | TYPE = DIFFERENTIAL EQUATION | NUMBER OF BLOCKS | 9 |
| CLASS = COMPONENT | TYPE = FOURIER SERIES | NUMBER OF BLOCKS | 4 |
| CLASS = COMPONENT | TYPE $=$ GUST | NUMBER OF BLOCKS | 5 |
| CLASS = COMPONENT | TYPE = LIFTING LINE WING | NUMBER OF BLOCKS | 4 |
| CLASS = COMPONENT | TYPE = ROTOR INFLOW | NUMBER OF BLOCKS = | 1 |
| CLASS = COMPONENT | TYPE = WING WAKE | NUMBER OF BLOCKS | 1 |
| CLASS = COMPONENT | TYPE = ROTOR WAKE GEOMETRY | NUMBER OF BLOCKS | 1 |
| CLASS = COMPONENT | TYPE = ROTOR PERFORMANCE | NUMBER OF BLOCKS | 1 |
| CLASS = COMPONENT | TYPE = ROTORCRAFT PERFORMANCE | NUMBER OF BLOCKS | 1 |
| CLASS = FRAME |  | NUMBER OF BLOCKS = | 7 |
| CLASS = INTERFACE | TYPE = STRUCTURAL DYNAMIC | NUMBER OF BLOCKS $=$ | 31 |
| CLASS = INTERFACE | TYPE = INPUT/OUTPUT | NUMBER OF BLOCKS | 1563 |
| CLASS $=$ OUTPUT |  | NUMBER OF BLOCKS = | 120 |
| CLASS $=$ INPUT |  | NUMBER OF BLOCKS = | 6 |
| CLASS $=$ WIND |  | NUMBER OF BLOCKS = | 1 |
| CLASS $=$ OPERATING CONDITION |  | NUMBER OF BLOCKS = | 1 |
| CLASS = PERIOD |  | NUMBER OF BLOCKS = | 3 |
| CLASS $=$ TRIM LOOP | TYPE $=$ NO SOLUTION | NUMBER OF BLOCKS | 1 |
| CLASS $=$ TRIM LOOP | TYPE = SUCCESSIVE SUBSTITUTION | NUMBER OF BLOCKS | 3 |
| CLASS $=$ TRIM LOOP | TYPE = NEWTON RAPHSON | NUMBER OF BLOCKS | 1 |
| CLASS $=$ TRIM PART | TYPE $=$ NO SOLUTION | NUMBER OF BLOCKS | 5 |
| CLASS $=$ TRIM PART | TYPE = IMPLICIT | NUMBER OF BLOCKS | 41 |
| CLASS $=$ TRIM PART | TYPE = HARMONIC | NUMBER OF BLOCKS | 3 |
| CLASS = FLUTTER |  | NUMBER OF BLOCKS = | 1 |
| CLASS = FLUTTER LOOP |  | NUMBER OF BLOCKS = | 3 |
| CLASS $=$ FLUTTER PART | TYPE $=$ NO SOLUTION | NUMBER OF BLOCKS | 861 |
| CLASS = FLUTTER PART | TYPE = INTERFACE | NUMBER OF BLOCKS | 2 |
| CLASS = FLUTTER PART | TYPE = DIFFERENTIAL EQUATIONS | NUMBER OF BLOCKS | 1 |
| CLASS = MODES |  | NUMBER OF BLOCKS = | 5 |


| CLASS = RESPONSE | TYPE = RIGID | NUMBER OF BLOCKS = | 32 |
| :---: | :---: | :---: | :---: |
| CLASS = RESPONSE | TYPE = VARIABLE | NUMBER OF BLOCKS | 1734 |
| CLASS = WEIGHTS |  | NUMBER OF BLOCKS = | 1 |

****************
TABLE INPUT DATA
****************



| INDEX | USAGE | CLASS | TYPE | NAME |
| :---: | :---: | :---: | :---: | :---: |
| 1 | YES | AIRFOIL | CAMRAD | BLADEAIRFOIL1 |
| INDEX |  | FILE NAME | FILE IDENTIFICATION | FILE UNIT NUMBER |
| 1 |  | BLADEAIRFOIL1 | HH:MM:SS DD-MMM-YY | 61 |
| INDEX |  | TITLE |  |  |
| 1 |  | ROTOR AIRFOIL |  |  |

## INITIALIZATION

GENERATE RECORDS FROM CORE BLOCKS, FIRST PASS
GENERATE RECORDS FROM CORE BLOCKS, SECOND PASS
GENERATE RECORDS FROM CORE BLOCKS, THIRD PASS
INITIALIZE RECORDS, FIRST PASS
INITIALIZE RECORDS, SECOND PASS
INITIALIZE RECORDS, THIRD PASS
INITIALIZE RECORDS, FOURTH PASS
INITIALIZE RESPONSE, FIRST PASS
INITIALIZE RESPONSE, SECOND PASS
INITIALIZE RESPONSE, THIRD PASS
GENERATE MATRICES AND VECTORS
INITIALIZE RESPONSE SUBVECTORS

SYSTEM AND FUNCTIONALITY ************************

TRIM SOLUTION PROCEDURE
---------------------------

```
FIRST LOOP = WAKE
            PARTS SOLVED IN LOOP
            PART = ROTOR 1 LIFTING LINE POSITION
            PART = ROTOR 1 COLLOCATION POINTS
            PART = ROTOR 1 DISK POSITION
            PART = ROTOR 1 DISK POSITION
            PART = ROTOR 1 WING
            PART = ROTOR 1 MEAN FRAME
            PART = ROTOR 1 WING LOAD
            PART = ROTOR 1 WING MEAN LOAD
            PART = ROTOR 1 WAKE GEOMETRY
            PART = ROTOR 1 WAKE GEOMETRY SENSOR
            PART = ROTOR 1 WAKE GEOMETRY SENSOR
            PART = ROTOR 1 WAKE INFLUENCE COEFF
                CHILD LOOP = TRIM
                    PARTS SOLVED AT END OF LOOP
                    PART = ROTOR 1 BLADE POSITION SENSOR
                    PART = ROTOR 1 WING SENSOR
                                    MODULES WRITTEN AT END OF LOOP
                                    MODULE = CONVERGENCE, LOOP = WAKE
                                    MODULE = SHELL 1000
                                    MODULE = SHELL 1
                                    MODULE = SHELL 2000
                                    MODULE = OUTPUT OF PART = ROTOR 1 BLADE POSITION SENSOR
                                    MODULE = OUTPUT OF PART = ROTOR 1 WING SENSOR
LOOP = TRIM 
                    PART = CONTROL
            PART = ROTOR 1 CONTROL
                CHILD LOOP = CIRCULATION
LOOP = CIRCULATION
            PARTS SOLVED IN LOOP
            PART = ROTOR 1 UNIFORM INFLOW
            PART = ROTOR 1 UNIFORM INFLOW SENSOR
            ART = ROTOR 1 UNIFORM INFLOW
            PART = ROTOR 1 WAKE VORTICITY 
            PART = ROTOR 1 INTVEL TRANSFORM
            PART = ROTOR 1 INTVEL ADDITION
            PART = ROTOR 1 INTVEL SPAN AVERAGE
            PART = ROTOR 1 INTVEL BLADE AVERAGE
            PART = ROTOR 1 INTVEL MEAN
            PART = ROTOR 1 INTVEL TOTAL
            CHILD LOOP = MOTION
                            PARTS SOLVED AT END OF LOOP
                    PART = ROTOR 1 TORQUE
```

PART = ROTOR 1 MEAN TORQUE
PART = ROTOR 1 PERFORMANCE
PART $=$ ROTORCRAFT PERFORMANCE

```
LOOP = MOTION
        AARTS SOLVED IN LOOP
        PART = ROTOR 1 HUB
        PART = ROTOR 1 BLADE 4
        PART = ROTOR 1 HUB
            CHILD LOOP = AIRFRAME
                    PARTS SOLVED AT END OF LOOP
                    PART = ROTOR 1 DISK POSITION
                    PART = ROTOR 1 TIP PATH PLANE
                    PART = ROTOR 1 MEAN TIP PATH PLANE
                    PART = ROTOR 1 WING LOAD
                    PART = ROTOR 1 WING MEAN LOAD
                    PART = ROTOR 1 WING FRAME
                    PART = ROTOR 1 WING MEAN FRAME
                    PART = ROTOR 1 FRAME
                    PART = ROTOR 1 MEAN FRAME
LAST LOOP = AIRFRAME
            PARTS SOLVED IN LOOP
            PARTS SOLVED IN
            PARTS SOLVED AT END OF LOOP
                    PART = AIRFRAME LOAD
                    PART = AIRFRAME MEAN LOAD
```


TRIM SYSTEM EQUATION

TOTAL NUMBER OF VECTORS
MOTION EQUATIONS AND DEGREES OF FREEDOM = ..... 60
CONSTRAINT EQUATIONS AND
OUTPUT EQUATIONS $=122$
NPUT VARIABLES = ..... 122
6

NUMBER OF LOOPS =
NUMBER OF PARTS = 49

INITIALIZATION FOR TRIM
GENERATE SUBMATRICES AND SUBVECTORS FOR TRIM PART RECORDS
GENERATE SUBMATRICES FOR TRIM MODES RECORDS
INITIALIZE SCRATCH DATA STRUCTURES FOR TRIM

```
SUCCESSIVE SUBSTITUTION ITERATION, LOOP = WAKE
```

LEVEL $1=$ UNIFORM INFLOW

| NEWTON RAPHSON ITERATION, LOOP = TRIM |  |  |  | LL |
| :---: | :---: | :---: | :---: | :---: |
| VARIABLES | QUANTITIES |  |  | LL |
| COLL LATCYC LNGCYC | CT/S | BETAS | BETAC | LL |
| TARGETS | 0.80000E-01 | $0.00000 \mathrm{E}+00$ | $0.00000 \mathrm{E}+00$ | LL |
| BEGINNING OF ITERATION, LOOP = TRIM |  |  |  | LL |
| 9.0000 3.0000 7.0000 | $0.85116 \mathrm{E}-01$ | -0.15321 | -0.17324 | LL |
| PERTURBATION IDENTIFICATION, COUNT $=1$, LOOP $=$ TRIM |  |  |  | LL |
| 8.5000 3.0000 7.0000 | $0.70264 \mathrm{E}-01$ | 0.69957 | 0.56290 | LL |
| PERTURBATION IDENTIFICATION, COUNT $=2$ 2, LOOP $=$ TRIM |  |  |  | LL |
| 9.0000 2.0000 7.0000 | $0.85132 \mathrm{E}-01$ | -1.2907 | -0.16100E-01 | LL |
| PERTURBATION IDENTIFICATION, COUNT $=3$, LOOP $=$ TRIM <br> $\begin{array}{lllllll}9.0000 & 3.0000 & 0.0000 & 0.72022 \mathrm{E}-01 & 1.3653\end{array}$ |  |  |  | LL |
|  |  |  |  | LL |
| PERTURBATION IDENTIFICATION |  |  |  | LL |
| DERIVATIVE MATRIX D |  |  |  | LL |
| VARIABLE $=$ COLL LATCYC | LNGCYC |  |  | LL |
| QUANTITY = CT/S 0.29704E-01 -0.16049E-04 | -0.13094E-01 |  |  | LL |
| QUANTITY = BETAS $\quad-1.7056$ 1.1375 | 0.74792 |  |  | LL |
| QUANTITY = BETAC -1.4723 -0.15714 | 1.5385 |  |  | LL |
| GAIN MATRIX $=($ RELAXATION FACTOR) $*$ D**-1 |  |  |  | LL |
| QUANTITY $=$ CT/S BETAS | BETAC |  |  | LL |
| VARIABLE $=$ COLL 12.445 0.13875E-01 | $0.99172 \mathrm{E}-01$ |  |  | LL |
| VARIABLE = LATCYC 10.148 0.17607 | $0.77625 \mathrm{E}-03$ |  |  | LL |
| VARIABLE = LNGCYC 12.945 0.31260E-01 | 0.22497 |  |  | LL |
| ITERATION, COUNT $=1$, LOOP $=$ TRIM |  |  |  |  |
|  |  |  |  | LL |
| 8.9556 2.9752 6.9775 | $0.85254 \mathrm{E}-01$ | -0.27723 | -0.17706 | LL |
| ITERATION, COUNT $=2$ L LOOP $=$ TRIM |  |  |  | LL |
| $8.9117 \quad 2.9708$ 6.9580 | $0.84040 \mathrm{E}-01$ | -0.27016 | -0.14817 | LL |
| ITERATION, COUNT $=3$ L LOOP $=$ TRIM |  |  |  | LL |
| 8.8798 2.9775 6.9475 | $0.83886 \mathrm{E}-01$ | -0.25342 | -0.14216 | LL |
| ITERATION, COUNT $=4$, LOOP $=$ TRIM |  |  |  | LL |
| 8.8491 2.9828 6.9371 | $0.83472 \mathrm{E}-01$ | -0.23580 | -0.13311 | LL |
| ITERATION, COUNT $=5$, LOOP $=$ TRIM |  |  |  | LL |
| 8.8223 2.9892 6.9295 | $0.82971 \mathrm{E}-01$ | -0.21302 | -0.11850 | LL |
| ITERATION, COUNT $=6$, LOOP $=$ TRIM |  |  |  | LL |
| 8.8001 2.9966 6.9243 | $0.82429 \mathrm{E}-01$ | -0.18291 | -0.98623E-01 | LL |


| $\begin{gathered} \text { ITERATION, COUNT = } \\ 8.7822 \end{gathered}$ | $\begin{aligned} & 7, \text { LO } \\ & 3.0043 \end{aligned}$ | $\begin{aligned} & \text { TRIM } \\ & 6.9208 \end{aligned}$ | 0.81910E-01 | -0.14614 | -0.76324E-01 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { ITERATION, COUNT }= \\ 8.7680 \end{gathered}$ | $\begin{gathered} 8, \mathrm{LO} \\ 3.0107 \end{gathered}$ | $\begin{aligned} & \text { TRIM } \\ & \quad 6.9178 \end{aligned}$ | 0.81438E-01 | -0.10594 | -0.54869E-01 |
| $\begin{gathered} \text { ITERATION, COUNT = } \\ 8.7570 \end{gathered}$ | $\begin{gathered} 9, \text { LO } \\ 3.0148 \end{gathered}$ | $\begin{aligned} & \text { TRIM } \\ & \quad 6.9148 \end{aligned}$ | 0.81026E-01 | -0.66360E | -0.36649E-01 |

SUCCESSIVE SUBSTITUTION ITERATION, NO VARIABLES, NUMBER OF LEVELS = 2 , LOOP = WAKE
ITERATION, COUNT = 1 , FOR LEVEL $1=$ UNIFORM INFLOW

SUCCESSIVE SUBSTITUTION ITERATION, LOOP = WAKE
LEVEL $2=$ NONUNIFORM INFLOW PRSC WAKE GEOM


| ITERATION, COUNT $=4$, LOOP $=$ TRIM |  |  |  |  |  | LL LL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9.6091 | 2.7959 | 7.8888 | $0.71883 \mathrm{E}-01$ | 0.50373 | $0.45100 \mathrm{E}-01$ | LL |
|  |  |  |  |  |  | LL |
| ITERATION, COUNT $=5$, LOOP $=$ TRIM |  |  |  |  |  | LL |
| 9.7211 | 2.7752 | 8.0103 | $0.73591 \mathrm{E}-01$ | 0.37796 | $0.38564 \mathrm{E}-01$ | LL |
| ITERATION, COUNT $=6$, LOOP $=$ TRIM |  |  |  |  |  | LL |
| 9.8093 | 2.7626 | 8.1060 | $0.73425 \mathrm{E}-01$ | 0.37165 | $0.42224 \mathrm{E}-01$ | LL |
|  |  |  |  |  |  | LL |
| ITERATION, COUNT $=7$, LOOP $=$ TRIM |  |  |  |  |  | LL |
| 9.8997 | 2.7528 | 8.2039 | $0.76389 \mathrm{E}-01$ | 0.16084 | $0.23948 \mathrm{E}-01$ | LL |
|  |  |  |  |  |  | LL |
| ITERATION, COUNT $=8$, LOOP $=$ TRIM |  |  |  |  |  | LL |
| 9.9496 | 2.7557 | 8.2586 | $0.76184 \mathrm{E}-01$ | 0.15632 | $0.25904 \mathrm{E}-01$ | LL |
|  |  |  |  |  |  | LL |
| ITERATION, COUNT $=$ 9, LOOP $=$ TRIM |  |  |  |  |  | LL |
| 10.002 | 2.7614 | 8.3167 | $0.77136 \mathrm{E}-01$ | 0.13893 | $0.28563 \mathrm{E}-01$ | LL |
|  |  |  |  |  |  | LL |
| ITERATION, COUNT = 10, LOOP = TRIM |  |  |  |  |  | LL |
| 10.041 | 2.7614 | 8.3574 | $0.78965 \mathrm{E}-01$ | $0.15718 \mathrm{E}-01$ | -0.25938E-01 | LL |
| SUCCESSIVE SUBSTITUTION ITERATION, NO VARIABLES, NUMBER OF LEVELS $=2$, LOOP = WAKE |  |  |  |  |  | L |
| ITERATION, COUNT = 1, FOR LEVEL 2 = NONUNIFORM INFLOW PRSC WAKE GEOM; LOOP = WAKE |  |  |  |  |  | L |

TRIM CONVERGENCE
****************
SUCCESSIVE SUBSTITUTION ITERATION, LOOP = WAKE
ITERATION 1, LEVEL $2=$ NONUNIFORM INFLOW PRSC WAKE GEOM

STATUS OF LOOP AND PART SOLUTIONS
LOOP $=$ WAKE
LOOP $=$ TRIM
LOOP $=$ CIRCULATION
LOOP $=$ MOTION
LOOP $=$ AIRFRAME

PART = ROTOR 1 HUB
PART = ROTOR 1 BLADE 4
PART = ROTOR 1 HUB
PART = AIRFRAME

TOTAL NUMBER
OF ITERATIONS
SUCCESSIVE SUBSTITUTION, NO VARIABLES NEWTON RAPHSON, CONVERGED SUCCESSIVE SUBSTITUTION, CONVERGED

```
HARMONIC SOLUTION, CONVERGED
SOLUIION, CONVERGED
HARMONIC SOLUTION, CONVERGED
```

HARMONIC SOLUTION, CONVERGED

CONVERGENCE OF LOOP AND PART SOLUTIONS

LOOP = WAKE SUCCESSIVE SUBSTITUTION, NO VARIABLES
NUMBER OF ITERATIONS = 1 (MAXIMUM 1), TOLERANCE $=0.000000 \mathrm{E}+00 \mathrm{FOR}$ LEVEL 1 = UNIFORM INFLOW
NUMBER OF ITERATIONS $=1$ (MAXIMUM 1), TOLERANCE $=0.000000 \mathrm{E}+00$ FOR LEVEL $2=$ NONUNIFORM INFLOW PRSC WAKE GEOM ERROR $=A B S(X-X O L D)$ OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE
LOOP = TRIM NEWTON RAPHSON, CONVERGED

NUMBER OF ITERATIONS $=10$ (MAXIMUM 40), TOLERANCE $=5.00000$
ERROR = ABS (TRIMMED-TARGET); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)

|  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :--- | :--- | :--- | :--- | :--- |
| VARIABLE |  |  | QUANTITY |  |  |  |  |
|  | TRIMMED | INITIAL |  |  | TRIMMED | TARGET | WEIGHT |
| COLL | 10.0408 | 8.75701 | CT/S | $0.789654 \mathrm{E}-01$ | $0.800000 \mathrm{E}-01$ | $0.80000 \mathrm{E}-03$ | 0.258662 |
| LATCYC | 2.76145 | 3.01478 | BETAS | $0.157181 \mathrm{E}-01$ | $0.000000 \mathrm{E}+00$ | $0.200000 \mathrm{E}-01$ | 0.157181 |
| LNGCYC | 8.35741 | 6.91484 | BETAC | $-0.259375 \mathrm{E}-01$ | $0.000000 \mathrm{E}+00$ | $0.200000 \mathrm{E}-01$ | 0.259375 |

SUCCESSIVE SUBSTITUTION, CONVERGED
NUMBER OF ITERATIONS $=17$ (MAXIMUM 200), TOLERANCE $=1.00000$
ERROR = ABS (X-XOLD) OR RMS (X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)


ERROR = ABS(X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)

PART = ROTOR 1 HUB HARMONIC SOLUTION, CONVERGED
NUMBER OF ITERATIONS = 1 (MAXIMUM 40), TOLERANCE = $2.00000 \quad$ FOR PERIOD $1=$ ROTOR 1

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ERROR $=$ ABS (X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)

*************
TRIM SOLUTION
*************

## 促

ROTORCRAFT PERFORMANCE

CONFIGURATION
WIND TUNNEL
ONE ROTOR
OPERATING CONDITION
WIND SPEED

| VELOCITY (KNOTS) | $=$ | 150.00 |
| :--- | :--- | ---: |
| $\mathrm{~V} /(\mathrm{OMEGA} * \mathrm{R})$ | $=$ | 0.3761 |
| MACH NUMBER | $=$ | 0.2268 |
| VELOCITY (FT/SEC) | $=$ | 253.17 |
| DYN PRESS (LB/FT**2) | $=$ | 76.21 |

DYN PRESS (LB/FT**2) = 76.21 REYNOLDS NO (/FT) $=0.161 \mathrm{E}+07$

CONTROL SETTINGS
ROTORCRAFT
PILOT CONTROL (DEG)

| COLLECTIVE | $=$ | 10.04 |
| :--- | :--- | ---: |
| LATERAL CYCLIC | $=$ | 2.76 |
| LONGITUDINAL CYCLIC | $=$ | 8.36 |

AIRFRAME
AERODYNAMIC CONTROLS (DEG)

| FLAPERON | $=$ | 0.00 |
| :--- | :--- | :--- |
|  | $=$ | 0.00 |

ROTOR 1
PRIMARY CONTROLS (DEG)
COLLECTIVE
$=0.00$
0.00
0.00

LONGITUDINAL CYCLIC =
0.00
0.00

| AIRFRAME ORIENTATION |  |  |
| :--- | :--- | ---: |
| YAW ANGLE (DEG) | $=$ | 0.00 |
| PITCH ANGLE (DEG) | $=$ | -7.00 |
| ROLL ANGLE (DEG) | $=$ | 0.00 |

AIRFRAME ORIENTATION
PITCH ANGLE (DEG) $=\quad-7.00$

ROTOR SPEED

|  |  |  |
| :--- | :--- | ---: |
| TIP SPEED |  | 673.16 |
| TIP MACH NUMBER | $=$ | 0.6030 |
| ROTATIONAL SPEED (RPM) | $=$ | 260.00 |
| OMEGA (RAD/SEC) | $=$ | 27.227 |
| REYNOLDS NO (/FT) | $=$ | $0.428 E+07$ |

AIRFRAME AND ROTOR CONTROLS ARE VALUES WITH ZERO PILOT AND GOVERNOR CONTROLS

ROTORCRAFT POWER
TOTAL
CLIMB + PARASITE
PROFILE + INDUCED

| $\mathrm{CP}=$ | 0.00071896 |
| ---: | ---: |
| $\mathrm{CPC}+\mathrm{CPP}=$ | 0.00033592 |
| $\mathrm{CPO}+\mathrm{CPI}=$ | 0.00038304 |
| $\mathrm{CP}=$ | 0.00071896 |
| $\mathrm{CPIND}=$ | 0.00019416 |
| $\mathrm{CPINT}=$ | 0.00000000 |
| $\mathrm{CPO}=$ | 0.00018888 |
| $\mathrm{CPP}=$ | 0.00033592 |
| $\mathrm{CPC}=$ | 0.00000000 |
| $\mathrm{CPO}=$ | 0.00018999 |
| $\mathrm{CPM}=$ | 0.00006841 |
| $\mathrm{CP}=$ | 0.00071896 |
| CPIDEAL $=$ | 0.00040433 |

## TOTAL

IDEAL
$C P=0.00071896$
CLIMB + PARASITE

## TOTAL

INDUCED
INTERFERENCE
PROFILE
PARASITE
CLIMB
PROFILE (SECTION) MINIMUM INDUCED

CPIDEAL $=0.00040433$

| CP/S $=$ | 0.0079007 | $\mathrm{P}=$ | 1820.972 | HP | = | 1001534.75 | LB-FT/SEC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CPC/S+CPP/S= | 0.0036914 | $\mathrm{PC}+\mathrm{PP}=$ | 850.818 | HP | = | 467949.75 | LB-FT/SEC |
| $\mathrm{CPO} / \mathrm{S}+\mathrm{CPI} / \mathrm{S}=$ | 0.0042092 | $\mathrm{PO}+\mathrm{PI}=$ | 970.155 | HP | = | 533585.00 | LB-FT/SEC |
| CP/S $=$ | 0.0079007 | $\mathrm{P}=$ | 1820.972 | HP | = | 1001534.75 | LB-FT/SEC |
| CPIND/S= | 0.0021336 | PIND= | 491.758 | HP | = | 270467.09 | LB-FT/SEC |
| CPINT/S= | 0.0000000 | PINT= | 0.000 | HP | = | 0.00 | LB-FT/SEC |
| CPO/S= | 0.0020756 | $\mathrm{PO}=$ | 478.396 | HP | = | 263117.91 | LB-FT/SEC |
| $\mathrm{CPP} / \mathrm{S}=$ | 0.0036914 | $\mathrm{PP}=$ | 850.818 | HP | = | 467949.75 | LB-FT/SEC |
| CPC/S $=$ | 0.0000000 | $\mathrm{PC}=$ | 0.000 | HP | = | 0.00 | LB-FT/SEC |
| CPO/S= | 0.0020878 | $\mathrm{PO}=$ | 481.211 | HP | = | 264666.03 | LB-FT/SEC |
| CPM/S $=$ | 0.0007518 | $\mathrm{PM}=$ | 173.274 | HP | = | 95300.67 | LB-FT/SEC |
| CP/S $=$ | 0.0079007 | $\mathrm{P}=$ | 1820.972 | HP | = | 1001534.75 | LB-FT/SEC |
| CPIDEAL/S= | 0.0044432 | PIDEAL= | 1024.092 | HP | = | 563250.44 | LB-FT/SEC |

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PERFORMANCE INDICE
EQUIVALENT
DRAG AREA
ROTOR LIFT-TO-DRAG RATIO
FIGURE OF MERIT
FIGURE OF MERIT
AIRCRAFT LIFT-TO-DRAG
$\mathrm{D}=$
L/DR = WEIGHT/D
$\mathrm{M}=\mathrm{PM} /(\mathrm{PO}+\mathrm{PI})$
$\mathrm{M}=\mathrm{PIDEAL} / \mathrm{P}$
$\mathrm{L} / \mathrm{D}=\mathrm{WEIGHT} * V R E F / \mathrm{P}$
107.6030 LB
$27.6552 \mathrm{FT} * * 2$
0.0000
0.1786
0.5624
0.0000

ROTORCRAFT POWER = SUM OF POWER FOR ALL ROTORS; TOTAL POWER = SHAFT AND REACTION JET POWER ABSORBED BY ROTORS
INDUCED POWER FROM SELF-INDUCED VELOCITY OF ROTORS; INTERFERENCE POWER FROM ALL OTHER SOURCES; PI = PIND + PINT CLIMB POWER PC = VCLIMB*WEIGHT; PARASITE POWER PP = $(P P+P C)-P C$
PROFILE + INDUCED POWER PO+PI = P - (PP+PC); PROFILE POWER PO = (PO+PI) - PIND - PINT
PROFILE POWER (SECTION) FROM INTEGRATED SECTION DRAG; MINIMUM INDUCED POWER FROM MOMENTUM THEORY INDUCED VELOCITY IDEAL POWER PIDEAL = MINIMUM INDUCED + PARASITE + CLIMB
NONIDEAL POWER PN = P - PIDEAL = PROFILE + EXCESS INDUCED + INTERFERENCE

POWER COEFFICIENT CP = P / RHO*A*VTIP**3
AIR DENSITY RHO $=0.002378$ SLUG/FT**3, REFERENCE ROTOR RADIUS R $=24.7240$ FT, ROTOR AREA A $=$ PI*R**2
REFERENCE ROTATIONAL SPEED OMEGA $=260.000$ RPM, TIP SPEED VTIP $=$ OMEGA*R, SOLIDITY RATIO $S=0.09100$ GROSS WEIGHT $=0.000 \mathrm{LB}, \mathrm{CLIMB} \mathrm{SPEED}=0.000 \mathrm{FT} / \mathrm{SEC}=0.00 \mathrm{FT} / \mathrm{MIN}$ GROSS WEIGHT $=$
REFERENCE SPEED VREF $=$
$=$
253.171 FT/SEC

LOADS APPLIED TO AIRFRAME COMPONENT
FORCE (LB) AND MOMENT (FT-LB); IN AIRFRAME AXES, MOMENTS ABOUT ORIGIN OF AIRFRAME AXES
APPLIED LOAD = F FOR AUXILIARY FORCE, T FOR TAIL BOOM, L FOR SLUNG LOAD; A FOR NONROTATING SWASHPLATE ACTUATOR AIRFRAME COMPONENT MASS $=0.00000 \mathrm{E}+00 \mathrm{SLUG}=0.00000 \mathrm{E}+00 \mathrm{LB} ;($ WEIGHT+ACCELERATION)/(AIRFRAME WEIGHT) $=1.000 \mathrm{G}$


AVAILABLE TRIM QUANTITIES
ROTORCRAFT PERFORMANCE, TRIM SENSOR
TOTAL POWER CP/S = 0.0079007
ROTOR 1 PERFORMANCE, TRIM SENSOR

## HAFT AXES

THRUST CT/S $=0.078965$ DRAG FORCE $\mathrm{CH} / \mathrm{S}=-0.000193$ SIDE FORCE CY/S $=0.001204$ POWER CP/S = 0.0079007

TIP-PATH PLANE AXES THRUST CT/S
DRAG FORCE CH/S $=-0.000229$
SIDE FORCE CY/S =
0.0012

WIND AXES
LIFT CL/S $=0.078362$
DRAG CX/S $=-0.009815$

TIP-PATH PLANE TILT (DEG)
LONGITUDINAL, BETAC $=-0.026$ LATERAL, BETAS $=0.016$

ROTORCRAFT COEFFICIENT/SOLIDITY FORM: POWER CP/S = P / RHO*A*VTIP**3*S (BASED ON ROTOR 1 PARAMETERS AIR DENSITY RHO $=0.002378$ SLUG/FT**3, REFERENCE ROTOR RADIUS R = 24.7240 FT, ROTOR AREA A $=$ PI*R**2 REFERENCE ROTATIONAL SPEED OMEGA $=260.000$ RPM, TIP SPEED VTIP $=$ OMEGA*R, SOLIDITY RATIO $S=0.09100$

ROTOR 1 COEFFICIENT/SOLIDITY FORM: FORCE CF/S = F / RHO*A*VTIP**2*S; POWER CP/S = P / RHO*A*VTIP**3*S AIR DENSITY RHO $=0.002378$ SLUG/FT**3, REFERENCE ROTOR RADIUS $R=\quad 24.7240$ FT, ROTOR AREA A $=$ PI*R**2

## ROTOR 1 PERFORMA

ROTOR 1 PERFORMANCE

CONFIGURATION
ONE ROTOR
OPERATING CONDITION

|  | MUX |
| :--- | :---: |
| HUB PLANE | 0.3733 |
| TIP-PATH PLANE | 0.3733 |
| CONTROL PLANE | 0.3627 |
|  |  |
| ROTOR VELOCITY |  |
| ADVANCE RATIO | $=0.3761$ |
| MACH NUMBER | $=0.2268$ |

VELOCITY OF AIR RELATIVE ROTOR MU = (-MUX,MUY,MUZ); MUX + AFT, MUY + FROM RIGHT, MUZ + DOWN THROUGH ROTOR DISK ALPHA = ANGLE OF ATTACK OF ROTOR PLANE RELATIVE VELOCITY, + AFT; ALFHP = ALFCP - T1S = ALFTPP + BC PSI = YAW ANGLE OF VELOCITY IN ROTOR PLANE, + FROM FORWARD/RIGHT QUADRANT
LONGITUDINAL TIP-PATH PLANE TILT BC + FORWARD, LATERAL TILT BS + TOWARD RETREATING SIDE
TIP-PATH PLANE TILT (BC,BS) AND CYCLIC CONTROL (T1C,T1S) ARE COEFFICIENTS OF POSITIVE FOURIER SERIES IN BLADE AZIMUTH $(\mathrm{BC}+\mathrm{T} 1 \mathrm{~S}) \mathrm{HP}=\mathrm{BCCP}=\mathrm{T} 1 \mathrm{STPP},(\mathrm{BS}-\mathrm{T} 1 \mathrm{C}) \mathrm{HP}=\mathrm{BSCP}=-\mathrm{T} 1 \mathrm{CTPP}$

ADVANCE RATIO = VELOCITY / VTIP
REFERENCE ROTOR RADIUS $\mathrm{R}=24.7240 \mathrm{FT}$, ROTATIONAL SPEED OMEGA $=260.000 \mathrm{RPM}$; TIP SPEED VTIP $=$ OMEGA*R SOUND SPEED $=1116.45 \mathrm{FT} / \mathrm{SEC}$

```
HUB PLANE (NONROTATING SHAFT AXES): Z IS SHAFT AXIS (POSITIVE THRUST DIRECTION) X-AXIS DOWNSTREAM; AZIMUTH ANGLE MEASURED FROM X-AXIS, IN DIRECTION OF ROTOR ROTATION ROTOR 1 ROTATES ABOUT Z-AXIS CLOCKWISE; Y-AXIS TOWARDS RETREATING SIDE
TIP-PATH PLANE TILTED RELATIVE HUB PLANE, FORWARD FOR + BC AND TOWARD RETREATING SIDE FOR + BS
``` CONTROL PLANE TILTED RELATIVE HUB PLANE, AFT FOR + T1S AND TOWARD RETREATING SIDE FOR + T1C
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{7}{|l|}{SHAFT AXES} \\
\hline THRUST & \(C T=0.00718585\) & CT/S \(=0.0789653\) & \(T=\) & 14870.263 & LB & \\
\hline DRAG FORCE & \(\mathrm{CH}=-0.00001759\) & \(\mathrm{CH} / \mathrm{S}=-0.0001933\) & H & -36.394 & LB & ANGLE \(=-0.14\) DEG \\
\hline SIDE FORCE & \(\mathrm{CY}=0.00010953\) & \(\mathrm{CY} / \mathrm{S}=0.0012037\) & Y & 226.666 & LB & ANGLE \(=0.87 \mathrm{DEG}\) \\
\hline ROLL MOMENT & CMX \(=-0.00001341\) & CMX \(/ \mathrm{S}=-0.0001474\) & MX= & -686.146 & FT-LB & OFFSET \(=-0.002 \mathrm{R}\) \\
\hline PITCH MOMENT & CMY \(=0.00000985\) & \(\mathrm{CMY} / \mathrm{S}=0.0001082\) & MY= & 503.994 & FT-LB & OFFSET \(=0.001 \mathrm{R}\) \\
\hline YAW MOMENT & \(\mathrm{CMZ}=0.00071986\) & \(\mathrm{CMZ} / \mathrm{S}=0.0079105\) & \(M Z=\) & 36830.453 & FT-LB & \\
\hline \multicolumn{7}{|l|}{TIP-PATH PLANE AXES} \\
\hline THRUST & \(C T=0.00718587\) & CT/S \(=0.0789656\) & \(T=\) & 14870.307 & LB & \\
\hline DRAG FORCE & \(\mathrm{CH}=-0.00002084\) & \(\mathrm{CH} / \mathrm{S}=-0.0002290\) & & -43.126 & LB & ANGLE \(=-0.17\) DEG \\
\hline SIDE FORCE & \(\mathrm{CY}=0.00010756\) & \(\mathrm{CY} / \mathrm{S}=0.0011820\) & Y & 222.586 & LB & ANGLE \(=0.86 \mathrm{DEG}\) \\
\hline ROLL MOMENT & CMX \(=-0.00001374\) & CMX \(/ \mathrm{S}=-0.0001510\) & \(\mathrm{MX}=\) & -702.819 & FT-LB & OFFSET \(=-0.002 \mathrm{R}\) \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline PITCH MOMENT & CMY= & 0.00000965 & CMY/S \(=\) & 0.0001061 & MY= & 493.890 & FT-LB & OFFSET \(=0.001 \mathrm{R}\) \\
\hline YAW MOMENT & \(\mathrm{CMZ}=\) & 0.00071986 & CMZ/S = & 0.0079105 & \(M Z=\) & 36830.277 & FT-LB & \\
\hline \multicolumn{9}{|l|}{WIND AXES} \\
\hline LIFT & \(\mathrm{CL}=\) & 0.00713098 & \(\mathrm{CL} / \mathrm{S}=\) & 0.0783625 & L = & 14756.729 & LB & \\
\hline DRAG & \(\mathrm{CX}=\) & -0.00089319 & \(\mathrm{cX} / \mathrm{S}=\) & -0.0098153 & \(\mathrm{x}=\) & -1848.351 & LB & ANGLE \(=-7.14 \mathrm{DEG}\) \\
\hline \multicolumn{9}{|l|}{TORQUE} \\
\hline SHAFT TORQUE & \(\mathrm{CQ}=\) & -0.00071896 & \(\mathrm{CQ} / \mathrm{S}=\) & -0.0079007 & \(\mathrm{Q}=\) & -36784.430 & FT-LB & \\
\hline
\end{tabular}

AERODYNAMIC ROTOR FORCE \(=(H, Y, T)\) AND MOMENT \(=(M X, M Y, M Z)\) ACTING ON HUB
FORCE ANGLES = PITCH AND ROLL ANGLES OF TOTAL FORCE VECTOR RELATIVE ROTOR PLANE
MOMENT OFFSETS = PITCH AND ROLL MOMENTS DIVIDED BY THRUST TIMES RADIUS
WIND AXES: DRAG IS COMPONENT OF TOTAL FORCE IN VELOCITY DIRECTION (NEGATIVE FOR PROPULSIVE FORCE)
WIND AXES: LIFT IS COMPONENT OF TOTAL FORCE PERPENDICULAR TO VELOCITY (ALWAYS POSITIVE)
SWASHPLATE TORQUE = SHAFT-AXIS COMPONENT OF MOMENT AT HUB NODE

ROTOR COEFFICIENT FORM: FORCE CF = F RHO*A*VTIP**2; MOMENT CM = M / RHO*A*VTIP**2*R
AIR DENSITY RHO \(=0.002378\) SLUG/FT**3, REFERENCE ROTOR RADIUS R \(=24.7240\) FT, ROTOR AREA A \(=\) PI*R**2
REFERENCE ROTATIONAL SPEED OMEGA \(=260.000\) RPM, TIP SPEED VTIP \(=\) OMEGA*R, SOLIDITY RATIO \(S=0.09100\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline ROTOR POWER & & & & & & & & & & \\
\hline TOTAL & \(\mathrm{CP}=\) & 0.00071896 & \(\mathrm{CP} / \mathrm{S}=\) & 0.0079007 & \(\mathrm{P}=\) & 1820.972 & HP & = & 1001534.75 & LB-FT/SEC \\
\hline CLIMB + PARASITE & \(\mathrm{CPC}+\mathrm{CPP}=\) & 0.00033592 & CPC/S+CPP/S= & 0.0036914 & \(\mathrm{PC}+\mathrm{PP}=\) & 850.818 & HP & = & 467949.75 & LB-FT/SEC \\
\hline PROFILE + INDUCED & \(\mathrm{CPO}+\mathrm{CPI}=\) & 0.00038304 & CPO/S+CPI/S= & 0.0042092 & \(\mathrm{PO}+\mathrm{PI}=\) & 970.155 & HP & = & 533585.00 & LB-FT/SEC \\
\hline INDUCED & CPIND= & 0.00019416 & CPIND/S= & 0.0021336 & PIND= & 491.758 & HP & = & 270467.09 & LB-FT/SEC \\
\hline INTERFERENCE & CPINT= & 0.00000000 & CPINT/S= & 0.0000000 & PINT= & 0.000 & HP & = & 0.00 & LB-FT/SEC \\
\hline PROFILE & \(\mathrm{CPO}=\) & 0.00018888 & CPO/S \(=\) & 0.0020756 & \(\mathrm{PO}=\) & 478.396 & HP & = & 263117.91 & LB-FT/SEC \\
\hline PROFILE (SECTION) & \(\mathrm{CPO}=\) & 0.00018999 & \(\mathrm{CPO} / \mathrm{S}=\) & 0.0020878 & \(\mathrm{PO}=\) & 481.211 & HP & = & 264666.03 & LB-FT/SEC \\
\hline MINIMUM INDUCED & CPM= & 0.00006841 & CPM/S= & 0.0007518 & \(\mathrm{PM}=\) & 173.274 & HP & = & 95300.67 & LB-FT/SEC \\
\hline IDEAL & CPIDEAL= & 0.00040433 & CPIDEAL/S \(=\) & 0.0044432 & PIDEAL= & 1024.092 & HP & = & 563250.44 & LB-FT/SEC \\
\hline NONIDEAL & CPN= & 0.00031463 & CPN/S= & 0.0034574 & \(\mathrm{PN}=\) & 796.881 & HP & = & 438284.31 & LB-FT/SEC \\
\hline
\end{tabular}

TOTAL POWER = SHAFT, SWASHPLATE, AND REACTION JET POWER ABSORBED BY ROTOR
SWASHPLATE POWER PS = MEAN POWER AT JOINT (TILTED BY CYCLIC)
CLIMB + PARASITE POWER PC+PP = - VELOCITY*DRAG; PROFILE + INDUCED POWER PO+PI = P - (PC+PP)
INDUCED POWER FROM SELF-INDUCED VELOCITY OF ROTOR; INTERFERENCE POWER FROM ALL OTHER SOURCES; PI = PIND + PINT PROFILE POWER PO = (PO+PI) - PIND - PINT
PROFILE POWER (SECTION) FROM INTEGRATED SECTION DRAG; MINIMUM INDUCED POWER FROM MOMENTUM THEORY INDUCED VELOCITY IDEAL POWER PIDEAL = MINIMUM INDUCED + INTERFERENCE + PARASITE + CLIMB NONIDEAL POWER PN = P - PIDEAL = PROFILE + EXCESS INDUCED

POWER COEFFICIENT CP \(=\mathrm{P} / \mathrm{RHO} \mathrm{A}^{2} * V T I P * *\)
AIR DENSITY RHO = 0.002378 SLUG/FT**3, REFERENCE ROTOR RADIUS R = 24.7240 FT, ROTOR AREA A = PI*R**2
REFERENCE ROTATIONAL SPEED OMEGA \(=260.000\) RPM, TIP SPEED VTIP = OMEGA*R, SOLIDITY RATIO S \(=0.0910\)


MEAN INFLOW RATIO: VELOCITY MAGNITUDE AND COMPONENTS IN SHAFT AXES (AVERAGE OVER ROTOR DISK)
FIGURE OF MERIT IS MEASURE OF ROTOR EFFICIENCY APPROPRIATE FOR AXIAL FLOW
BOTH DEFINITIONS INCLUDE CONVENTIONAL HOVER FIGURE OF MERIT, M = T*VIDEAL/P = T*SQRT(T/2*RHO*A)/P
\(\mathrm{M}=(\mathrm{PM}+\mathrm{PINT}) /(\mathrm{PO}+\mathrm{PI})=\mathrm{T} *(\mathrm{VIDEAL}+\mathrm{VINT}) /(\mathrm{PO}+\mathrm{PI})\)
\(M=P I D E A L / P=1-P N / P=T *(V+V I D E A L) / P\) FOR AXIAL FLOW (NOT VALID FOR ZERO POWER)
SECOND DEFINITION NEARLY PROPULSIVE EFFICIENCY ETA \(=T V / P\) FOR HIGH INFLOW
PROPULSIVE EFFICIENCY IS MEASURE APPROPRIATE FOR HIGH SPEED PROPULSION
ETA \(=-X * V / P=(P C+P P) / P \quad\) (NOT VALID FOR ZERO VELOCITY OR ZERO POWER)
ROTOR LIFT-TO-DRAG RATIO IS MEASURE OF ROTOR LIFTING EFFICIENCY, APPROPRIATE FOR EDGEWISE FLOW
L/D = L*V/(PI+PO) (NOT USEFUL FOR ZERO VELOCITY); ROTOR EQUIVALENT DRAG D = P/V+X = (PI+PO)/V
WING LIFT-TO-DRAG RATIO AND INDUCED EFFICIENCY E ARE MEASURES APPROPRIATE FOR FIXED WING
\(D I / Q=(L / Q) * * 2 /(E * P I * B * * 2)\)

INFLOW RATIO, INFLOW = VELOCITY / VTIP
REFERENCE ROTOR RADIUS R \(=24.7240\) FT, ROTATIONAL SPEED OMEGA \(=260.000\) RPM; TIP SPEED VTIP \(=\) OMEGA*R
COEFFICIENT CF \(=F / Q A\), WING COEFFICIENT CFW \(=F / Q S\)

RADIUS R = 24.7240 FT, SOLIDITY RATIO SIGMA \(=0.09100\), AIR DENSITY RHO = 0.002378 SLUG/FT**3
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{16}{|l|}{BLADE SECTION ANGLE OF ATTACK (DEG)} \\
\hline RADIAL STATION & \(=0.560\) & 0.615 & 0.665 & 0.710 & 0.750 & 0.785 & 0.815 & 0.845 & 0.870 & 0.890 & 0.910 & 0.930 & 0.950 & 0.970 & 0.990 \\
\hline AZIMUTH (DEG) & ALPHA & & & & & & & & & & & & & & \\
\hline PSI \(=0.0\) & 8.1 & 7.9 & 8.0 & 8.1 & 8.1 & 8.1 & 7.9 & 7.2 & 6.3 & 5.9 & 6.2 & 6.7 & 6.6 & 6.9 & 6.0 \\
\hline PSI \(=15.0\) & 6.1 & 6.6 & 6.8 & 6.8 & 6.8 & 6.7 & 6.5 & 5.9 & 5.1 & 4.8 & 5.1 & 5.4 & 5.4 & 5.7 & 5.0 \\
\hline PSI \(=30.0\) & 4.9 & 5.0 & 5.2 & 5.3 & 5.3 & 5.3 & 5.1 & 4.7 & 4.1 & 3.8 & 3.9 & 4.2 & 4.2 & 4.4 & 3.9 \\
\hline PSI \(=45.0\) & 3.8 & 3.9 & 4.0 & 4.0 & 4.0 & 3.9 & 3.8 & 3.5 & 3.1 & 2.8 & 2.9 & 3.1 & 3.0 & 3.2 & 2.8 \\
\hline PSI \(=60.0\) & 2.6 & 2.7 & 2.8 & 2.8 & 2.8 & 2.7 & 2.6 & 2.4 & 2.1 & 1.9 & 1.9 & 2.0 & 2.0 & 2.1 & 1.8 \\
\hline PSI \(=75.0\) & 1.8 & 1.8 & 1.8 & 1.8 & 1.7 & 1.6 & 1.6 & 1.4 & 1.2 & 1.1 & 1.1 & 1.1 & 1.1 & 1.1 & 1.0 \\
\hline PSI \(=90.0\) & 1.3 & 1.2 & 1.1 & 1.0 & 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.4 & 0.4 & 0.3 & 0.3 & 0.2 \\
\hline PSI \(=105.0\) & 1.1 & 1.0 & 0.8 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.0 & -0.1 & -0.1 & -0.1 & -0.2 & -0.2 & -0.2 \\
\hline PSI \(=120.0\) & 1.3 & 1.0 & 0.7 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & -0.1 & -0.1 & -0.1 & -0.2 & -0.2 & -0.3 & -0.3 \\
\hline PSI \(=135.0\) & 1.4 & 1.1 & 1.0 & 0.9 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.1 & 0.1 & 0.0 & 0.0 & -0.1 & -0.1 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline PSI \(=150.0\) & 2.5 & 2.4 & 2.3 & 2.2 & 2.0 & 1.8 & 1.7 & 1.6 & 1.4 & 1.0 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 \\
\hline PSI \(=165.0\) & 4.4 & 4.3 & 4.2 & 3.9 & 3.7 & 3.4 & 3.3 & 3.2 & 3.0 & 2.3 & 1.8 & 1.6 & 1.4 & 1.3 & 1.0 \\
\hline PSI \(=180.0\) & 6.7 & 6.5 & 6.2 & 5.8 & 5.5 & 5.2 & 5.0 & 4.9 & 4.7 & 3.7 & 3.0 & 2.6 & 2.3 & 2.1 & 1.7 \\
\hline PSI \(=195.0\) & 8.8 & 8.4 & 8.0 & 7.6 & 7.2 & 6.8 & 6.5 & 6.4 & 6.1 & 4.9 & 3.9 & 3.5 & 3.1 & 2.8 & 2.2 \\
\hline PSI \(=210.0\) & 10.5 & 9.9 & 9.4 & 8.8 & 8.4 & 7.9 & 7.6 & 7.4 & 7.1 & 5.7 & 4.6 & 4.1 & 3.6 & 3.2 & 2.5 \\
\hline \(\mathrm{PSI}=225.0\) & 11.9 & 11.1 & 10.4 & 9.8 & 9.3 & 8.8 & 8.5 & 8.3 & 7.8 & 6.4 & 5.2 & 4.7 & 4.2 & 3.7 & 2.9 \\
\hline \(\mathrm{PSI}=240.0\) & 12.8 & 11.9 & 11.2 & 10.6 & 10.1 & 9.6 & 9.2 & 9.0 & 8.5 & 7.0 & 6.0 & 5.4 & 4.9 & 4.4 & 3.5 \\
\hline \(\mathrm{PSI}=255.0\) & 12.6 & 12.0 & 11.5 & 11.0 & 10.6 & 10.2 & 9.9 & 9.7 & 9.0 & 7.6 & 6.7 & 6.3 & 5.7 & 5.3 & 4.3 \\
\hline PSI \(=270.0\) & 11.7 & 11.4 & 11.3 & 11.1 & 10.8 & 10.6 & 10.4 & 10.1 & 9.3 & 8.1 & 7.5 & 7.2 & 6.7 & 6.3 & 5.2 \\
\hline PSI \(=285.0\) & 10.5 & 10.6 & 10.8 & 10.9 & 10.9 & 10.9 & 10.9 & 10.4 & 9.4 & 8.5 & 8.2 & 8.1 & 7.6 & 7.4 & 6.2 \\
\hline PSI \(=300.0\) & 9.4 & 10.0 & 10.4 & 10.7 & 10.9 & 11.0 & 11.0 & 10.4 & 9.4 & 8.6 & 8.6 & 8.8 & 8.4 & 8.3 & 7.0 \\
\hline \(\mathrm{PSI}=315.0\) & 8.5 & 9.2 & 9.7 & 10.0 & 10.2 & 10.2 & 10.1 & 9.4 & 8.3 & 7.8 & 8.0 & 8.4 & 8.1 & 8.3 & 7.1 \\
\hline PSI \(=330.0\) & 7.4 & 8.5 & 9.1 & 9.4 & 9.6 & 9.7 & 9.5 & 8.8 & 7.8 & 7.3 & 7.6 & 8.0 & 7.8 & 8.0 & 7.0 \\
\hline \(\mathrm{PSI}=345.0\) & 6.8 & 7.9 & 8.6 & 8.9 & 9.1 & 9.2 & 9.1 & 8.3 & 7.2 & 6.8 & 7.2 & 7.7 & 7.5 & 7.7 & 6.8 \\
\hline \(\mathrm{PSI}=360.0\) & 8.1 & 7.9 & 8.0 & 8.1 & 8.1 & 8.1 & 7.9 & 7.2 & 6.3 & 5.9 & 6.2 & 6.7 & 6.6 & 6.9 & 6.0 \\
\hline RADIAL STATION & \(=0.250\) & 0.340 & 0.420 & 0.495 & & & & & & & & & & & \\
\hline AZIMUTH (DEG) & ALPHA & & & & & & & & & & & & & & \\
\hline PSI \(=0.0\) & 6.5 & -5.2 & 6.1 & 8.5 & & & & & & & & & & & \\
\hline PSI \(=15.0\) & 0.9 & 6.1 & 3.7 & 4.9 & & & & & & & & & & & \\
\hline PSI \(=30.0\) & 2.9 & 3.0 & 4.5 & 4.7 & & & & & & & & & & & \\
\hline \(\mathrm{PSI}=45.0\) & 1.9 & 3.0 & 3.3 & 3.6 & & & & & & & & & & & \\
\hline \(\mathrm{PSI}=60.0\) & 1.7 & 1.6 & 2.1 & 2.4 & & & & & & & & & & & \\
\hline PSI \(=75.0\) & 0.9 & 1.2 & 1.5 & 1.7 & & & & & & & & & & & \\
\hline PSI \(=90.0\) & 1.2 & 1.3 & 1.4 & 1.3 & & & & & & & & & & & \\
\hline PSI \(=105.0\) & 1.2 & 1.4 & 1.3 & 1.2 & & & & & & & & & & & \\
\hline PSI \(=120.0\) & 1.5 & 1.8 & 1.8 & 1.6 & & & & & & & & & & & \\
\hline PSI \(=135.0\) & 2.2 & 2.4 & 2.1 & 1.7 & & & & & & & & & & & \\
\hline PSI \(=150.0\) & 3.1 & 3.2 & 3.0 & 2.7 & & & & & & & & & & & \\
\hline PSI \(=165.0\) & 3.8 & 4.2 & 4.3 & 4.4 & & & & & & & & & & & \\
\hline PSI \(=180.0\) & 4.8 & 5.7 & 6.2 & 6.6 & & & & & & & & & & & \\
\hline PSI \(=195.0\) & 6.4 & 7.9 & 8.8 & 9.0 & & & & & & & & & & & \\
\hline PSI \(=210.0\) & 9.9 & 10.9 & 11.5 & 11.1 & & & & & & & & & & & \\
\hline PSI \(=225.0\) & 26.7 & 17.3 & 14.3 & 12.9 & & & & & & & & & & & \\
\hline PSI \(=240.0\) & 90.1 & 26.8 & 16.9 & 14.0 & & & & & & & & & & & \\
\hline PSI \(=255.0\) & -155.8 & 41.8 & 19.0 & 14.1 & & & & & & & & & & & \\
\hline PSI \(=270.0\) & -123.1 & 6.5 & 15.9 & 12.7 & & & & & & & & & & & \\
\hline PSI \(=285.0\) & -130.8 & -89.7 & 16.7 & 10.7 & & & & & & & & & & & \\
\hline PSI \(=300.0\) & -134.6 & -28.0 & 5.9 & 8.6 & & & & & & & & & & & \\
\hline PSI \(=315.0\) & -147.7 & -18.2 & -0.7 & 7.4 & & & & & & & & & & & \\
\hline PSI \(=330.0\) & -135.5 & 4.5 & -4.7 & 3.8 & & & & & & & & & & & \\
\hline PSI \(=345.0\) & -50.4 & 16.5 & 10.7 & 5.8 & & & & & & & & & & & \\
\hline PSI \(=360.0\) & 6.5 & -5.2 & 6.1 & 8.5 & & & & & & & & & & & \\
\hline
\end{tabular}

BOUND CIRCULATION PEAKS (ENTIRE BLADE)
\begin{tabular}{|c|c|c|c|}
\hline & \multicolumn{3}{|r|}{BOUND CIRCULATION} \\
\hline AZIMUTH (DEG) & GMAX & GO & GI \\
\hline PSI \(=0.0\) & 0.02614 & 0.02614 & -0.00595 \\
\hline PSI \(=15.0\) & 0.02496 & 0.02496 & 0.00000 \\
\hline PSI \(=30.0\) & 0.02294 & 0.02294 & 0.00000 \\
\hline PSI \(=45.0\) & 0.01916 & 0.01916 & 0.00000 \\
\hline PSI \(=60.0\) & 0.01445 & 0.01445 & 0.00000 \\
\hline PSI \(=75.0\) & 0.00913 & 0.00913 & 0.00000 \\
\hline PSI \(=90.0\) & 0.00540 & 0.00540 & 0.00000 \\
\hline PSI \(=105.0\) & 0.00501 & -0.00190 & 0.00501 \\
\hline PSI \(=120.0\) & 0.00678 & -0.00190 & 0.00678 \\
\hline PSI \(=135.0\) & 0.00782 & 0.00782 & 0.00000 \\
\hline PSI \(=150.0\) & 0.01200 & 0.01200 & 0.00000 \\
\hline PSI \(=165.0\) & 0.01810 & 0.01810 & 0.00000 \\
\hline PSI \(=180.0\) & 0.02258 & 0.02258 & 0.00000 \\
\hline PSI \(=195.0\) & 0.02401 & 0.02401 & 0.00000 \\
\hline PSI \(=210.0\) & 0.02321 & 0.02321 & 0.00000 \\
\hline PSI \(=225.0\) & 0.02237 & 0.02237 & 0.00000 \\
\hline PSI \(=240.0\) & 0.02187 & 0.02187 & 0.00000 \\
\hline PSI \(=255.0\) & 0.02179 & 0.02179 & 0.00000 \\
\hline PSI \(=270.0\) & 0.02188 & 0.02188 & 0.00000 \\
\hline PSI \(=285.0\) & 0.02240 & 0.02240 & 0.00000 \\
\hline PSI \(=300.0\) & 0.02318 & 0.02318 & -0.00682 \\
\hline PSI \(=315.0\) & 0.02236 & 0.02236 & -0.00591 \\
\hline PSI \(=330.0\) & 0.02376 & 0.02376 & -0.00342 \\
\hline PSI \(=345.0\) & 0.02578 & 0.02578 & -0.00919 \\
\hline PSI \(=360.0\) & 0.02614 & 0.02614 & -0.00595 \\
\hline
\end{tabular}
\begin{tabular}{crrr}
\multicolumn{3}{c}{ RADIAL STATION } & \multicolumn{1}{c}{ RAKE GEOMETRY } \\
RMAX & RO & \multicolumn{1}{c}{ RI } & \multicolumn{1}{c}{ RGI } \\
0.87000 & 0.87000 & 0.34000 & 0.25000 \\
0.87000 & 0.87000 & -0.60000 & 0.25000 \\
0.87000 & 0.87000 & -0.60000 & -0.60000 \\
0.87000 & 0.87000 & -0.60000 & -0.60000 \\
0.87000 & 0.87000 & -0.60000 & -0.60000 \\
0.81500 & 0.81500 & -0.60000 & -0.60000 \\
0.61500 & 0.61500 & -0.60000 & 0.56000 \\
0.56000 & 0.97000 & 0.56000 & 0.25000 \\
0.42000 & 0.97000 & 0.42000 & 0.25000 \\
0.42000 & 0.42000 & -0.60000 & 0.34000 \\
0.71000 & 0.71000 & -0.60000 & -0.60000 \\
0.75000 & 0.75000 & -0.60000 & -0.60000 \\
0.78500 & 0.78500 & -0.60000 & -0.60000 \\
0.89000 & 0.89000 & -0.60000 & -0.60000 \\
0.89000 & 0.89000 & -0.60000 & 0.25000 \\
0.91000 & 0.91000 & -0.60000 & 0.25000 \\
0.91000 & 0.91000 & -0.60000 & 0.34000 \\
0.91000 & 0.91000 & -0.60000 & 0.34000 \\
0.91000 & 0.91000 & -0.60000 & 0.49500 \\
0.89000 & 0.89000 & -0.60000 & 0.49500 \\
0.89000 & 0.89000 & 0.34000 & 0.42000 \\
0.89000 & 0.89000 & 0.34000 & 0.34000 \\
0.89000 & 0.89000 & 0.42000 & 0.25000 \\
0.87000 & 0.87000 & 0.25000 & 0.25000 \\
0.87000 & 0.87000 & 0.34000 & 0.25000
\end{tabular}

GO = OUTBOARD PEAK, GI = INBOARD PEAK, GMAX = PEAK WITH MAXIMUM MAGNITUDE
CORRESPONDING RADIAL STATIONS OF PEAKS ARE RO,RI,RMAX; VALUE OUTSIDE BLADE IF PEAK NOT FOUND RGI = SPAN STATION OF INBOARD CIRCULATION PEAK
FOR DUAL PEAK WAKE MODEL: CONVERGENCE OF WAKE GEOMETRY ITERATION REQUIRES RGI = RI

GAMMA \(=\) BOUND CIRCULATION / OMEGA*R**2
REFERENCE ROTOR RADIUS R = 24.7240 FT, ROTATIONAL SPEED OMEGA = 260.000 RPM; TIP SPEED VTIP = OMEGA*R

UNIFORM INFLOW
ROTOR VELOCITY MUX \(=0.3733\) MUY \(=-0.0001\) \(M U Z=0.0458\)

\section*{ROTOR LOADS \\ \(\mathrm{CT}=0.0071050\) CMY \(=0.0000143\)} CMX \(=-0.0000142\)

INDUCED VELOCITY LAMBDAI \(=0.01641\) LAMBDAX \(=0.02282\) LAMBDAY \(=0.01218\)

INFLOW FACTOR
TIP PATH PLANE NORMAL
\(\mathrm{UX}=0.1217\)
UY \(=0.0016\)
\(\mathrm{UZ}=-0.9926\)

COSINE ANGLE BETWEEN WAKE AND GROUND NORMAL, COSE \(=0.0000\) WAKE VELOCITY RATIO, W(FAR)/V(DISK) \(=2.0000\) INFLOW STATE
MUZ/LH =

MU /LH =
0.768
6.263

LAMBDAI/LH = 0.275
\begin{tabular}{ll}
\((\) MUZ + LAMBDAI \() / L H=\) & 1.044 \\
\((\) MUZ \(+2 *\) LAMBDAI \() /\) LH \(=\) & 1.319
\end{tabular}

LH \(=0.05960\)

ROTOR VELOCITY AND ROTOR LOADS IN TIP-PATH PLANE AXES; TIP PATH PLANE NORMAL IN INERTIAL AXES
ROTOR LOADS: THRUST COEFFICIENT CT, PITCH MOMENT COEFFICIENT CMY, ROLL MOMENT COEFFICIENT CMX

INDUCED VELOCITY: MEAN LAMBDAI, LONGITUDINAL GRADIENT LAMBDAX=KX*LAMBDAI, LATERAL GRADIENT LAMBDAY=KY*LAMBDAI INFLOW FACTOR: LONGITUDINAL KX, LATERAL KY
INFLOW STATE: LH \(=\operatorname{SQRT}(C T / 2), \mathrm{MU}=\operatorname{SQRT}(\) MUX** \(2+\mathrm{MUY} * * 2)\)

ADVANCE RATIO MU, INFLOW RATIO LAMBDA = VELOCITY / VTIP
REFERENCE ROTOR RADIUS \(R=24.7240\) FT, ROTATIONAL SPEED OMEGA \(=260.000\) RPM; TIP SPEED VTIP \(=\) OMEGA*R

RIGID WAKE GEOMETRY
WAKE CONVECTION VELOCITY, TIP-PATH PLANE AXES DMUX \(=0.0000\) DMUY \(=0.0000\) DMUZ \(=-0.0171\) WAKE CONVECTION VELOCITY INERTIAI AXES

TRIM SOLUTION
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ANALYSIS FRAMES
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\begin{tabular}{|c|c|}
\hline FRAME = INERTIAL & GRAVITY In +Z DIRECTION ( +Z AXIS OF INERTIAL FRAME DOWNWARD) \\
\hline FRAME \(=\) AIRFRAME & \begin{tabular}{l}
AXES OF TEST MODULE (CONSTRAINED) \\
X-AXIS FORWARD (WIND POSITIVE FROM X DIRECTION); ORIGIN AT CENTER OF TEST MODULE
\end{tabular} \\
\hline FRAME \(=\) ROTOR N & \begin{tabular}{l}
NONROTATING AXES OF ROTOR \\
Z IS SHAFT AXIS (POSITIVE THRUST DIRECTION); ORIGIN AT CENTER OF ROTATION (HUB) X-AXIS DOWNSTREAM; AZIMUTH ANGLE MEASURED FROM X-AXIS, IN DIRECTION OF ROTOR ROTATION ROTOR 1 ROTATES ABOUT Z-AXIS CLOCKWISE
\end{tabular} \\
\hline FRAME \(=\) ROTOR N ROTATING & \begin{tabular}{l}
ROTATING AXES OF ROTOR \\
Z IS SHAFT AXIS (POSITIVE THRUST DIRECTION); ORIGIN AT CENTER OF ROTATION (HUB) X-AXIS DOWNSTREAM FOR ZERO AZIMUTH
\end{tabular} \\
\hline FRAME \(=\) ROTOR N BLADE M & \begin{tabular}{l}
ROTATING AXES OF BLADE \\
Z IS SHAFT AXIS (POSITIVE THRUST DIRECTION); ORIGIN AT CENTER OF ROTATION (HUB) Y-AXIS SPANWISE (POSITIVE OUTWARD); X-AXIS CHORDWISE ROTOR 1 X-AXIS POSITIVE TOWARD LEADING EDGE
\end{tabular} \\
\hline AXeS Of blade element & \begin{tabular}{l}
z-AXIS IS PARALLEL TO SHAFT AXIS; ORIGIN AT INBOARD END OF ELEMENT \\
X-AXIS SPANWISE (POSITIVE OUTWARD); Y-AXIS CHORDWISE \\
ROTOR 1 Y-AXIS POSITIVE TOWARD TRAILING EDGE \\
(APPROXIMATE; ACTUALLY X IS BEAM AXIS OF ELEMENT, Y-AXIS IS IN hUB PLANE)
\end{tabular} \\
\hline AXES Of TRAILING edge flap & X-AXIS IS FLAP HINGE AXIS (SPANWISE, POSITIVE OUTWARD); ORIGIN AT FLAP HINGE Y-AXIS IN HUB PLANE (CHORDWISE); Z-AXIS APPROXIMATELY PARALLEL TO SHAFT AXIS ROTOR 1 y-AXIS POSITIVE TOWARD TRAILING EDGE \\
\hline
\end{tabular}
*************
OUTPUT OF PART = ROTOR 1 BLADE POSITION SENSOR
OUTPUT \(=\) ROTOR 1 BLADE 4 POS FLJOINT
    OUTPUT KIND \(=\) COMPONENT OUTPUT
STRUCTURAL DYNAMIC COMPONENT, SENSOR 4; BLADE POSITION SENSOR, AT JOINT
UNITS: DISPLACEMENT \(=\mathrm{X} /\) RADIUS (BLADE RADIUS \(R=24.724 \mathrm{FT}\) ), ANGLE = DEG
ELEMENT DEFINITION: FLAP AND LAG HINGE ROTATION
SOLUTION KIND = TIME DOMAIN PERIODIC; NUMBER OF PERIODS = 1
PERIOD 1 (PRIMARY) = ROTOR 1
AZIMUTH FOR ZERO TIME \(=0.000\) DEG
FREQUENCY \(=27.227\) RAD/SEC
NUMBER OF TIME STEPS IN SOLUTION
DIMENSIONLESS TIME PSI: AZIMUTH \(=\) PSI, TIME \(=(\) AZIMUTH - (AZIMUTH FOR ZERO TIME) \() / O M E G A\)
VARIABLE RESPONSE TYPE, LENGTH = 2; NOMINAL KIND = REST POSITION
MOTION \(=\) NOMINAL + DOF
\begin{tabular}{llllllll} 
ELEMENT & NAME & & & & & LABEL & NOMINAL \\
1 & RTR & 1 & BLD & 4 & POS & JOINT & FLAP \\
2 & RTR & 1 & BLD & 4 & POS & JOINT & LAG
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline MOTION VALUE & FLAP & LAG \\
\hline \multicolumn{3}{|l|}{TIME HISTORY} \\
\hline MEAN & 4.21460 & 11.1450 \\
\hline MAXIMUM & 5.41003 & 11.4647 \\
\hline MINIMUM & 2.85712 & 10.8017 \\
\hline 1/2 PEAK-TO-PEAK & 1.27646 & 0.331498 \\
\hline \multicolumn{3}{|l|}{TIME HISTORY (AZIMUTH IN DEG)} \\
\hline PSI \(=0.000\) & 3.54013 & 10.8481 \\
\hline PSI \(=15.000\) & 2.99181 & 10.8047 \\
\hline ............. & ....... & ....... \\
\hline PSI \(=345.000\) & 4.19055 & 10.8888 \\
\hline PSI \(=360.000\) & 3.54013 & 10.8481 \\
\hline HARMONICS (FROM & 24 TIME STEPS) & \\
\hline MEAN & 4.21460 & 11.1450 \\
\hline COSINE 1 & -0.259376E-01 & -0.307642 \\
\hline SINE 1 & \(0.157176 \mathrm{E}-01\) & 0.168354 \\
\hline -•••......... & ....... & ....... \\
\hline COSINE 10 & \(0.455621 \mathrm{E}-02\) & 0.139678 \\
\hline SINE 10 & -0.683559E-02 & -0.521005 \\
\hline
\end{tabular}

\section*{*****}

TRIM SOLUTIO
*************

OUTPUT OF PART = ROTOR 1 WING SENSOR
-••
....
OUTPUT = ROTOR 1 WING 4 SENSOR 6
OUTPUT KIND = COMPONENT OUTPUT
OUTPUT SEOUENCE
102
RESPONSE KIND = OUTPUT
OUTPUT = ROTOR 1 WING 4 SENSOR 6 RESPONSE SEOUENCE

1010
COMPONENT \(=\) ROTOR 1 WING 4

LIFTING LINE WING COMPONENT, AERODYNAMIC SENSOR 7
QUANTITY DEFINITION: SECTION FORCE; Z COMPONENT, WING SECTION AXES
UNITS: M**2 * SECTION COEFFICIENT FORM -- FORCE/LENGTH M**2*CF = F / .5*RHO*CS**2*C
VELOCITY SCALE = SOUND SPEED CS \(=1116.45 \mathrm{FT} / \mathrm{SEC}\); LENGTH SCALE \(=\) SECTION CHORD C; TIME SCALE TREF = CS/U
AIR DENSITY RHO \(=0.002378\) SLUG/FT**3; WING SPAN BREF \(=19.7792 \mathrm{FT}\); SPAN INTEGRAL SCALED WITH WING SPAN BREF ELEMENTS: QUANTITY AT 19 SPAN STATIONS, AND SPAN INTEGRAL

SOLUTION KIND = TIME DOMAIN PERIODIC; NUMBER OF PERIODS = 1
PERIOD 1 (PRIMARY) \(=\) ROTOR 1
AZIMUTH FOR ZERO TIME \(=0.000\) DEG
FREQUENCY \(=27.227\) RAD/SEC \(=4.3333 \quad \mathrm{HZ}=260.00 \quad \mathrm{RPM}\)
NUMBER OF TIME STEPS IN SOLUTION = 24
DIMENSIONLESS TIME PSI: AZIMUTH = PSI, TIME = (AZIMUTH - (AZIMUTH FOR ZERO TIME))/OMEGA



\section*{NUMBER OF SPAN STATIONS \(=19\), RAMIN \(=0.20000\), RAMAX \(=1.00000\)}


TRIM REFERENCE UPDATE
************************
SYSTEM AND FUNCTIONALITY ************************

FLUTT
FLUTTER SOLUTION PROCEDURE
FIRST LOOP = ROTORCRAFT
            PARTS SOLVED BEFORE CHILD LOOPS
            PART = GUST
                CHILD LOOP = ROTOR 1
                                    MODULES WRITTEN AT END OF LOOP
                                    MODULE \(=\) MODES \(=\) ROTOR 1 BLADE 4 FLUTTER
LOOP = ROTOR 1
CHILD LOOP = ROTOR 1 BLADE 4
LOOP = ROTOR 1 BLADE 4 RORE CHILD LOOP
            PARTS SOLVED BEFORE CHILD LOOPS
            PART = ROTOR 1 BLADE 4 CONTROL
            DE PART = ROTOR 1 BLADE 4
DE PART = ROTOR 1 BLADE 4

INITIALIZATION FOR FLUTTER
GENERATE SUBMATRICES FOR FLUTTER PART RECORDS GENERATE SUBMATRICES FOR FLUTTER MODES RECORDS
---------------------------------------------
FLUTTER LOOP ITERATION AND PART SOLUTION

FLUTTER SOLUTION

MODE SET = ROTOR 1 BLADE 4 FLUTTER

MODAL TRANSFORMATION FOR PART = ROTOR 1 BLADE 4
ONLY MODE SHAPES ARE USED IN SOLUTION PROCEDURE; FREQUENCY AND MODAL MASS ARE PRESENTED FOR INFORMATION MODE SHAPE DISPLACEMENT (FLAP, LAG, AXIAL, LINEAR JOINT) SCALED WITH ROTOR RADIUS; ROTATION (PITCH, ANGULAR JOINT) IN RADIANS ROTOR ROTATIONAL SPEED \(=27.227 \mathrm{RAD} / \mathrm{SEC}=4.3333 \mathrm{HZ}=260.000 \mathrm{RPM}\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{MODE NUMBER} & \multirow[b]{2}{*}{LABEL} & \multirow[b]{2}{*}{UTILIZATION} & \multicolumn{3}{|l|}{MODAL FREQUENCY-------------} & \multirow[t]{2}{*}{\begin{tabular}{l}
MODAL \\
MASS
\end{tabular}} & \multirow[t]{2}{*}{MODAL DAMPING \(\mathrm{G}=2\) *CRITICAL} & \multirow[t]{2}{*}{\[
\begin{aligned}
& \text { MODE } \\
& \text { FLAP }
\end{aligned}
\]} & \multirow[t]{2}{*}{SHAPE AT LAG} & \multirow[t]{2}{*}{\[
\begin{array}{r}
\text { TIP---- } \\
\text { PITCH }
\end{array}
\]} & \multirow[b]{2}{*}{AXIAL} \\
\hline & & & PER REV & RAD / SEC & HZ & & & & & & \\
\hline 1 & M5E1 & DYNAMIC & 0.2215 & 6.032 & 0.9600 & 1238.8 & 0.0000 & 0.000 & 1.000 & 0.047 & -0.246 \\
\hline 2 & M5E2 & DYNAMIC & 1.0218 & 27.822 & 4.4279 & 1189.1 & 0.0000 & 1.000 & -0.016 & 0.093 & -0.061 \\
\hline 3 & M5E3 & DYNAMIC & 2.8566 & 77.777 & 12.3785 & 100.78 & 0.0000 & -0.339 & -0.077 & 1.000 & 0.040 \\
\hline 4 & M5E4 & DYNAMIC & 4.7114 & 128.279 & 20.4163 & 57.449 & 0.0000 & -0.119 & 0.187 & 1.000 & -0.077 \\
\hline 5 & M5E5 & DYNAMIC & 5.1700 & 140.764 & 22.4033 & 0.64482 & 0.0000 & -0.049 & -0.013 & 1.000 & 0.007 \\
\hline 6 & M5E6 & DYNAMIC & 5.9562 & 162.170 & 25.8101 & 1.2431 & 0.0000 & -0.013 & -0.001 & 1.000 & 0.001 \\
\hline 7 & M5E7 & DYNAMIC & 10.0124 & 272.609 & 43.3871 & 12.790 & 0.0000 & 0.082 & 0.008 & 1.000 & -0.002 \\
\hline 8 & M5E8 & DYNAMIC & 13.9513 & 379.855 & 60.4558 & 225.49 & 0.0000 & -0.100 & 0.363 & 1.000 & -0.239 \\
\hline 9 & M5E9 & DYNAMIC & 15.3666 & 418.388 & 66.5886 & 5.9904 & 0.0000 & -0.102 & -0.018 & 1.000 & 0.009 \\
\hline 10 & M5E10 & DYNAMIC & 19.1520 & 521.455 & 82.9922 & 0.52240 & 0.0000 & -0.022 & -0.005 & 1.000 & 0.003 \\
\hline 11 & M5E11 & STATIC & 25.9464 & 706.445 & 112.4343 & 6.9881 & 0.0000 & 0.058 & 0.007 & 1.000 & -0.001 \\
\hline 12 & M5E12 & STATIC & 28.2659 & 769.599 & 122.4855 & 119.49 & 0.0000 & -0.078 & 0.205 & 1.000 & -0.368 \\
\hline 13 & M5E13 & STATIC & 29.8656 & 813.156 & 129.4177 & 2733.3 & 0.0000 & -0.144 & 1.000 & 0.742 & 0.438 \\
\hline 14 & M5E14 & STATIC & 34.3678 & 935.738 & 148.9273 & 0.57636 & 0.0000 & -0.031 & -0.005 & 1.000 & 0.003 \\
\hline 15 & M5E15 & STATIC & 46.3886 & 1263.029 & 201.0173 & 6.2591 & 0.0000 & -0.120 & 0.013 & 1.000 & -0.023 \\
\hline 16 & M5E16 & STATIC & 46.6911 & 1271.266 & 202.3282 & 37.387 & 0.0000 & -0.098 & -0.166 & 1.000 & 0.157 \\
\hline 17 & M5E17 & STATIC & 57.2403 & 1558.488 & 248.0411 & 2.2187 & 0.0000 & -0.003 & -0.002 & 1.000 & 0.002 \\
\hline 18 & M5E18 & STATIC & 62.4466 & 1700.241 & 270.6017 & 0.64587 & 0.0000 & 0.027 & 0.004 & 1.000 & -0.002 \\
\hline 19 & M5E19 & STATIC & 75.9098 & 2066.807 & 328.9425 & 7.3313 & 0.0000 & 0.016 & -0.068 & 1.000 & 0.100 \\
\hline 20 & M5E20 & STATIC & 79.0744 & 2152.970 & 342.6559 & 0.14960 & 0.0000 & -0.002 & 0.002 & 1.000 & -0.003 \\
\hline & . . . . & -•.... & . . . . . & -........ & . . . . . . & . . . . . & . . . . . & .... & .... & ..... & . . . \\
\hline 38 & M5E38 & STATIC & 897.9336 & 24448.162 & 3891.0457 & 45.895 & 0.0000 & 0.018 & .0 .3
-0.312 & \(\underline{1.000}\) & - 0.277
-0.27 \\
\hline 39 & M5E39 & STATIC & 1840.1981 & 50103.328 & 7974.1919 & 3.0502 & 0.0000 & -0.030 & 0.251 & 1.000 & 0.305 \\
\hline
\end{tabular}

OUT-OF-PLANE COMPONENT OF MODE SHAPE

************
FLUTTER SOLUTION
****************
---------------------------
DESCRIPTION OF VARIABLES
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{12}{|l|}{DEGREES OF FREEDOM} \\
\hline VARIABLE & LABEL & TRANSFORMS & COMPONENT & \multicolumn{3}{|l|}{VECTOR} & \multicolumn{5}{|l|}{ELEMENT} \\
\hline 1 & R1MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 1 \\
\hline 2 & R2MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 2 \\
\hline 3 & R3MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 3 \\
\hline 4 & R4MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 4 \\
\hline 5 & R5MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 5 \\
\hline 6 & R6MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 6 \\
\hline 7 & R7MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 7 \\
\hline 8 & R8MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 8 \\
\hline 9 & R9MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 9 \\
\hline 10 & R10MB4 & & MODE & ROTOR & 1 BLADE & 4 FLUTTER & ROTOR & 1 BLADE & 4 & MODE & 10 \\
\hline
\end{tabular}

TRANSFORMS: MULTIBLADE COORDINATES (0,1C,1S,...,NC,NS,N/2), ROTATING TO NONROTATING (COS,SIN), SYMMETRIC/ANTISYMMETRIC
\(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~\)
ANALYSIS OF SYSTEM OF CONSTANT COEFFICIENT LINEAR DIFFERENTIAL EQUATIONS梀 FLUTTER SOLUTION

SECOND ORDER FORM
NUMBER OF DEGREES OF FREEDOM, MX = 10
UMBER OF CONTROLS, NUMBER OF OUTPUT
\(\mathrm{MY}=0\)
REQUENCY SCALE FACTOR (RAD/SEC) = 27.2271
OUASISTATIC REDUCTION PERFORMED WITH OUTPUT RESIDUAL
MATRIX INPUT FORM: DIMENSIONAL TIME
MATRIX OUTPUT FORM: DIMENSIONAL TIME
EIGENVALUES WRITTEN TO PLOT FILE
NUMBER OF EQUATION SETS ANALYZED = 1

DEGREES OF FREEDOM
\begin{tabular}{|c|c|c|c|c|c|}
\hline NAME & UTILIZATION & SYMMETRY & RIGID & ORDER & DEGREE OF FREEDOM AS OUTPUT \\
\hline R1MB4 & DYNAMIC & Вотн & No & FROM EQUATIONS & \\
\hline R2MB4 & DYNAMIC & BOTH & NO & FROM EQUATIONS & \\
\hline R3MB4 & DYNAMIC & ВОтн & NO & FROM EQUATIONS & \\
\hline R4MB4 & DYNAMIC & вотн & NO & FROM EQUATIONS & \\
\hline R5MB4 & DYNAMIC & Вотн & NO & FROM EQUATIONS & \\
\hline R6MB4 & DYNAMIC & Bот & No & FROM EQUATIONS & \\
\hline R7MB4 & DYNAMIC & BOTH & NO & FROM EQUATIONS & \\
\hline R8MB4 & DYNAMIC & Вотн & NO & FROM EQUATIONS & \\
\hline R9MB4 & DYNAMIC & BOTH & NO & FROM EQUATIONS & \\
\hline R10MB4 & DYNAMIC & BOTH & NO & FROM EQUATIONS & \\
\hline
\end{tabular}
ANALYSIS OF SYSTEM OF CONSTANT COEFFICIENT LINEAR DIFFERENTIAL EQUATIONS
EQUATION SET NUMBER 4: COMPLETE EQUATIONS,

FLUTTER SOLUTION

FIRST ORDER FORM
\(\begin{array}{lllll}\text { NUMBER OF STATES, MXX }= & 20 & \\ \text { NUMBER OF CONTROLS, MVV }= & 0 & \\ \text { NUMBER OF OUTPUT, MYY }= & 0 & \\ \text { NUMBER OF QUASISTATIC DEGREES OF } & \text { FREEDOM }= & 0 \\ \text { NUMBER OF FIRST ORDER STATES }= & 0\end{array}\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{4}{*}{NAMES OF STATES:} & D (R1MB4 & )/DT & D (R2MB4 & )/DT & D (R3MB4 & )/DT & D (R4MB4 & )/DT & D (R5MB4 & )/DT \\
\hline & D (R6MB4 & )/DT & D (R7MB4 & )/DT & D (R8MB4 & )/DT & D (R9MB4 & )/DT & D (R10MB4 & )/DT \\
\hline & R1MB4 & & R2MB4 & & R3MB4 & & R4MB4 & & R5MB4 & \\
\hline & R6MB4 & & R7MB4 & & R8MB4 & & R9MB4 & & R10MB4 & \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline \multicolumn{2}{|r|}{LAMBDA} & NATURAL \\
\hline REAL & IMAG & FREQUENCY \\
\hline PER/REV & PER/REV & PER/REV \\
\hline -0.08655 & 17.24194 & 17.24216 \\
\hline -0.08655 & -17.24194 & 17.24216 \\
\hline -0.19559 & 14.36533 & 14.36666 \\
\hline -0.19559 & -14.36533 & 14.36666 \\
\hline -1.96705 & 11.05222 & 11.22590 \\
\hline -1.96705 & -11.05222 & 11.22590 \\
\hline -0.17269 & 8.70923 & 8.71094 \\
\hline -0.17269 & -8.70923 & 8.71094 \\
\hline -0.00692 & 5.69705 & 5.69705 \\
\hline -0.00692 & -5.69705 & 5.69705 \\
\hline -0.00605 & 4.65397 & 4.65398 \\
\hline -0.00605 & -4.65397 & 4.65398 \\
\hline -1.26894 & 4.39969 & 4.57903 \\
\hline -1.26894 & -4.39969 & 4.57903 \\
\hline -0.04942 & 2.73792 & 2.73837 \\
\hline -0.04942 & -2.73792 & 2.73837 \\
\hline -0.00241 & 1.03347 & 1.03347 \\
\hline -0.00241 & -1.03347 & 1.03347 \\
\hline -0.10787 & 0.19612 & 0.22383 \\
\hline -0.10787 & -0.19612 & 0.22383 \\
\hline
\end{tabular}

LAMBDA \(=\) REAL \(+I *\) IMAG, FREQUENCY \(=\) IMAG, NATURAL FREQUENCY \(=\) MAG(LAMBDA) DAMPING RATIO \(=\) - REAL (LAMBDA) / MAG(LAMBDA)
PERIOD \(=1 /\) FREQUENCY(HZ), TIME CONSTANT \(=-1 / \operatorname{REAL}(L A M B D A)\)
TIME TO HALF (DOUBLE) AMPLITUDE \(=0.6931472\) * TIME CONSTANT FREQUENCY SCALE FACTOR \(=27.2271\) RAD/SEC
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|r|}{LAMBDA} & DAMPING & FREQUENCY & PERIOD & TIME & TIME HALF \\
\hline REAL & IMAG & RATIO & & & CONSTANT & AMPLITUDE \\
\hline 1/SEC & RAD/SEC & & HZ & SEC & SEC & SEC \\
\hline -2.3566 & 469.4488 & 0.0050 & 74.715 & 0.0134 & 0.4243 & 0.2941 \\
\hline -2.3566 & -469.4488 & 0.0050 & 74.715 & 0.0134 & 0.4243 & 0.2941 \\
\hline -5.3253 & 391.1267 & 0.0136 & 62.250 & 0.0161 & 0.1878 & 0.1302 \\
\hline -5.3253 & -391.1267 & 0.0136 & 62.250 & 0.0161 & 0.1878 & 0.1302 \\
\hline -53.5572 & 300.9203 & 0.1752 & 47.893 & 0.0209 & 0.0187 & 0.0129 \\
\hline -53.5572 & -300.9203 & 0.1752 & 47.893 & 0.0209 & 0.0187 & 0.0129 \\
\hline -4.7018 & 237.1274 & 0.0198 & 37.740 & 0.0265 & 0.2127 & 0.1474 \\
\hline -4.7018 & -237.1274 & 0.0198 & 37.740 & 0.0265 & 0.2127 & 0.1474 \\
\hline -0.1883 & 155.1142 & 0.0012 & 24.687 & 0.0405 & 5.3113 & 3.6815 \\
\hline -0.1883 & -155.1142 & 0.0012 & 24.687 & 0.0405 & 5.3113 & 3.6815 \\
\hline -0.1647 & 126.7144 & 0.0013 & 20.167 & 0.0496 & 6.0701 & 4.2075 \\
\hline -0.1647 & -126.7144 & 0.0013 & 20.167 & 0.0496 & 6.0701 & 4.2075 \\
\hline -34.5495 & 119.7911 & 0.2771 & 19.065 & 0.0525 & 0.0289 & 0.0201 \\
\hline -34.5495 & -119.7911 & 0.2771 & 19.065 & 0.0525 & 0.0289 & 0.0201 \\
\hline -1.3456 & 74.5458 & 0.0180 & 11.864 & 0.0843 & 0.7432 & 0.5151 \\
\hline -1.3456 & -74.5458 & 0.0180 & 11.864 & 0.0843 & 0.7432 & 0.5151 \\
\hline -0.0655 & 28.1383 & 0.0023 & 4.478 & 0.2233 & 15.2683 & 10.5832 \\
\hline -0.0655 & -28.1383 & 0.0023 & 4.478 & 0.2233 & 15.2683 & 10.5832 \\
\hline -2.9370 & 5.3399 & 0.4819 & 0.850 & 1.1767 & 0.3405 & 0.2360 \\
\hline -2.9370 & -5.3399 & 0.4819 & 0.850 & 1.1767 & 0.3405 & 0.2360 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|}
\hline & CPU TIME (SEC) & PERCENT OF CASE & NUMBER OF CALLS & TIME PER CALL (SEC) \\
\hline CASE & xxx.xxxx & 100.00 & 1 & xxx. xxxxx \\
\hline INPUT & xxx.xxxx & 0.74 & 1 & xxx. xxxxx \\
\hline INITIALIZE & xxx. xxxx & 0.55 & 3 & xxx.xxxxx \\
\hline TRIM & xxx.xxxx & 97.15 & 1 & xxx.xxxxx \\
\hline FLUTTER & xxx.xxxx & 1.56 & 1 & xxx.xxxxx \\
\hline TRIM PART SOLUTION & xxx.xxxx & 97.08 & 9938 & xxx.xxxxx \\
\hline TRIM DIFFERENTIAL EQUATIONS & xxx. xxxx & 0.78 & 5 & xxx. xxxxx \\
\hline TRIM OUTPUT & xxx.xxxx & 0.02 & 6 & xxx.xxxxx \\
\hline FLUTTER PART SOLUTION & xxx. xxxx & 1.55 & 36 & xxx. xxxxx \\
\hline FLUTTER DIFFERENTIAL EQUATIONS & xxx.xxxx & 1.50 & 12 & xxx. xxxxx \\
\hline FLUTTER OUTPUT & xxx.xxxx & 0.00 & 2 & xxx. xxxxx \\
\hline MODES & xxx.xxxx & 0.59 & 3 & xxx.xxxxx \\
\hline
\end{tabular}

DATA VECTOR STATISTICS
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline DATA & NUMBER OF & & DIRECTORY & MAXIMUM & \multicolumn{2}{|l|}{MINIMUM} & TOTAL & NUMBER \\
\hline VECTOR & SECTIONS & & SPACE & USED SPACE & FREE SP & ACE & LENGTH & OF PACKS \\
\hline DTVCCM & 35972 & REAL & & 1930981 (19\%) & 6899298 & (69\%) & 10000000 & 1 \\
\hline DTVCCM & 35972 & INTEGER & 431695 & 1129210 (11\%) & 8347586 & (83\%) & 10000000 & 1 \\
\hline DTVCCM & 35972 & CHARACTER & & 258996 (26\%) & 737684 & (74\%) & 1000000 & 1 \\
\hline CORECM & 4763 & REAL & & 64751 ( 6\%) & 935249 & (94\%) & 1000000 & 0 \\
\hline CORECM & 4763 & INTEGER & 57187 & 160820 (16\%) & 839180 & (84\%) & 1000000 & 0 \\
\hline CORECM & 4763 & CHARACTER & & 402336 (20\%) & 1519331 & (76\%) & 2000000 & 0 \\
\hline TABLCM & 1 & REAL & & 7267 ( 1\%) & 992733 & (99\%) & 1000000 & 0 \\
\hline TABLCM & 1 & INTEGER & 43 & 259 ( 0\%) & 99741 & (99\%) & 100000 & 0 \\
\hline TABLCM & 1 & CHARACTER & & 106 ( 0\%) & 99894 & (99\%) & 100000 & 0 \\
\hline SHLLCM & 14 & REAL & & 31863 ( 8\%) & 368137 & (92\%) & 400000 & 0 \\
\hline SHLLCM & 14 & INTEGER & 199 & 9595 (10\%) & 90405 & (90\%) & 100000 & 0 \\
\hline SHLLCM & 14 & CHARACTER & & 11591 (12\%) & 88409 & (88\%) & 100000 & 0 \\
\hline
\end{tabular}

END OF CASE NUMBER 1
CLOSE FILE 43

ANALYSIS EXIT (END OF PROGRAM)

\section*{7-2 Trim Convergence}

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapter "Convergence and Efficiency"

PARTITIONED, ITERATIVE PROCEDURE CONSTRUCTED FOR TRIM TASK
system divided into parts, which solve subset of equations for periodic or constant response
loops iterate between part solutions, until converged system solution obtained
figure 1 shows basic loops and principal interface variables

TRIM LOOPS:
WAKE, TRIM, CIRCULATION, AND MOTION
regulator loop can be introduced, between trim and circulation loops

TRIM PARTS USING HARMONIC OR TIME FINITE ELEMENT SOLUTION METHOD:

ROTOR, AIRFRAME, AND DRIVE TRAIN


Figure 7-1a Basic loops of trim analysis (optional regulator loop not shown).


Figure 7-1b Levels of wake loop.

\section*{EACH LOOP AND EACH PART MUST BE CONVERGED RESULTS ARE UNRELIABLE IF ANY ITERATION IS NOT CONVERGED}
warnings are produced if iteration does not converge
trace during loop or part solution (input parameters TRACEL and TRACEP) can be used to obtain more information about convergence

SEVERE DIVERGENCE CAN PRODUCE FLOATING POINT OVERFLOW

\section*{ITERATIVE LOOP AND PART SOLUTIONS} FOR EACH ITERATION, CONVERGENCE CONTROLLED BY

RELAXATION FACTOR
TOLERANCE
NUMBER OF ITERATIONS
trim loop also depends on initial conditions, and derivative matrix identification

\section*{FIRST ACTION TO IMPROVE OR ACHIEVE CONVERGENCE IS TO REDUCE RELAXATION FACTOR}
for efficiency, relaxation factor should be as large as possible
good practice to establish required value for any new class of problems
if relaxation factor is reduced significantly, it may also be necessary to:
increase number of iterations allowed
reduce tolerance

\title{
IF USUAL RANGE OF RELAXATION FACTOR VALUES DOES NOT PRODUCE CONVERGENCE, THEN OTHER PARAMETERS CAN BE CONSIDERED
}

\section*{TOLERANCE IN TEST FOR CONVERGENCE CONTROLS ACCURACY OF SOLUTION, AFTER CONVERGENCE HAS BEEN ACHIEVED}
interpretation of tolerance value depends on weights in convergence test
for efficiency, tolerance should be as large as possible
establish required value for any new class of problems
by reducing tolerance until answer of interest is no longer affected, based on appropriate engineering judgement
tolerance required for accurate solution can be affected by other parameters
relaxation factor functions by reducing difference between successive iterations
so if convergence test is applied to that difference, may need to reduce tolerance whenever relaxation factor is reduced
otherwise convergence test satisfied and iteration stops simply because small relaxation factor slows rate of convergence

CONVERGENCE OF LOOP CAN DEPEND ON ACCURACY OF SOLUTION OF ITS CHILD LOOPS AND PARTS
so may be necessary to use small tolerance in order to achieve convergence of some parent loop

IF SOLUTION DOES NOT CONVERGE BUT DOES NOT EXHIBIT OSCILLATORY OR DIVERGENT BEHAVIOR
may be sufficient to just increase maximum number of iterations allowed

CONVERGENCE PROBLEMS CAN ALSO REFLECT PHYSICAL LIMITATIONS OF SYSTEM DEFINED

POSSIBLE TO DEFINE PROBLEM THAT DOES NOT HAVE A SOLUTION
must change definition of system, or change definition of solution task

\section*{WAKE LOOP}

\section*{LEVELS OR STAGES}

\section*{LEVEL 1: UNIFORM INFLOW}

LEVEL 2: NONUNIFORM INFLOW WITH PRESCRIBED/RIGID WAKE GEOMETRY

LEVEL 3: NONUNIFORM INFLOW WITH FREE WAKE GEOMETRY

\section*{NO TEST FOR CONVERGENCE}
execute specified number of iterations for each stage required number of stages and number of iterations should be established for any new class of problems
increase wake level and number of iterations until answer of interest is no longer affected, based on appropriate engineering judgement
results of all wake levels can be examined in single job

ANALYSIS LEVEL
increase analysis level to improve accuracy
nonuniform inflow often affects results
free wake geometry generally important at low speed (below advance ratio of about \(\mu=0.25\) )

\section*{WAKE GEOMETRY ITERATIONS}
often just one iteration required for each stage
increase number of iterations of last stage to improve convergence
increasing number of iterations of earlier stages is less effective
more than one iteration for uniform inflow has no effect
iteration is required if anything is being calculated by inner loops that will change wake geometry
usually not necessary to iterate on wake geometry if trim loop specifies rotor speed, thrust, and tippath plane angle-of-attack
so in practice, single iteration of each required stage often sufficient
some problems need iteration in wake geometry loop (and relaxation factor on wake geometry)
if rotor is at fixed collective pitch (rather than trimmed to specified thrust), then overall geometry of wake is not known in advance, only after thrust calculated
or in hover and at low speeds, where rotor loading is very sensitive to wake geometry
then 4-6 iterations often required
momentum or vortex theory suggests value of 0.5 for relaxation factor
or with dual-peak wake model
span station of inboard peak \(r_{G I}\) used to calculate influence coefficients
\(r_{G I}\) is wake geometry feature, potentially requiring additional wake loop iterations for convergence typically one more iteration of last stage required with dual-peak model
but effect of \(r_{G I}\) is not large, and other wake geometry features updated at same time
convergence of \(r_{G I}\) assessed by comparing value used to calculate influence coefficients with value found in current circulation solution

TRIM LOOP

\section*{RELAXATION FACTOR}
reduce relaxation factor to achieve convergence
use value just small enough to prevent oscillation of Newton-Raphson solution
for simple problem of trimming thrust in hover, relaxation factor of 0.7 can be satisfactory
in extreme operating conditions, value of 0.25 or less may be required

\section*{PERTURBATION IDENTIFICATION}
improve perturbation identification of derivative matrix in order to achieve convergence
using second-order difference often very effective
or change sign or magnitude of perturbation;
or recalculate matrix more often

\section*{RECURSIVE IDENTIFICATION}
use recursive identification of derivative matrix in order to improve convergence
or change weight parameter
if parameter variance matrix of recursive update diverges, then perturbation identification should be performed more often, to re-initialize recursive identification

\section*{TRIM ITERATIONS}
increase maximum number of iterations in order to achieve convergence

\section*{INITIAL CONDITIONS}
use better initial conditions for trim variables to achieve convergence
trim loop unlikely to converge if started too far from the solution
these initial conditions are for uniform inflow stage, even if nonuniform inflow is being used (uniform inflow solution initializes nonuniform inflow stage)
first task in project dealing with new problem is to establish set of initial conditions for operating conditions of interest
initial conditions can be obtained from nearby operating condition
can perform operating condition sweep of several cases in a single job
starting from some case that easily converges, such as hover with analysis using trim solution from previous case as initial conditions
can also use solution obtained by turning off stall in rotor model as initial conditions for full solution

\section*{CONVERGENCE OF INNER LOOPS}
may need to decrease tolerance on child loops and parts in order to achieve convergence of trim loop
if inner loop is not sufficiently converged, trim iteration may become erratic or even diverge
inner loops may not be converged during perturbation process to identify derivative matrix
resulting derivative matrix can be bad, so trim iteration diverges even if inner loops finally converge
then must achieve better convergence of inner loops during perturbation process

\section*{PHYSICAL LIMITATIONS}

CONVERGENCE PROBLEMS CAN ALSO REFLECT PHYSICAL LIMITATIONS OF ROTORCRAFT DEFINED; FOR EXAMPLE:
operating condition specified may be beyond capabilities of rotor to produce thrust or propulsive force
in free flight at high gross weight, high turn rate, or high speed
or in wind tunnel at high thrust
trim problem should be changed: use specified power (free flight) or fixed collective pitch (wind tunnel)
operating condition specified may involve high main rotor power, and hence be beyond capabilities of tail rotor to produce thrust to balance main rotor torque
tail rotor thrust limit can be increased, perhaps by suppressing stall
or tail rotor can be replaced by auxiliary force
connection of pilot's controls to rotor controls may not be consistent with specified trim problem
for example: tiltrotor aircraft in wind tunnel, not possible to trim lateral flapping with lateral stick since pilot's controls are not connected to rotor lateral cyclic
control matrix should be changed
for trim with yaw angle in free flight, be sure to use velocity yaw angle or sideslip (SIDE), not yaw of body axes relative inertial frame (YAW)

\section*{REGULATOR LOOP}

REGULATOR ALGORITHM IS SIMILAR TO NEWTONRAPHSON METHOD USED FOR TRIM LOOP

SO INFLUENCES OF PARAMETERS ON CONVERGENCE AND ACCURACY ARE SIMILAR

PHYSICAL LIMITATIONS
CONVERGENCE PROBLEMS CAN ALSO REFLECT PHYSICAL LIMITATIONS OF ROTORCRAFT DEFINED; FOR EXAMPLE:
controls might be used by both trim loop and regulator loop
cost function objective defined by tolerance may not be physically achievable

\section*{CIRCULATION LOOP}

\section*{RELAXATION FACTOR}
reduce relaxation factor to achieve convergence
relaxation factor of \(\lambda=0.10\) or 0.05 often required for nonuniform inflow models
smaller relaxation factor required with three-quarter chord collocation points than with quarter chord points

\section*{CIRCULATION ITERATIONS}
increase maximum number of iterations in order to achieve convergence

\section*{AERODYNAMIC SPAN STATIONS}
revise aerodynamic span stations to achieve convergence spanwise oscillation in sign of angle-of-attack can be produced if small panel is next to large panel panel widths should vary smoothly along blade span
in nonuniform inflow model, can have tip vortex rollup occur inboard of blade tip, at span station \(r_{T V}\) any aerodynamic panels outboard of this rollup location will see large upwash, with significant influence on loads and perhaps convergence problems as well
generally \(r_{T V}=e_{\text {tip }}\) should be used, unless inboard rollup is physically reasonable (as with highly tapered tip)

\section*{NEAR WAKE CORE SIZE}
with highly distorted geometry, some close encounters between collocation points and vortex lines of near wake can still occur
especially near a reverse flow boundary can increase core size of near wake line segments in order to avoid singularities and achieve convergence
but this core size has no physical significance
its value must not influence solution for loading

\section*{HOVERING ROTOR AT LOW THRUST}
circulation convergence problems will be encountered for hovering rotor at low thrust
circulation iteration then only asymptotically convergent with normal inflow and wake models avoid problem by fixing wake geometry in calculation of induced velocity at low thrust
for uniform inflow stage: change induced velocity model, using special inflow equation
for nonuniform inflow stage: wake geometry specified directly, rather than using prescribed geometry (which depends on thrust)
resulting inflow is not correct, but is small at low thrust

\section*{CIRCULATION CONVERGENCE IN NONUNIFORM INFLOW CALCULATION}
if use relatively small value for maximum number of circulation iterations (for efficiency), circulation may not be converged for uniform inflow stage, or in early trim iterations of nonuniform inflow stage
acceptable as long as trim converges, and circulation is converged at end of nonuniform inflow stage
uniform inflow gives approximate solution, which may be reasonable even with circulation not converged
check ratio of induced power to ideal power
uniform inflow result initializes prescribed wake geometry stage
mean induced velocity determines prescribed/rigid wake geometry
if uniform inflow not sufficiently converged, may need more wake iterations in nonuniform inflow stage
if wake geometry is important, will need free wake geometry stage in any case

RESULTS ARE UNRELIABLE IF CIRCULATION LOOP NOT CONVERGED AT END OF ANY ITERATION OF NONUNIFORM INFLOW STAGE IN WAKE LOOP

\section*{MOTION LOOP}

\section*{RELAXATION FACTOR}
reduce relaxation factor to achieve convergence relaxation factor \(\lambda<1\) may be required when airframe vibration is being calculated
if airframe does not produce vibratory hub motion, no iteration is required, and relaxation factor should be 1.0

\section*{MOTION ITERATIONS}
increase maximum number of iterations in order to achieve convergence

\section*{ROTOR, AIRFRAME, AND DRIVE TRAIN PARTS}

\section*{RELAXATION FACTOR}
reduce relaxation factor to achieve convergence
small relaxation factor may be required for rotor with significant elastic motion, or operating in dynamic stall

\section*{DAMPING FACTOR}
estimate of aerodynamic damping can be added to both sides of differential equations, in order to improve convergence
harmonic solution procedure treats this damping same on both sides of equations, so damping value does not affect solution
typical value is 0.5 (fraction critical)

\section*{PART SOLUTIONS}
increase maximum number of iterations of part solution in order to achieve convergence
convergence can also be affected by strategy for update of reference and matrices for differential equation solutions

\section*{7-3 Trim Efficiency}

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapter "Convergence and Efficiency"

\section*{KEY TO EFFICIENT UTILIZATION OF ANALYSIS:}

USE MINIMUM COMPLEXITY AND SIZE REQUIRED FOR PARTICULAR CONFIGURATION AND PROBLEM
first phase of an investigation is best devoted to establishing minimum model required

RECOMMENDATIONS REGARDING MODELLING REQUIREMENTS ARE BASED ON PAST EXPERIENCE WITH ANALYSIS
the more a new configuration or new problem differs from that experience, the more important it is to check these recommendations

\section*{PARAMETERS CONTROLLING SOLUTION PROCEDURE CAN INFLUENCE EFFICIENCY}
relaxation factor should be as large as possible
tolerance should be as large as possible
in order to achieve converged and accurate solution with minimum number of iterations

\section*{MODEL SIZE}

\title{
ROTORCRAFT SHELL PROVIDES NUMBER OF PARAMETERS THAT CONTROL MODEL SIZE
}

\section*{AZIMUTH STEP}
azimuth step of 15 degrees typically used in solution procedures (24 azimuth steps per revolution)

\section*{NUMBER OF HARMONICS}
typically 7 to 10 harmonics of motion required to calculate rotor structural loads
airframe vibration occurs at harmonics \(p N\) ( \(N=\) number of blades
typically \(p=1\) or 2 required to calculate vibration generally 10 per-rev is limit of harmonic content required for performance and loads problems, consistent with:
frequency content of excitation
bandwidth of solution (one-half number of azimuth steps
frequency range of rotor modes used
fewer harmonics, perhaps just 1/rev, may be sufficient for some problems
zero harmonics (just the mean motion) appropriate and efficient for ideal hover
but rotorcraft in hover can still have tip-path plane tilt relative shaft, so at least \(1 / \mathrm{rev}\) needed
and \(1 /\) rev solution in rotating frame required to obtain mean nonrotating frame forces and moments acting on airframe

\section*{NUMBER OF BLADE MODES AND ELEMENTS}
when blade modes are used in solution procedure:
typically 4 or 5 bending modes required to calculate structural loads, and 1 or 2 torsion modes
so typically 3 to 5 elastic elements needed to model blade (whether modes used or not)
generally modes with natural frequency below 10 per-rev may be important
but selecting modes based on frequency alone gives many flap modes, less lag modes, and few torsion modes
fewer modes and elements, perhaps just rigid flapping, may be sufficient for some problems

\section*{AIRFRAME ELASTIC MODES}
airframe elastic modes required to calculate vibration typically need those modes with natural frequency beyond \(p N /\) rev vibration of interest

\section*{MODEL COMPLEXITY}

\title{
ROTORCRAFT SHELL PROVIDES NUMBER OF PARAMETERS THAT CONTROL MODEL COMPLEXITY
}
low complexity
airframe structure one rotor in wind tunnel constant rotational speed no elastic modes
airframe aerodynamics
no aerodynamics
no interference velocity
rotor structure
blade pitch control no control flexibility
pitch link flexibility
flap hinge
rigid blade elements
few elastic elements
few degrees of freedom
rigid wing
rotor aerodynamics
no aerodynamics
few blade panels
static stall
rotor wake
uniform inflow
rigid or prescribed geometry
single-peak wake model
few wake spirals
high complexity
rotorcraft in free flight
drive train model
airframe elastic modes
airframe aerodynamics
airframe flow field at rotor
swashplate mechanism
pitch link flexibility swashplate flexibility
flap and lag hinges, pitch bearing elastic blade elements many elastic blade elements many elastic degrees of freedom general wing
rotor blade aerodynamics many aerodynamic panels dynamic stall
nonuniform inflow
free wake geometry dual-peak wake model many wake spirals

\section*{7-4 Trim Parameters}

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapters "Class = TRIM" and "Class = TRIM ROTOR"

\section*{TABLE SUMMARIZES PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF TRIM SOLUTION}
and default values in analysis
\begin{tabular}{llllll}
\hline \hline loop & iterations & tolerance & relaxation & perturbation & parameters \\
\hline \hline wake & ITERP \(=1\) & & RELAXP \(=1\). & & LEVEL=2*1 \\
& ITERF \(=1\) & & RELAXF \(=1\). & & \\
\hline trim & ITERT \(=40\) & TOLERT \(=1\). & RELAXT \(=.5\) & DELTA \(=1\). & OPPID \(=1\) \\
& & & & MPID=40 \\
& & & & OPRID \(=0\) \\
& & & & ALPHA \(=.5\) \\
& & & & initial conditions \\
\hline circulation & ITERC \(=60\) & TOLERC \(=1\). & RELAXC \(=.1\) & \\
\hline motion & ITERM \(=40\) & TOLERM \(=2\). & RELAXM \(=1\). & \\
\hline \hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline part & iterations & tolerance & relaxation & perturbation & parameters \\
\hline \multirow[t]{3}{*}{all} & & & & & MPSI \(=\) MPSIH \(=24\) \\
\hline & & & & & MPSIAV=4 \\
\hline & & & & & OPHRMP \(=2\) \\
\hline \multirow[t]{7}{*}{rotor} & \(\mathrm{MREVR}=40\) & TOLERR=2. & RELAXR=1. & DELTAR=. 01 & MHARMR \(=10\) \\
\hline & & & & & DAMPR=. 5 \\
\hline & & & & & OPUPDT=1101 \\
\hline & & & & & NUPDTR=4 \\
\hline & & & & & OPPART=1 \\
\hline & & & & & METHOD=1 \\
\hline & & & & & OPMODE=0 \\
\hline \multirow[t]{2}{*}{airframe} & MREVA \(=40\) & TOLERA \(=2\). & RELAXA \(=1\). & DELTAA \(=.01\) & MHARMA \(=10\) \\
\hline & & & & & DAMPA \(=.5\) \\
\hline \multirow[t]{2}{*}{drive train} & MREVD \(=40\) & TOLERD \(=2\). & RELAXD=1. & DELTAD \(=.01\) & MHARMD \(=10\) \\
\hline & & & & & DAMPD \(=.5\) \\
\hline
\end{tabular}

\section*{7-5 Transient Parameters}

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapters "Class = TRANSIENT" and "Class = TRANSIENT ROTOR"

TABLE SUMMARIZES PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF TRANSIENT SOLUTION
and default values in analysis
\begin{tabular}{|c|c|c|c|c|c|}
\hline part & iterations & tolerance & relaxation & perturbation & parameters \\
\hline \multirow[t]{6}{*}{integration} & & & & & TRESP \(=.02\) \\
\hline & & & & & OPUPDT=101 \\
\hline & & & & & MPSIAV \(=4\) \\
\hline & & & & & METHOD=1 \\
\hline & & & & & ALPHA \(=-.05\) \\
\hline & & & & & OPINIT=1 \\
\hline \multirow[t]{4}{*}{rotorcraft} & MITERS \(=100\) & TOLERS \(=2\). & RELAXS=1.,. 5 & DELTAS \(=.01\) & TSTEPS \(=.01\) \\
\hline & & & & & DAMPS \(=0\). \\
\hline & & & & & OPAVS \(=1\) \\
\hline & & & & & NUPDTS \(=4\) \\
\hline
\end{tabular}

\section*{7-6 Flutter Parameters}

REFERENCE: CAMRAD II Documentation, Volume VI, Rotorcraft Input; Chapters "Class = FLUTTER" and "Class = FLUTTER ROTOR"

TABLE SUMMARIZES PARAMETERS CONTROLLING CONVERGENCE AND ACCURACY OF FLUTTER SOLUTION and default values in analysis
\begin{tabular}{lll}
\hline \hline loop or part & perturbation & parameters \\
\hline \hline loops & & MPSIAV \(=12\) \\
rotor & DELTAR \(=.01\) & OPMODE \(=0\) \\
airframe & DELTAA \(=.01\) & \\
drive train & DELTAD \(=.01\) & \\
\hline \hline
\end{tabular}

\section*{7-7 CAMRAD II Experience}

\section*{7-7.1 Convergence and Accuracy of Trim Loops}

\section*{GENERAL}
if analysis converges:
circulation loop and parts require many iterations first time they are executed, but few iterations during convergence of outer loops
if analysis does not converge:
inner loops always run to maximum number of iterations
so symptom of divergence is job taking much more computational time than expected

TRIM
relaxation factor RELAXT
hover, trim thrust with collective: 0.7 usually good
otherwise 0.25 may be needed
tolerance TOLERT
sometimes need smaller value than default (1.), in order to obtain accurate propulsive force and parasite power
for trim convergence, may need one or more of following parameter changes:

OPPID \(=2\)
DELTA \(=-1\).
OPRID \(=1\) and ALPHA \(=0.9\)
MPID \(=5\)
operating condition sweep in multicase job
second-order difference (OPPID = 2) often very effective
initial conditions
good initial conditions may be required for convergence (especially in forward flight, or with elastic blade)
and more efficient (fewer trim iterations needed)
needed for uniform inflow solution (which then initializes the nonuniform inflow level)
can obtain by running uniform inflow cases first
multicase jobs with OPINIT \(=1\) or 7
then run nonuniform inflow, with good initial conditions for uniform inflow level
with new configuration, must obtain control setting for uniform inflow trim
use simplified model, to get approximate results with minimum computational time
some of simplifications to consider: uniform inflow trim
trim input: LEVEL \(=1\), RELAXC \(=0.5\) few azimuth stations, few harmonics
trim input: \(\mathrm{MPSI}=8\), MHARMR \(=1\), MHARMA \(=0\)
two blades modes or no blade elasticity
trim rotor input: DOFM \(=2^{*} 1,38^{*} 2\), DOFB \(=3^{*} 1,9^{*} 0\)
simple blade structure
rotor structure input: \(\mathrm{OPBEAM}=0, \mathrm{KNODE}=0\), LOCKPL \(=0\)
few aerodynamic panels
rotor aerodynamics input: NPANEL \(=10\), REDGE =
\(.12, .28, .42, .54, .64, .73, .81, .87, .92, .96,1\).
or 5 panels, or even 2 panels for fast (but approximate) result
resulting control settings can be used to start trim calculations without these simplifications

\section*{CIRCULATION}
relaxation factor RELAXC
uniform inflow: 0.5 usually good
nonuniform inflow: need 0.10 (default) or 0.05
number of iterations ITERC
20 will be faster (if trim converges, and circulation is converged at end of solution)
tolerance TOLERC (default value 1.0)
windmilling rotor: 0.05 needed (uniform inflow)
vibratory hub loads calculations: may need smaller tolerance to improve consistency and repeatability of results (nonuniform inflow)
more sensitive than performance calculations to convergence of aerodynamics
for example, TOLERC \(=0.2\) and \(\operatorname{ITERC}=200\), instead of default TOLERC \(=1.0\)
ducted fan: 0.01 needed (and RELAXC \(=0.1\) )
if large number of iterations (say several hundred) are needed, it may be symptom of bad wake model
circulation convergence can be essential to convergence of trim iteration
erratic trim iteration can be symptom of insufficient circulation convergence

\section*{MOTION}
rotor part or motion loop convergence can be more difficult with airframe degrees of freedom in the trim solution (for vibration calculation)
relaxation factor RELAXM: may need value less than 1.0
or run multicase job with OPINIT \(=7\)
vary parameters that control vibration level
for example, vary airframe moments of inertia and elastic mode generalized masses, from very large down to actual values

\section*{CONVERGENCE WITH ELASTIC BLADE IN FORWARD FLIGHT} may need one or more of following parameter changes for convergence of trim loops and parts
\begin{tabular}{llll} 
& parameter & default value & for convergence \\
\hline trim & RELAXT & 0.5 & 0.2 \\
\hline circulation & RELAXC & 0.1 & 0.05 \\
& ITERC & 60 & 200 \\
\hline rotor & RELAXR & 1.0 & \(0.5-0.3\) \\
& DAMPR & 0.5 & 1.0 \\
& blade modes & no & yes \\
\hline
\end{tabular}

\section*{WHEN CONVERGENCE PROBLEMS ARE ENCOUNTERED, ALWAYS TRY SMALLER RELAXATION FACTORS FIRST}
use of trim blade modes and RELAXR \(=0.5-0.3\) often required for convergence with elastic blade in forward flight
blade modes help by reducing order (neglect very high frequency modes, quasistatic solution for moderate frequency modes)
and by allowing introduction of modal damping GDAMPM if eventually converges, faster with smaller ITERC (say 20) default for uniform inflow (RELAXC \(=0.5\) ) often satisfactory (RELAXC \(=0.1\) is default for nonuniform inflow level)
smaller tolerances (TOLERC and TOLERR) may be needed to improve convergence of outer loop
convergence can also be affected by strategy for update of reference and matrices for differential equation solutions
for example, \(\operatorname{OPUPDT}(3)=1\) and NUPDTR \(=10\) might help a case where the rotor part solution diverges after many iterations of the circulation loop
rotor part solution usually require linear interpolation to evaluate harmonics

OPHRMP = 2 (default), not Fourier interpolation
can use time domain finite element solution method for rotor part, instead of harmonic solution method

METHOD \(=2\), with blade modes and RELAXR \(=0.5-0.2\);
and perhaps smaller RELAXN (set by core input)
initialization of response by simpler previous case (using multicase job with OPINIT = 6 or 7 ) often effective for very difficult convergence problems

WITH ELASTIC BLADE IN HOVER, TYPICALLY CAN USE DEFAULT VALUES

\section*{7-7.2 Structural Dynamics of Rotor Blade}

\section*{NUMERICAL PROBLEMS CAN BE ENCOUNTERED WHEN USING BLADE MODES WITH QUASISTATIC ELIMINATION OF HIGH FREQUENCY MODES}

GENERAL SYMPTOMS:
negative eigenvalues of modal mass or spring matrix ; perhaps singular spring matrix of quasistatic modes (KLL)

SYMPTOMS IN TRIM TASK: divergence
SYMPTOMS IN FLUTTER TASK: erratic or incorrect eigenvalues

\section*{MODAL ANALYSIS SET UP SUCH THAT LOW FREQUENCY MODES ARE MOST ACCURATE}
theoretical approach allows good results to be obtained from high frequency modes, as long as they are not so inaccurate that information is lost in the modal transformation

CAN AVOID THESE NUMERICAL PROBLEMS BY NEGLECTING THE HIGH FREQUENCY MODES
use DOFM \(=0\) instead of DOFM \(=2\)
in trim rotor, transient rotor, and flutter rotor input as required
may still need quasistatic response for moderate frequency modes, in order to retain control response

\section*{TRIM SOLUTION MAY CONVERGE WITHOUT USING BLADE MODES}
avoid these issues, as well as any question of accuracy with modal truncation; but can not use modal damping without modes

\section*{7-7.3 Selection of Modes for Elastic Rotor Blade}

WHEN ANALYZING ELASTIC BLADE, NUMBER OF BLADE MODES REQUIRED FOR ACCURACY SHOULD BE DETERMINED FOR ANY NEW PROBLEM
often blade modes needed to improve convergence
then DOFM \(=1\) (dynamic), 2 (quasistatic), or 0 (neglect) must be specified

\section*{USE OF QUASISTATIC MODES CAN AFFECT TRIM AND PERFORMANCE SOLUTION}
since the blade is rotating, quasistatic approximation can neglect inertial and Coriolis loads that would cancel spring and centrifugal loads in rotor total mean loads
locked pitch bearing (LOCKP \(=0\) ) means a quasistatic pitch degree of freedom, if blade modes are not used

\section*{FREE FLIGHT TRIM:}
quasistatic approximation can lead to hub force \(\neq\) rotor aerodynamic propulsive force (plus blade weight term)
then trim loop convergence (hub force = airframe drag) does not give rotor propulsive force \(=\) airframe drag

\section*{WIND TUNNEL TRIM:}
even with rotor aerodynamic thrust and/or propulsive force trimmed to specified values, rotor power (from torque at hub node) can be wrong
then profile power ( \(P_{o}=P-P_{i}-P_{p}\) ) also wrong, and very different from \(P_{o}\) calculated from blade section forces

\section*{WITH CONTROLLED PITCH JOINT, PITCH MODE MAY BE AT HIGH FREQUENCY}
especially with locked pitch joint (rotorcraft shell creates large pitch spring)
if neglect pitch mode, control loads will be large; and beam elements inboard of pitch bearing may twist, in place of pitch joint motion
check pitch motion in mode shapes used (flutter task)
check trim pitch joint motion using position sensor
NEED ENOUGH MODES (MAYBE QUASISTATIC) TO CAPTURE CONTROL INPUT
with elastic blade model, can use realistic pitch spring and unlocked pitch joint

\section*{ACCURATE BLADE LOADS CALCULATION MAY REQUIRE MANY MODES}
neglecting modes may make structure appear too stiff, and hence blade loads too high
particularly with multiple load path
example: stiff pitch case of bearingless rotor
PROBLEM ELIMINATED BY USING MORE MODES (MAYBE QUASISTATIC)

\section*{7-7.4 Bearingless Rotors}

\section*{STRUCTURAL DYNAMIC CHARACTERISTICS OF BEARINGLESS ROTOR BLADE}

MANY ELASTIC VARIABLES WITH SMALL INERTIA (PARTICULARLY IN FLEXBEAM) mass matrix is poorly-conditioned or nearly singular so numerical problems may be encountered in calculating eigenvalues for flutter analysis

STATIC REDUCTION APPROPRIATE can use modal transform and modal truncation (zero or quasistatic) to implement static reduction calculating modal transform itself requires eigenanalysis

\section*{FREE VIBRATION CALCULATIONS}
use to check physical and numerical aspects of structural dynamics

\section*{PARAMETERS THAT MAY IMPROVE NUMERICAL CHARACTERISTICS}
neglect high frequency modes (DOFM \(=0\) not 2 )
NDOFU \(=1\) for flexbeam; consistent with constant tension force along element length, which nearly true with small flexbeam mass
flexbeam MASS and ITHETA not too small
NINTEG large enough for accurate integration

\section*{CONSTRAINT PROVIDED BY PITCH LINK ELIMINATES A TORSION DEGREE OF FREEDOM}
structural dynamics poorly-conditioned if choose wrong torsion degree of freedom to eliminate
shell identifies torsion degree of freedom of softest flexbeam element (largest \(\int_{0}^{\ell} d x / G J\) )
user should define flexbeam nodes so shell finds correct element (or use core input to specify correct element)

\section*{TYPICAL MODEL FOR STRUCTURAL DYNAMIC CALCULATIONS}

\section*{BLADE ELEMENTS}

1 element for cuff or torque tube (inboard of EFB)
4 elements for blade (outboard of EFB)
standard elastic degrees of freedom (3222)

\section*{FLEXBEAM ELEMENTS}

2 elastic elements
plus rigid elements at hub and blade attachment
standard elastic degrees of freedom (3222 or 1222)
FEWER BLADE AND FLEXBEAM ELASTIC ELEMENTS MAY GIVE ACCURATE RESULTS FOR FUNDAMENTAL FLAP AND LAG MODES

\section*{7-7.5 Rotor Wake Model}

\section*{CONSIDER DUAL-PEAK MODELS OF FAR WAKE ROLLUP (OPFW = 1 OR 2)}

CAN HAVE LOADING DISTRIBUTION WHERE BLADE LIFT IS NEGATIVE JUST ON LAST AERODYNAMIC PANEL
depending on blade twist and operating condition then not appropriate to assume that tip vortex strength is determined by outboard peak
and calculations can be sensitive to very small changes in tip loading

CAN HAVE LOCALIZED NEGATIVE LOADING PRODUCED BY STRONG BLADE-VORTEX INTERACTIONS

\section*{DUAL-PEAK MODEL NOT APPROPRIATE IN SUCH CASES}
can produce convergence problems for circulation loop can produce poor performance calculations

CONTROL IDENTIFICATION OF DUAL-PEAK LOADING BY LIMITING SEARCH FOR CIRCULATION PEAKS
use RGMAX \(=0.95\) or so

\section*{BEST FOR USER TO CHOOSE WAKE MODEL}
make initial runs with OPFW \(=0\) (single-peak model using maximum circulation)
examine each job for extensive negative tip loading, and rerun with OPFW \(=2\) (dual-peak model) as required

\section*{7-7.6 Geometry Definition for Structural Dynamic Components}

CAN CREATE NEW COMPONENTS AND INTERFACES USING CORE INPUT

MAY ENCOUNTER ERROR MESSAGE:
matrix singular
part differential equations
inverting PHIXOB
MAY BE CAUSED BY 180 DEGREE MISMATCH OF CONNECTION AXES (REST POSITION) ON TWO SIDES OF STRUCTURAL DYNAMIC INTERFACE
constraint equation singular since constructed using Rodrigues parameters

USE input PROGRAM TO DRAW AXES AT STRUCTURAL DYNAMIC INTERFACES
shows rest position of components

\section*{7-8 CAMRAD II Projects}

TYPICAL CAMRAD II PROJECT

\author{
DEFINE BASELINE INPUT FOR A ROTORCRAFT \\ SERIES OF CAMRAD II JOBS, VARYING: \\ operating condition \\ analysis parameters \\ rotorcraft parameters
}

\section*{BEGIN WITH CORRELATION: CALCULATIONS FOR SOMETHING THAT HAS ALREADY BEEN TESTED}
determine how to use analysis to perform required tasks define empirical models, establish analysis complexity

\section*{THEN PREDICTION: CALCULATIONS FOR NEW DESIGN}
must be aware of limits
analysis limitations determined by correlation
new designs may go beyond correlation
near limits: use analysis to establish sensitivity
past limits: need new tests (for correlation), perhaps new analysis methods

THEN VERIFICATION: CHECK PREDICTIONS AFTER TEST NEW ROTORCRAFT
usually develops into new correlation activity

FUNDAMENTAL APPROACH TO DEVELOPING CAMRAD II CALCULATION:

\author{
START SIMPLE, SMALL, FAST \\ BUILD UP TO COMPLEX, LARGE, SLOW
}
try new task or new configuration as soon as identify requirement
so have time to work on problems encounter

\section*{ASSISTANCE AVAILABLE FROM MAINTENANCE/SUPPORT CONTRACTOR}

ORGANIZE THE PROJECT
SET UP SYSTEM TO ORGANIZE ALL INPUT, JOB, OUTPUT FILES
need conventions for directories, file names, file extensions
airfoil and input files created by INPUT program have unique identification (time/date)

\section*{AIDS TO ORGANIZATION}

USE NAMELIST FILES OF BASELINE PARAMETERS FOR EACH ROTOR OR ROTORCRAFT
maintain standard files for organization
use these namelist files to generate shell input file, for configuration control of input data and to avoid long job namelists
keep namelist files readable
several scalars per line; matrices and arrays in appropriate block format
annotate namelist files document changes (leave superseded data in file)

USE CODE PARAMETER TO IDENTIFY CASES every case has unique identification (time/date)

\section*{KEEP INPUT AND JOBS FOR OLD PROJECTS}
computer files (backed up) and paper documentation, sufficient to duplicate work several years later

\section*{STEPS IN DEVELOPING CAMRAD II CALCULATION}
(1) PREPARE INPUT
template available for preparation of input data (file ztemplate.list)
previous work or sample jobs provide starting point CHECK INPUT
(2) DRAW GEOMETRY
script available to extract data needed to draw planform (file zplanform.com, runs INPUT program)
draw 3D geometry
CHECK GEOMETRY

\section*{(3) FREE VIBRATION CALCULATIONS}
flutter analysis, no aerodynamics
first independent blade
then entire rotor, drive train, etc
typical job for blade frequency calculation available (file zfrequency.list)

CHECK PHYSICAL AND NUMERICAL ASPECTS OF STRUCTURAL DYNAMICS
check frequencies
establish CAMRAD II model for accurate blade frequencies (with fewest possible elastic elements)

\section*{(4) RUN BASIC CASE}
start with no trim, uniform inflow
perhaps simplified model, or wind tunnel case (files zsimple.list and zwindtunnel.list)
hover then forward flight (file zhover.list)
(5) DETERMINE REQUIRED MODEL

\section*{FOR TYPICAL CASE, TO ESTABLISH PARAMETERS VALUES FOR CONVERGENCE AND ACCURACY}
investigate input parameters for which value or importance is not clear
turn things on to check assumptions about what is important
such as nonuniform inflow, free wake, degrees of freedom
turn things off for efficiency
use minimum model possible
vary tolerances, wake loop level and number of wake iterations to establish accuracy
based on engineering judgement regarding answer of interest

\section*{BUILD UP COMPLEXITY OF JOB}
add trim, nonuniform inflow, elastic motion as appropriate establish parameters required to achieve convergence
(6) UNIFORM INFLOW FOR REQUIRED OPERATING CONDITIONS establish initial conditions for trim loop
(7) RUN JOBS OF PROJECT
multicase jobs, varying:
operating condition
analysis parameters
rotorcraft parameters

CONVERGENCE PROBLEMS ARE OFTEN ENCOUNTERED for new problem or new configuration, analysis may not work at first
engineer must help the solution procedure ASSISTANCE AVAILABLE FROM MAINTENANCE/SUPPORT CONTRACTOR

\&END

!=========================
\&NLDEF class='TRIM', \&END
\&NLVAL

\&NLDEF class='TRIM ROTOR', name='ROTOR n', \&END
\&NLVAL
    OPMODE=0, DOFB=12*\#
    OPMODE \(=1\), DOFM \(=40 * \#\), GDAMPM \(=40 * \#\),
    MHSEN \(=1\), MCSEN \(=1\), MBSEN \(=1\), MPSEN \(=1\), MASEN \(=1\), MWSEN \(=1\),
! no blade modes
no blade modes
with blade mode
l rotor output
\&END
\& END
\(1=============================================================================================================================================1\)
\&NLDEF class='TRANSIENT',\&END
\&NLVAL

TIMEB=\#,TIMEE=\#,
METHOD=1,OPINIT=1
TRESP=\#,TSTEPS=\#
DOFA=6*\#, DOFM=\#, DOFD=8*\#
MSSEN=1,
\&END
\&NLDEF class='TRANSIENT ROTOR', name='ROTOR n',\&END \&NLVAL

OPWAKE=2, OPGEOM=2, OPVATR=1,OPVRTA=1,DOFL=3*1,
OPWAKE \(=3\), OPGEOM \(=2\), OPVATR \(=1\), OPVRTA \(=1\), DOFL \(=3 * 1\),
OPWAKE \(=4\), OPGEOM \(=2\),OPVATR \(=2\), OPVRTA \(=2\), DOFL \(=3 * 1\),
OPWAKE \(=4\), OPGEOM \(=2\),
OPMODE \(=0\), DOFB \(=12 * \#\)
OPMODE \(=1\), DOFM \(=40 * \#\), GDAMPM \(=40 * \#\),
MHSEN \(=1\), MCSEN \(=1\), MBSEN \(=1\), MPSEN \(=1\), MASEN \(=1\), MWSEN \(=1\),
! time range
! time range
! integration method
! time step
! time step
degrees of freedom
! airframe sensors
! nonuniform inflow
! nonuniform inf
! dynamic inflo
trim inflow
no blade modes
! with blade modes
! rotor output
\&NLDEF class='FLUTTER',\&END
\&NLVAL
    OPFLUT=1,OPMEAN=0,
    OPFLUT \(=0\), OPMEAN \(=1\),
    PPSTAB=1, OPMEAN=1,NRPRNT=2*1,NAPRNT=1,NDPRNT=1,
    DOFA=6*\#, DOFM=\#, DOFD=8*\#,
    CONP=5*\#, GUST=3*\#,
    MSSEN=1,
\&END
\&NLDEF class='FLUTTER ROTOR', name='ROTOR n',\&END
\&NLVAL
OPWAKE=3, OPVATR=1, OPVRTA=1, DOFL=3*1,
OPWAKE=4, OPVATR=2,OPVRTA=2,DOFL=3*1,
OPMODE \(=0\), DOFB \(=12 * \#\)
\(\operatorname{MHSEN}=1, \operatorname{MCSEN}=1, \operatorname{MBSEN}=1, \operatorname{MPSEN}=1, \operatorname{MASEN}=1\),
\&END
\(!=============================================\)
\(\& N L D E F\) class='AIRFRAME', type='STRUCTURE', \&END
\&NLVAL
TITLE='xxx',
CONFIG=0,OPFREE=0, OPAERO \(=0\),
\(\operatorname{CONFIG}=1, \operatorname{RGEAR}(2)=\#\),
CONFIG=2,
CONFIG=3,
CONFIG=4, ATILT=\#,
WEIGHT=\#, IXX=\#, IYY=\#, IZZ=\#, IXY=\#, IXZ=\#, IYZ=\#,
MASSR=2*\#,
FSCG=\#, BLCG=\#, WLCG=\#,
FSRTR=2*\#, BLRTR=2*\#, WLRTR=2*\#, ASHAFT=2*\#, ACANT=2*\#,
    FSWB=\#, BLWB=\#, WLWB=\#,
FSHT=\#, BLHT=\#, WLHT=\#,
    FSHT=\#, BLHT=\#,WLHT=\#,FSVT=\#,BLVT=\#,WLVT=\#,
    FSPIV=\#, BLPIV=\#, WLPIV=\#,ASPIV=\#,ADPIV=\#,
\&END
\&END
\&NLVAL
\&END
\&NLDEF class='AIRFRAME',type='CONTROL', \&END
\&NLVAL \&END
dynamic inflow
trim inflow
- no blade modes
! with blade mor
NLDEF
OPFLUT=1, OPMEAN=0,
Floquet theory, periodic coefficients
time invariant, averaged coefficients
flight dynamics
! degrees of freedom
airframe sensors
. variables as output
\&NLDEF clas
! description
! wind tunnel
! single mr/tr
! tandem
! coaxial
! tiltrotor
! inertia
! geometry
! tiltrotor
! CAMRAD/JA: IWB=>IWBL, IWBM=IWB+57.3*MOMOW/MOMAW SIDEA \(=>\) SIDEDA, ROLLA \(=>\) ROLLDA, YAWA \(=>\) YAWDA IHT=>IHTL; IVT=>IVTL; FETAIL=>1/EHTAIL
\&NLVAL \&END ! ==========
```

\&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR n',\&END
\&NLVAL
TITLE='xxx',
VTIPN=\#,
CONTRL=0,HINGE=3,PITCH=2,
CONTRL=0,}\mathrm{ ,HINGE=0,PITCH=2,
CONE=\#, EFLAP=\#, ELAG=\#,EPITCH=\#,
DROOP=\#, SWEEP=\#, EDS=\#
DLAG=\#,KPITCH=\#, LOCKP=\#
OPBEAM=2,DRELST=.04,KNODE=2,RNODE=.28,.59
OPBEAM=0,KNODE=0,OPWING=0,
GDAMPU=.01,GDAMPV=.01,GDAMPW=.01,GDAMPT=.01,
WIN=1,TWISIL=\#
NPROP=51
RPROP=0.,.02,.04,.06,.08,.10,.12,.14,.16,.18,.20,
.22,.24,.26,.28,.30,.32,.34,.36,.38,.40
.42,.44,.46,.48,.50,.52,.54,.56,.58,.60
.62,.64,.66,.68,.70,.72,.74,.76,.78,.80
.82,.84,.86,.88,.90,.92,.94,.96,.98,1.0
EA=51*0.,ZQC=51*0., ZC=51*0., ZI=51*0.,
XEA=51*0.,XQC=51*0.,XC=51*O.,XI=51*0.,
N-S1*)
KP=51*\#,KT=51*\#
EA=51*\#,
IFLAP=51*\#,EILAG=51**,GJ=51**
MASS=51*\#,ITHETA=51*\#,IPOLAR=51*\#,
\&END
\&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR n',\&END
\&NLVAL
NPANEL=20,
REDGE=.12,.20,.28,.35,.42,.48,.54,.59,.64,.69,
.anG=.12,.20,.28,.35,.42,.48,.54,.59,.64,.69,
NPROP=\#,
RPROP=0.,\#,1.,
CHORD=\#,ASWEEP=\#
SEN=5,OPREF=5*4,
QUANT= 5,25,35,82,82
DENT= 1, 0, 0, 0, 0,
AXIS= 3, 0, 0, 1, 3,
NAPLOT=1, 4, 1, 1, 1,
END

```

```

! description
! blade root (flap and lag hinges)
! blade root (hingeless)
! with PITCH=2, use LOCKP=0 or KPITCH=\# ( not DOFB=0)
! elastic blade analysis (align RNODE with REDGE)
! rigid blade analysis
! linear twist
! section properties
! TWISTA at inertial stations, not aerodynamic stations
! sqrt(Itheta/mR**2); CAMRAD/JA: KP=KT=sqrt(KP2)
! EI $\left(\mathrm{m} /\right.$ Itheta) $=\mathrm{EI} /{ }^{\prime}(\mathrm{R} * \mathrm{KP}) * * 2$
! aerodynamic panels
! aero properties (0, panel midpoints, 1)
! chord in ft or m
aerodynamic sensors
lambda
alpha, theta
$\mathrm{Fx}, \mathrm{Fz}$
PSCL= 2, 1, 1, 2, 2

```
\&NLDEF class='ROTOR',type='INFLOW', name='ROTOR n', \&END
\&NLVAL \&END
\&NLDEF class='ROTOR',type='WAKE',name='ROTOR n',\&END
\&NLVAL
OPSCEN=1,
OPSCEN \(=2\),TWIST=\#,RICWG=\#,
! forward flight wake
! hover wake
\&END
\&NLDEF class='ROTOR',type='WAKE', name='ROTOR n', \&END \&NLVAL

OPSCEN=0,
- turn off scenario
\(\mathrm{OPFW}=2\),
RFW=3., MFWG=3
! dual peak wake model
! wake extent

\section*{\&END}
\(!======================\)
\&NLDEF class='TABLES',\&END
\&NLVAL
AFTABL=0,
! same table all rotors
AFTABL \(=1\),
. separate tables
\&END
\(!===============================\)
\&NLDEF action='end of shell', \&END
\&NLDEF action='end of core',\&END
\(!\)
! ===================================================
\(!\) swashplate control and kinematics (shell input)
CAMRAD/JA: parameters xph,rph,phiph,phipl
\(\mathrm{XSP}=\mathrm{XPH}+\mathrm{HSP} / \mathrm{R} \tan (\mathrm{phipl}) \quad \mathrm{XPH}=\mathrm{XTO}+\mathrm{xph} \cos (\mathrm{phiph})\)
\(\begin{array}{ll}\text { YSP }=\mathrm{YPH} & \text { YPH } \\ \text { YSP rph }\end{array}\)
\(\mathrm{ZSP}=-\mathrm{HSP} / \mathrm{R}\)
\(\mathrm{KPL}=\mathrm{KPITCH} /(\mathrm{xph} * \mathrm{R}) * * 2\)
KPL \(=\) KPITCH \(\left(\right.\) xph * R) \({ }^{* * 2}\)
\&NLDEF class='AIRFRAME', type='STRUCTURE', \&END
\&NLVAL
HSP=\#, OPSPM=0, ! swashplate node
\&END
\&NLDEF class='ROTOR',type='STRUCTURE', name='ROTOR n', \&END
\&NLVAL
CONTRL=2,PITCH=1,KPITCH=0.,LOCKP=1, ! control system
\(\mathrm{XSP}=\#, \mathrm{YSP}=\#, \mathrm{ZSP}=\#, \mathrm{XPH}=\#, \mathrm{YPH}=\#, \mathrm{ZPH}=\#, \mathrm{EPH}=\#\), KPL=\#, LOCKPL=\#, LOCKSP=0,
\&END
\(!=============================================================================================================================1\)
\(!\)
hydraulic lag damper (core input; see Training Manual, Volume VII, Chapter 9)
! hydraulic lag damper (core input; see Training Manual, Volume VII, Chapter
! identify component ( \(n, m, k\) ), and spring/damper for lag joint (j)
\&NLDEF class='COMPONENT',type='BEAM', name='ROTOR \(n\) BLADE m ELEMENT \(k ', ~ \& E N D ~\)
\&NLVAL
\(\operatorname{CTYPE}(j)=3, \operatorname{CLIN}(j)=0 .\),
! linear + hydraulic damper
\(\operatorname{NCHYDA}(j)=0, \operatorname{CHYDA}(1, j)=x\),
\(\operatorname{NCHYDB}(j)=2, \operatorname{CHYDB}(1, j)=0 ., 0 ., y\),
! lag moment \(M=\min (x, y * r * * 2), r=\) lag rate
\({ }^{\text {END }}\)
! CAMRAD/JA: \(x=L D A M P M, y=L D A M P M / L D A M P R * * 2\)
```

file: zfrequency.list
!=====================================================================================
! blade frequency calculation
!====================================================================================
\&NLJOB \&END
!====================================================================================
\&NLDEF class='CASE',\&END
\&NLVAL FLTASK=1,CODE='FREQUENCY',\&END ! flutter task
\&NLDEF class='TRIM',\&END
\&NLVAL
VTIPIN=1,VTIP=\#,COLL=\#, ! operating condition
VTIPIN=2,RPM=\#,COLL=\#, ! operating condition
LEVEL=2*1,OPTRIM=0,
MHARMR=2*0,MHARMA=2*0,MHARMD=2*0, ! harmonics
MPSI=4,MPSIAV=1, ! axisymmetric
\&END
\&NLDEF class='TRIM ROTOR',name='ROTOR n',\&END
\&NLVAL
OPMODE=0,DOFB=12*1, ! no blade modes
OPMODE=1,DOFM=40*\#,GDAMPM=40*\#, ! with blade modes
MPSEN=1,
! blade position
\&END

```

```

\&NLDEF class='FLUTTER',\&END
\&NLVAL
OPBLD=1, ! independent blade
MPSIAV=1, ! no average
\&END
\&NLDEF class='FLUTTER ROTOR',name='ROTOR n',\&END
\&NLVAL
OPMODE=1,DOFM=10*1,30*0,GDAMPM=40*\#, ! blade modes
\&END
l=====================================================================================
\&NLDEF class='AIRFRAME',type='STRUCTURE', \&END
\&NLVAL OPAERO=0,\&END ! no aerodynamics
\&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR n',\&END
\&NLVAL OPAERO=0,\&END ! no aerodynamics
!=====================================================================================
\&NLDEF action='end of shell',\&END
\&NLDEF action='end of core',\&END

```
```

file: zhover.list
l========================================================================================
! hover analysis
!===================================================================================
\&NLDEF class='TRIM',\&END
\&NLVAL
LEVEL=2*1, ! uniform inflow
LEVEL=2*2, ! nonuniform inflow
CTTRIM=.08,MTRIM=1, ! wind tunnel trim
MNAME='CT/S ',VNAME='COLL ',
COLL=10.,LNGCYC=0., LATCYC=0.,
TOLERT=1.,RELAXT=.7, ! trim loop
MHARMR=2*0,MHARMA =2*0,MHARMD=2*0,MPSIAV=1, ! harmonicS
MPSI=4,
! uniform inflow only
NxPRNT=1,NxFILE=1,MxTIME=1, ! output
\&END
\&NLDEF class='TRIM ROTOR',name='ROTOR n',\&END
\&NLVAL
OPMODE=1,DOFM=40*\#,GDAMPM=40*\#, ! with blade modes
OPMODE=0,DOFB=12*\#, ! no blade modes
NxPRNT=1,NXFILE=1,MXTIME=1, ! output
\&END
l=====================================================================================
\&NLDEF class='FLUTTER',\&END
\&NLVAL MPSIAV=1,\&END ! no average of eqns
l======================================================================================
\&NLDEF class='AIRFRAME',type='STRUCTURE',\&END
\&NLVAL OPAERO=0,\&END ! no aerodynamics
!====================================================================================
!====================================================================================
! hover free wake
\&NLDEF class='TRIM',\&END
\&NLVAL
LEVEL=3,NWPRNT=1, ! free wake geometry
TOLERC=.2,ITERF=4,RELAXF=.5, ! wake convergence
MPSI=24, ! nonuniform inflow
\&END
\&NLDEF class='ROTOR',type='WAKE',name='ROTOR n',action='init',\&END
\&NLVAL
OPSCEN=2,TWIST=\#,RICWG=\#, ! hover scenario
\&END
\&NLDEF class='ROTOR',type='WAKE',name='ROTOR n',\&END
\&NLVAL
OPSCEN=0, ! turn off scenario
OPFWG=3, ! general method
! OPMCRC=0, ! mean circulation
\&END
!=====================================================================================

```
```

file: zsimple.list
!=====================================================================================
! simplified models
!===================================================================================
\&NLDEF class='TRIM',\&END
\&NLVAL
LEVEL=1,RELAXC=.5, ! uniform inflow
OPTRIM=0,
MPSI=8,
MHARMR=1,MHARMA=1,MHARMD=1, ! few harmonics
\&END
! ====================================================================================
\&NLDEF class='TRIM ROTOR',name='ROTOR n',\&END
\&NLVAL
DOFM=2*1,38*0,DOFB=3*1,9*0, ! degrees of freedom
\&END
!====================================================================================
\&NLDEF class='ROTOR',type='STRUCTURE',name='ROTOR n',\&END
\&NLVAL
OPBEAM=0,KNODE=0, ! rigid blade
OPWING=0, ! rigid wing
LOCKPL=0, ! locked pitch link
\&END
l=====================================================================================
\&NLDEF class='ROTOR',type='AERODYNAMICS', name='ROTOR n',\&END
\&NLVAL NPANEL=10,}\mathrm{ REDGE=.12,.28,.42,.54,.64,.73,.81,.87,.92,.96,1.,
\&END
\&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR n',\&END
\&NLVAL NPANEL=5,
\&END
\&NLDEF class='ROTOR',type='AERODYNAMICS',name='ROTOR n',\&END
\&NLVAL NPANEL=2, ! two wing panels
REDGE=.3,.7,1.,
\&END
! ====================================================================================

```
```

file: zwindtunnel.list
!======================================================================================
! conversion of free flight case to single rotor in wind tunnel
l==========================================================================================
\&NLDEF class='TRIM',\&END
\&NLVAL
WINDIN=1,WKTS=\#, ! operating condition
COLL=\#,CTTRIM=\#,MTRIM=3, ! wind tunnel trim
MNAME='CT/S ','BETAS ','BETAC ',
\&END
!==================================================================================
\&NLDEF class='AIRFRAME',type='STRUCTURE',\&END
\&NLVAL
TITLE='WIND TUNNEL SUPPORT', ! description
CONFIG=0,OPFREE=0,OPAERO=0, ! wind tunnel
OPTRAN=0, ! no drive train
ASHAFT=0.,ACANT=0., ! geometry
\&END
! ===================================================================================

```

\section*{WHEN PROBLEMS ARE ENCOUNTERED:}

FOLLOW STEPS RECOMMENDED ABOVE FOR DEVELOPING CAMRAD II CALCULATION

REVIEW GUIDELINES REGARDING CONVERGENCE

TWO WAYS TO ATTACK A PROBLEM:
consider physics and solution procedure, to identify possible source of problem, and hence revised parameters
try changing whatever parameters are available

\section*{SIMPLIFY JOB: TASKS, ANALYSIS PARAMETERS, SYSTEM CONFIGURATION}
find simplest case such that switching one parameter controls problem
in order to understand what is happening, and hence identify what modifications to make to analysis or input
and to reduce computation time while investigate problem

CAN USE tRACEP and TRACEL PARAMETERS OF CASE INPUT TO GET MORE INFORMATION ABOUT CONVERGENCE

\section*{KEEP TRYING}

\section*{ASSISTANCE AVAILABLE FROM MAINTENANCE/SUPPORT CONTRACTOR}

\section*{7-9 Checking CAMRAD II Results}

\section*{DEVELOP HABIT OF CHECKING RESULTS}

NEVER ACCEPT RESULTS OF COMPUTER PROGRAM WITHOUT SCRUTINY
usually possible to establish that results are satisfactory for intended purpose
must question code in order to recognize cases when results are unreliable
in process, develop better understanding of computer program

CONSIDER LIMITATIONS OF ANALYSIS job can always be done better recognize when must be done better

\section*{CHECK AIRFOIL TABLE FILE}

\section*{PLOT TABLE DATA}
graphical display is best way to assess table data typographical errors interpolation and extrapolation problems inadequate resolution
use INPUT program to extract data from unformatted CAMRAD airfoil file
not necessarily same information as in source files

\section*{POSSIBLE PROBLEMS} angle-of-attack and Mach number resolution too small bad patches between real airfoil data (low angle-of-attack) and generic airfoil data (high angle-of-attack)
attempt to go beyond data in source files (no extrapolation)
spanwise interpolation can give bad characteristics
stall behavior, and effect of zero lift angle

\section*{ANALYSIS MAY USE AIRFOIL TABLE BEYOND RANGE OF GOOD DATA}
data at high angle-of-attack often not available for true airfoil (so tables contain generic data for high angle-ofattack, typically for NACA 0012)
no extrapolation beyond maximum Mach number in table (or in original source file)

\section*{CHECK OTHER TABLE FILES}

\section*{CHECK SHELL INPUT FILE}

\section*{WHEN THINK INPUT IS COMPLETE, RE-EXAMINE ALL NUMBERS}
checking entire set of data, and looking for things forgotten
use listing of input parameters (one line per parameter), refer to input manual as required
for each parameter, consider job and ask:
is variable important?
is value correct?
product of this work is changes, and list of parameters not sure of for parameter uncertainties:
refer to training and input manuals for recommendations
run CAMRAD II to determine proper values

\section*{PLOT INPUT DATA: RADIAL DISTRIBUTION OF BLADE PROPERTIES}
graphical display is best way to assess blade data compare blade properties with original information use INPUT program to extract data from namelist files or from shell input file

\section*{DRAW GEOMETRY}
graphical display is best way to assess geometry of system constructed by rotorcraft shell
use INPUT program to extract data from namelist files or from shell input file
analysis job can be read as a namelist input file, after read shell input file
if use core input to change system, essential to check definition of geometry
including axes at structural dynamic interfaces

\section*{FREE VIBRATION CALCULATIONS}
checks physical and numerical aspects of structural dynamics
frequencies represent integrated inertia and stiffness
often know what calculated frequencies should be

\section*{CHECK CAMRAD II ANALYSIS}

CHECK INPUT PARAMETERS OF JOB
project requires series of jobs, varying parameters
define reference job, save its output file
print shell input at beginning of each case
so difference between current output and reference job output will list changes in shell input parameters
make sure that input intended to change was in fact changed

THINK ABOUT PHYSICS OF PROBLEM
formulate expectations, and check them
CORRELATE CALCULATED RESULTS WITH EXPERIMENT
perhaps with other analyses

\section*{BE SURE OF DEFINITIONS}
units, sign conventions, normalization, frames
documented in output

\section*{CHECK CONVERGENCE OF EVERY CASE \\ RESULTS ARE UNRELIABLE IF ANY ITERATION IS NOT CONVERGED}
usually each wake level must be converged, since it initializes the next level
search for "TRIM CONVERGENCE"
search for "BOUND CIRCULATION PEAKS"

\section*{CHECK RESULTS FOR REASONABLE ROTOR BEHAVIOR}
controls, flapping, rotor angles
search for "ROTOR n PERFORMANCE"
performance indices, angle of attack
search for "PERFORMANCE INDICES"
induced power factor \(\kappa=P / P_{m}\) will show problems with inflow or wake model
equivalent section drag \(c_{d o}=8 C_{P o} / \sigma\) will show problems with aerodynamics
angle of attack will show problems with aerodynamic model
radial oscillations can occur with nonuniform inflow
large \(\alpha\) at tip station possible with bad RTVTX figure of merit, propulsive efficiency, rotor \(L / D\) will show problems with total performance
plot aerodynamic sensors
polar plot of angle of attack
span plot of bound circulation time history of section thrust time history of induced velocity
printer-plot may be adequate to identify problems
wake geometry

SUCCESSIVE SUBSTITUTION ITERATION, LOOP = WAKE
LEVEL 1 = UNIFORM INFLOW

\begin{tabular}{|c|c|c|}
\hline ITERATION, COUNT = 10.846 & \[
\begin{gathered}
7, \text { LOC } \\
3.2171
\end{gathered}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.2884
\end{aligned}
\] \\
\hline ITERATION, COUNT = 10.949 & \[
\begin{gathered}
8, \text { LOC } \\
3.2290
\end{gathered}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.3092
\end{aligned}
\] \\
\hline ITERATION, COUNT = 11.033 & \[
\begin{gathered}
9, \text { LOC } \\
3.2387
\end{gathered}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.3278
\end{aligned}
\] \\
\hline ITERATION, COUNT = 11.100 & \[
\begin{aligned}
& 10, \text { LOO } \\
& 3.2462
\end{aligned}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.3437
\end{aligned}
\] \\
\hline ITERATION, COUNT = 11.156 & \[
\begin{aligned}
& 11, \text { LOO } \\
& 3.2526
\end{aligned}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.3581
\end{aligned}
\] \\
\hline ITERATION, COUNT = 11.202 & \[
\begin{aligned}
& 12, ~ \mathrm{LOOI} \\
& 3.2579
\end{aligned}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.3710
\end{aligned}
\] \\
\hline ITERATION, COUNT = 11.240 & \[
\begin{gathered}
13, ~ L O C \\
3.2624
\end{gathered}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.3827
\end{aligned}
\] \\
\hline ITERATION, COUNT = 11.271 & \[
\begin{aligned}
& 14, \text { LOOI } \\
& 3.2661
\end{aligned}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& 8.3930
\end{aligned}
\] \\
\hline ITERATION, COUNT = 11.296 & \[
\begin{gathered}
15, \text { LOO } \\
3.2693
\end{gathered}
\] & \[
\begin{aligned}
& \text { TRIM } \\
& \quad 8.4024
\end{aligned}
\] \\
\hline
\end{tabular}
\begin{tabular}{lll}
\(0.71354 \mathrm{E}-01\) & 0.31072 & 0.52580 \\
\(0.73016 \mathrm{E}-01\) & 0.25047 & 0.41753 \\
\(0.74375 \mathrm{E}-01\) & 0.20306 & 0.33292 \\
\(0.75439 \mathrm{E}-01\) & 0.16376 & 0.26355 \\
\(0.76301 \mathrm{E}-01\) & 0.13216 & 0.20835 \\
\(0.76999 \mathrm{E}-01\) & 0.10668 & 0.16454 \\
\(0.77562 \mathrm{E}-01\) & \(0.86092 \mathrm{E}-01\) & 0.12974 \\
\(0.78029 \mathrm{E}-01\) & \(0.68738 \mathrm{E}-01\) & 0.10099 \\
\(0.78396 \mathrm{E}-01\) & \(0.55531 \mathrm{E}-01\) & \(0.79517 \mathrm{E}-01\)
\end{tabular}
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******************
TRIM CONVERGENCE

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SUCCESSIVE SUBSTITUTION ITERATION, LOOP = WAKE ITERATION 1, LEVEL \(1=\) UNIFORM INFLOW

\section*{STATUS OF LOOP AND PART SOLUTIONS}
2 TOTAL NUMBER
```

LOOP = TRIM
LOOP = CIRCULATION
LOOP = MOTION
LOOP = AIRFRAME

```
PART \(=\) ROTOR 1 HUB
PART \(=\) ROTOR 1 BLADE 4
PART \(=\) ROTOR 1 HUB
\begin{tabular}{lr} 
NEWTON RAPHSON, CONVERGED & OF ITERATIONS \\
SUCCESSIVE SUBSTITUTION, CONVERGED & 19 \\
SUCCESSIVE SUBSTITUTION, CONVERGED & 80 \\
NO SOLUTION & 80 \\
& \\
HARMONIC SOLUTION, CONVERGED & 184 \\
HARMONIC SOLUTION, CONVERGED & 103 \\
HARMONIC SOLUTION, CONVERGED & 94 \\
HARMONIC SOLUTION, CONVERGED & 9
\end{tabular}

COnvere
CONVERGENCE OF LOOP AND PART SOLUTIONS

LOOP = TRIM NEWTON RAPHSON, CONVERGED
NUMBER OF ITERATIONS \(=15\) (MAXIMUM 40), TOLERANCE \(=5.00000\)
ERROR = ABS(TRIMMED-TARGET); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)


LOOP = CIRCULATION SUCCESSIVE SUBSTITUTION, CONVERGED
NUMBER OF ITERATIONS \(=2\) (MAXIMUM 200), TOLERANCE \(=1.00000\)
ERROR = ABS(X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)
LOOP = MOTION SUCCESSIVE SUBSTITUTION, CONVERGED
NUMBER OF ITERATIONS \(=1\) (MAXIMUM 40), TOLERANCE \(=2.00000\)
ERROR = ABS (X-XOLD) OR RMS (X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)

PART = ROTOR 1 HUB
NUMBER OF ITERATIONS = 1 (MAXIMUM 40), TOLERANCE = 2.00000 FOR PERIOD \(1=\) ROTOR 1
ERROR = ABS(X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)



\title{
PART = ROTOR 1 HUB \\ HARMONIC SOLUTION, CONVERGED \\ NUMBER OF ITERATIONS \(=1\) (MAXIMUM 40),\(~\) TOLERANCE \(=2.00000 \quad\) FOR PERIOD \(1=\) ROTOR 1
ERROR \(=\) ABS (X-XOLD) OR RMS (X-XOLD) ; ERROR RATIO \(=\) ERROR \(/(\) TOLER*WEIGHT) (LE 1 FOR CONVERGENCE) \\ ABS (X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)
}

PART = AIRFRAME 1 (MAXIMUM 40) HARMONIC SOLUTION, CONVERGED ERROR = ABS(X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)


PERTURBATION IDENTIFICATION
DERIVATIVE MATRIX D

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TRIM CONVERGENCE
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SUCCESSIVE SUBSTITUTION ITERATION, LOOP = WAKE ITERATION 1, LEVEL 1 = UNIFORM INFLOW

\section*{STATUS OF LOOP AND PART SOLUTIONS}
```

LOOP = TRIM
LOOP = CIRCULATION
LOOP = MOTION
LOOP = AIRFRAME

```
PART \(=\) ROTOR 1 HUB
PART \(=\) ROTOR 1 BLADE 4
PART \(=\) ROTOR 1 HUB
PART \(=\) AIRFRAME
*************************************
**************************************
**
** WARNING **
** LOOP NOT CONVERGED **
** PART NOT CONVERGED **
** RESULTS UNRELIABLE **
** **
相
**************************************
CONVERGENCE OF LOOP AND PART SOLUTIONS

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multicolumn{3}{|l|}{VARIABLE} & \multicolumn{5}{|l|}{QUANTITY} \\
\hline & TRIMMED & INITIAL & & TRIMMED & TARGET & WEIGHT & ERROR RATIO \\
\hline COLL & 9.62809 & 9.00000 & CT/S & \(0.488627 \mathrm{E}-01\) & \(0.800000 \mathrm{E}-01\) & \(0.800000 \mathrm{E}-03\) & 7.78432 \\
\hline LATCYC & 2.50043 & 3.00000 & BETAS & 0.553251 & \(0.000000 \mathrm{E}+00\) & \(0.200000 \mathrm{E}-01\) & 5.53251 \\
\hline LNGCYC & 8.30013 & 8.00000 & BETAC & 2.18360 & \(0.000000 \mathrm{E}+00\) & \(0.200000 \mathrm{E}-01\) & 21.8360 \\
\hline \multicolumn{8}{|l|}{LOOP \(=\) CIRCULATION SUCCESSIVE SUBSTITUTION, NOT CONVERGED} \\
\hline \multicolumn{8}{|l|}{NUMBER OF ITERATIONS \(=2\) (MAXIMUM 2), TOLERANCE \(=1.00000\)} \\
\hline \multicolumn{8}{|l|}{ERROR \(=\) ABS (X-XOLD) OR RMS (X-XOLD) ; ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)} \\
\hline VARIABLE & & & ELEMENT & ERROR & & WEIGHT & ERROR RATIO \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline ROTO & 1 MEAN & FORCE & (TPP & AXES) & INTERFACE & 4 & ABS ( X -XOLD \()\) & 2973.4 & 467.07 & 6.3660 \\
\hline ROTO & 1 MEAN & FORCE & (TPP & AXES) & INTERFACE & 5 & ABS ( X-XOLD) & 1995.4 & 467.07 & 4.2722 \\
\hline ROTO & 1 MEAN & FORCE & (TPP & AXES) & INTERFACE & 6 & ABS ( X-XOLD) & 526.72 & 467.07 & 1.1277 \\
\hline
\end{tabular}

LOOP = MOTION
SUCCESSIVE SUBSTITUTION, CONVERGED
NUMBER OF ITERATIONS \(=1\) (MAXIMUM 40), TOLERANCE \(=2.00000\)
ERROR = ABS(X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)
\begin{tabular}{|c|c|c|c|c|}
\hline PART = ROTOR 1 HUB & \multicolumn{4}{|c|}{HARMONIC SOLUTION, NOT CONVERGED} \\
\hline NUMBER OF ITERATIONS & 2 (MAXIMUM & 2), TOLERANCE = & FOR PERIOD & 1 = ROTOR 1 \\
\hline ERROR = ABS (X-XOLD) OR & (X-XOLD) ; E & R RATIO = ERRO & (LE 1 FOR & NVERGENCE) \\
\hline
\end{tabular}

PART = ROTOR 1 BLADE 4 HARMONIC SOLUTION, NOT CONVERGED
NUMBER OF ITERATIONS = 2 (MAXIMUM 2), TOLERANCE \(=2.00000\) FOR PERIOD \(1=\) ROTOR 1
ERROR = ABS(X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)
\begin{tabular}{llll} 
VARIABLE & & QUANTITY & ELEMENT \\
ROTOR 1 ROT HUB/BLADE 4 ROOT & INTERFACE & 4 \\
ROTOR 1 BLADE 4 ROOT/BLADE & INTERFACE & 4 \\
ROTOR 1 BLADE 4 EL 1/EL 2 & INTERFACE & 4 \\
ROTOR 1 BLADE 4 EL 2/EL 3 & INTERFACE & 4
\end{tabular}
\begin{tabular}{lrr} 
ERROR & & WEIGHT \\
RMS (X-XOLD) & \(=1491.3\) & 467.0 \\
RMS (X-XOLD) & \(=1491.3\) & 467.0 \\
RMS (X-XOLD) & \(=1408.2\) & 467.0 \\
RMS (X-XOLD) & \(=1378.8\) & 467.0
\end{tabular}

ROTOR 1 ROT HUB/BLADE 4 ROOT
ROTOR 1 BLADE 4 ROOT/BLADE
ROTOR 1 BLADE 4 EL \(2 / E L 3\)

HARMONIC SOLUTION, NOT CONVERGED
PART = ROTOR 1 HUB
NUMBER OF ITERATIONS \(=2\) (MAXIMUM 2), TOLERANCE = 2.00000 FOR PERIOD \(1=\) ROTOR 1
ERROR = ABS(X-XOLD) OR RMS(X-XOLD); ERROR RATIO = ERROR/(TOLER*WEIGHT) (LE 1 FOR CONVERGENCE)


PERFORMANCE INDICE


MEAN INFLOW RATIO: VELOCITY MAGNITUDE AND COMPONENTS IN SHAFT AXES (AVERAGE OVER ROTOR DISK) FIGURE OF MERIT IS MEASURE OF ROTOR EFFICIENCY APPROPRIATE FOR AXIAL FLOW

BOTH DEFINITIONS INCLUDE CONVENTIONAL HOVER FIGURE OF MERIT, M = T*VIDEAL/P = T*SQRT(T/2*RHO*A)/P
\(\mathrm{M}=(\mathrm{PM}+\mathrm{PINT}) /(\mathrm{PO}+\mathrm{PI})=\mathrm{T} *(\mathrm{VIDEAL}+\mathrm{VINT}) /(\mathrm{PO}+\mathrm{PI})\)
\(M=P I D E A L / P=1-P N / P=T *(V+V I D E A L) / P\) FOR AXIAL FLOW (NOT VALID FOR ZERO POWER)
SECOND DEFINITION NEARLY PROPULSIVE EFFICIENCY ETA = TV/P FOR HIGH INFLOW
PROPULSIVE EFFICIENCY IS MEASURE APPROPRIATE FOR HIGH SPEED PROPULSION
ETA \(=-X * V / P=(P C+P P) / P\) (NOT VALID FOR ZERO VELOCITY OR ZERO POWER)
ROTOR LIFT-TO-DRAG RATIO IS MEASURE OF ROTOR LIFTING EFFICIENCY, APPROPRIATE FOR EDGEWISE FLOW
L/D \(=\mathrm{L} * \mathrm{~V} /(\mathrm{PI}+\mathrm{PO})\) (NOT USEFUL FOR ZERO VELOCITY); ROTOR EQUIVALENT DRAG D \(=\mathrm{P} / \mathrm{V}+\mathrm{X}=(\mathrm{PI}+\mathrm{PO}) / \mathrm{V}\)
WING LIFT-TO-DRAG RATIO AND INDUCED EFFICIENCY E ARE MEASURES APPROPRIATE FOR FIXED WING
\(D I / Q=(L / Q) * * 2 /(E * P I * B * * 2)\)

INFLOW RATIO, INFLOW = VELOCITY / VTIP
REFERENCE ROTOR RADIUS \(R=24.7240 \mathrm{FT}\), ROTATIONAL SPEED OMEGA \(=260.000 \mathrm{RPM}\); TIP SPEED VTIP \(=\) OMEGA*R COEFFICIENT CF = F/QA, WING COEFFICIENT CFW \(=\mathrm{F} / \mathrm{QS}\)

ROTOR AREA A \(=P I * R * * 2\); BLADE AREA \(S=A * S I G M A ;\) WING SPAN B \(=R\); DYNAMIC PRESSURE \(0=.5 * R H O * V * * 2\)
ROTOR AREA A
RADIUS \(\mathrm{R}=24.7240 \mathrm{FT}\), SOLIDITY RATIO SIGMA \(=0.09100\), AIR DENSITY RHO \(=0.001918 \mathrm{SLUG} / \mathrm{FT} * * 3\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{16}{|l|}{BLADE SECTION ANGLE OF ATTACK (DEG)} \\
\hline RADIAL STATION & \(=0.560\) & 0.615 & 0.665 & 0.710 & 0.750 & 0.785 & 0.815 & 0.845 & 0.870 & 0.890 & 0.910 & 0.930 & 0.950 & 0.970 & 0.990 \\
\hline AZIMUTH (DEG) & ALPHA & & & & & & & & & & & & & & \\
\hline PSI \(=0.0\) & 7.1 & 7.5 & 7.5 & 7.4 & 7.2 & 7.0 & 6.8 & 6.1 & 5.1 & 4.7 & 5.0 & 5.4 & 5.3 & 5.6 & 4.9 \\
\hline PSI \(=15.0\) & 5.6 & 5.6 & 5.6 & 5.5 & 5.4 & 5.2 & 4.9 & 4.3 & 3.7 & 3.3 & 3.5 & 3.8 & 3.7 & 3.9 & 3.5 \\
\hline PSI \(=30.0\) & 4.2 & 4.0 & 3.9 & 3.7 & 3.5 & 3.3 & 3.1 & 2.7 & 2.3 & 2.0 & 2.1 & 2.3 & 2.2 & 2.4 & 2.1 \\
\hline PSI \(=45.0\) & 2.6 & 2.4 & 2.2 & 2.0 & 1.9 & 1.7 & 1.5 & 1.3 & 1.0 & 0.9 & 0.9 & 1.0 & 1.0 & 1.1 & 0.9 \\
\hline PSI \(=60.0\) & 1.7 & 1.6 & 1.4 & 1.2 & 1.0 & 0.9 & 0.7 & 0.5 & 0.4 & 0.3 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\
\hline PSI \(=75.0\) & 1.2 & 1.0 & 0.8 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & -0.1 & -0.1 & -0.1 & -0.1 & -0.1 \\
\hline PSI \(=90.0\) & 1.1 & 0.9 & 0.8 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
\hline PSI \(=105.0\) & 1.5 & 1.3 & 1.1 & 0.9 & 0.8 & 0.7 & 0.6 & 0.6 & 0.5 & 0.4 & 0.3 & 0.3 & 0.2 & 0.2 & 0.1 \\
\hline PSI \(=120.0\) & 2.0 & 1.8 & 1.7 & 1.6 & 1.5 & 1.4 & 1.3 & 1.2 & 1.1 & 0.8 & 0.6 & 0.5 & 0.5 & 0.4 & 0.3 \\
\hline PSI \(=135.0\) & 2.9 & 2.9 & 2.8 & 2.6 & 2.4 & 2.3 & 2.1 & 2.0 & 1.8 & 1.4 & 1.1 & 0.9 & 0.8 & 0.8 & 0.6 \\
\hline PSI \(=150.0\) & 4.2 & 4.2 & 4.0 & 3.7 & 3.5 & 3.3 & 3.1 & 2.9 & 2.7 & 2.2 & 1.7 & 1.5 & 1.3 & 1.2 & 1.0 \\
\hline PSI \(=165.0\) & 5.9 & 5.6 & 5.4 & 5.0 & 4.7 & 4.4 & 4.2 & 4.1 & 3.9 & 3.1 & 2.5 & 2.2 & 1.9 & 1.8 & 1.4 \\
\hline PSI \(=180.0\) & 7.5 & 7.2 & 6.8 & 6.4 & 6.1 & 5.8 & 5.5 & 5.4 & 5.2 & 4.2 & 3.4 & 3.0 & 2.7 & 2.4 & 1.9 \\
\hline PSI \(=195.0\) & 9.1 & 8.7 & 8.3 & 7.9 & 7.6 & 7.2 & 6.9 & 6.9 & 6.7 & 5.4 & 4.4 & 3.9 & 3.5 & 3.2 & 2.5 \\
\hline PSI \(=210.0\) & 10.7 & 10.2 & 9.7 & 9.3 & 9.0 & 8.6 & 8.4 & 8.3 & 8.0 & 6.6 & 5.5 & 4.9 & 4.4 & 4.0 & 3.2 \\
\hline PSI \(=225.0\) & 12.2 & 11.6 & 11.1 & 10.7 & 10.3 & 9.9 & 9.7 & 9.6 & 9.2 & 7.7 & 6.6 & 6.0 & 5.4 & 4.9 & 4.0 \\
\hline PSI \(=240.0\) & 13.2 & 12.7 & 12.3 & 11.8 & 11.4 & 11.1 & 10.9 & 10.7 & 10.1 & 8.6 & 7.6 & 7.1 & 6.5 & 6.0 & 4.9 \\
\hline PSI \(=255.0\) & 13.8 & 13.4 & 13.1 & 12.8 & 12.5 & 12.2 & 11.9 & 11.5 & 10.6 & 9.3 & 8.5 & 8.1 & 7.5 & 7.0 & 5.8 \\
\hline PSI \(=270.0\) & 18.4 & 16.4 & 15.1 & 12.7 & 12.7 & 11.3 & 21.1 & 10.5 & 10.1 & 9.3 & 9.0 & 8.9 & 8.3 & 8.0 & 6.6 \\
\hline PSI \(=285.0\) & 12.8 & 12.9 & 12.8 & 12.1 & 11.8 & 10.7 & 22.8 & 10.2 & 9.8 & 9.2 & 9.2 & 9.3 & 8.9 & 8.7 & 7.4 \\
\hline PSI \(=300.0\) & 11.5 & 11.7 & 11.8 & 11.9 & 11.7 & 11.7 & 12.4 & 10.6 & 9.3 & 8.5 & 8.7 & 9.0 & 8.7 & 8.8 & 7.5 \\
\hline PSI \(=315.0\) & 10.1 & 10.6 & 10.8 & 10.8 & 10.8 & 10.7 & 10.3 & 9.6 & 8.4 & 7.8 & 8.1 & 8.4 & 8.2 & 8.4 & 7.2 \\
\hline PSI \(=330.0\) & 8.1 & 9.3 & 9.7 & 9.9 & 9.9 & 9.9 & 9.7 & 8.9 & 7.6 & 7.0 & 7.5 & 8.0 & 7.8 & 8.1 & 7.1 \\
\hline PSI \(=345.0\) & 8.7 & 8.6 & 8.7 & 8.8 & 8.8 & 8.7 & 8.5 & 7.6 & 6.5 & 6.0 & 6.4 & 6.9 & 6.8 & 7.1 & 6.2 \\
\hline PSI \(=360.0\) & 7.1 & 7.5 & 7.5 & 7.4 & 7.2 & 7.0 & 6.8 & 6.1 & 5.1 & 4.7 & 5.0 & 5.4 & 5.3 & 5.6 & 4.9 \\
\hline
\end{tabular}



MEAN INFLOW RATIO: VELOCITY MAGNITUDE AND COMPONENTS IN SHAFT AXES (AVERAGE OVER ROTOR DISK)
FIGURE OF MFRTT IS MFASURE OF ROTOR EFFICIENCY APPROPRTATE FOR AXIAI FIOW
BOTH DEFINITIONS INCLUDE CONVENTIONAL HOVER FIGURE OF MERIT, M = T*VIDEAL/P \(=T * S Q R T(T / 2 * R H O * A) / P\)
\(\mathrm{M}=(\mathrm{PM}+\mathrm{PINT}) /(\mathrm{PO}+\mathrm{PI})=\mathrm{T} *(\mathrm{VIDEAL}+\mathrm{VINT}) /(\mathrm{PO}+\mathrm{PI})\)
\(M=P I D E A L / P=1-P N / P=T *(V+V I D E A L) / P\) FOR AXIAL FLOW (NOT VALID FOR ZERO POWER)
SECOND DEFINITION NEARLY PROPULSIVE EFFICIENCY ETA = TV/P FOR HIGH INFLOW
PROPULSIVE EFFICIENCY IS MEASURE APPROPRIATE FOR HIGH SPEED PROPULSION
ETA \(=-X * V / P=(P C+P P) / P \quad\) (NOT VALID FOR ZERO VELOCITY OR ZERO POWER)
ROTOR LIFT-TO-DRAG RATIO IS MEASURE OF ROTOR LIFTING EFFICIENCY, APPROPRIATE FOR EDGEWISE FLOW
L/D = L*V/(PI+PO) (NOT USEFUL FOR ZERO VELOCITY); ROTOR EQUIVALENT DRAG D = P/V+X = (PI+PO)/V
WING LIFT-TO-DRAG RATIO AND INDUCED EFFICIENCY E ARE MEASURES APPROPRIATE FOR FIXED WING
\(\mathrm{DI} / \mathrm{Q}=(\mathrm{L} / \mathrm{Q}) * * 2 /(\mathrm{E} * \mathrm{PI} * \mathrm{~B} * * 2)\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline BLADE SECTION ANGI & 0.5 & ( DEG) & & & & & & & & & & & & & \\
\hline RADIAL STATION & 0.560 & 0.615 & 0.665 & 0.710 & 0.750 & 0.785 & 0.815 & 0.845 & 0.870 & 0.890 & 0.910 & 0.930 & 0.950 & 0.970 & 0.990 \\
\hline AZIMUTH (DEG) & ALPHA & & & & & & & & & & & & & & \\
\hline PSI \(=0.0\) & 7.2 & 7.5 & 7.5 & 7.4 & 7.2 & 7.0 & 6.7 & 6.0 & 5.1 & 4.6 & 4.9 & 5.3 & 5.3 & 5.6 & 4.9 \\
\hline PSI \(=15.0\) & 5.1 & 5.3 & 5.3 & 5.3 & 5.2 & 5.0 & 4.8 & 4.3 & 3.6 & 3.3 & 3.5 & 3.7 & 3.7 & 3.9 & 3.5 \\
\hline PSI \(=30.0\) & 4.0 & 4.0 & 3.8 & 3.7 & 3.5 & 3.4 & 3.2 & 2.8 & 2.3 & 2.1 & 2.2 & 2.3 & 2.3 & 2.5 & 2.2 \\
\hline PSI \(=45.0\) & 2.7 & 2.5 & 2.4 & 2.2 & 2.0 & 1.8 & 1.7 & 1.4 & 1.1 & 1.0 & 1.0 & 1.1 & 1.1 & 1.2 & 1.0 \\
\hline PSI \(=60.0\) & 1.8 & 1.8 & 1.7 & 1.6 & 1.5 & 1.3 & 1.2 & 1.0 & 0.7 & 0.6 & 0.5 & 0.5 & 0.5 & 0.5 & 0.4 \\
\hline PSI \(=75.0\) & 1.4 & 1.1 & 0.9 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & -0.1 & 0.0 & -0.3 & -0.2 & -0.3 & -0.3 & -0.3 \\
\hline PSI \(=90.0\) & 1.8 & 1.7 & 1.6 & 1.4 & 1.3 & 1.1 & 0.9 & 0.8 & -0.5 & 2.2 & -1.7 & 1.1 & -0.4 & -0.1 & -0.2 \\
\hline PSI \(=105.0\) & 1.4 & 1.2 & 0.9 & 0.7 & 0.6 & 0.4 & 0.3 & 0.4 & -1.5 & 2.8 & -3.1 & 2.2 & -1.2 & 0.1 & -0.2 \\
\hline PSI \(=120.0\) & 2.0 & 1.8 & 1.6 & 1.5 & 1.4 & 1.2 & 1.1 & 1.2 & -0.8 & 3.8 & -2.7 & 3.4 & -1.4 & 0.8 & 0.1 \\
\hline PSI \(=135.0\) & 2.9 & 2.9 & 2.8 & 2.6 & 2.4 & 2.2 & 2.0 & 2.1 & 0.4 & 4.2 & -2.0 & 4.1 & -1.0 & 1.4 & 0.5 \\
\hline PSI \(=150.0\) & 4.0 & 4.1 & 4.0 & 3.8 & 3.6 & 3.4 & 3.2 & 3.2 & 1.8 & 4.4 & -0.6 & 4.0 & 0.0 & 1.8 & 0.9 \\
\hline PSI \(=165.0\) & 6.0 & 5.9 & 5.6 & 5.3 & 5.0 & 4.7 & 4.4 & 4.4 & 3.4 & 4.8 & 1.0 & 3.9 & 1.2 & 2.2 & 1.4 \\
\hline PSI \(=180.0\) & 7.9 & 7.6 & 7.2 & 6.8 & 6.4 & 6.1 & 5.9 & 5.8 & 5.1 & 5.4 & 2.5 & 4.2 & 2.4 & 2.8 & 2.0 \\
\hline PSI \(=195.0\) & 9.5 & 9.1 & 8.7 & 8.3 & 7.9 & 7.5 & 7.3 & 7.3 & 6.7 & 6.2 & 4.0 & 4.7 & 3.4 & 3.4 & 2.6 \\
\hline PSI \(=210.0\) & 11.1 & 10.6 & 10.1 & 9.7 & 9.3 & 9.0 & 8.8 & 8.7 & 8.2 & 7.2 & 5.4 & 5.4 & 4.5 & 4.2 & 3.3 \\
\hline PSI \(=225.0\) & 12.5 & 11.9 & 11.4 & 11.0 & 10.6 & 10.3 & 10.1 & 10.0 & 9.5 & 8.2 & 6.7 & 6.4 & 5.6 & 5.1 & 4.1 \\
\hline PSI \(=240.0\) & 13.6 & 13.0 & 12.6 & 12.2 & 11.8 & 11.5 & 11.3 & 11.1 & 10.5 & 9.0 & 7.8 & 7.4 & 6.7 & 6.2 & 5.0 \\
\hline PSI \(=255.0\) & 14.3 & 13.8 & 13.6 & 13.3 & 13.0 & 12.7 & 12.4 & 12.0 & 11.0 & 9.7 & 8.8 & 8.4 & 7.8 & 7.3 & 6.0 \\
\hline PSI \(=270.0\) & 19.5 & 16.3 & 19.2 & 12.2 & 12.7 & 11.6 & 23.8 & 10.8 & 10.4 & 9.6 & 9.3 & 9.1 & 8.6 & 8.2 & 6.8 \\
\hline PSI \(=285.0\) & 13.1 & 12.4 & 14.0 & 12.2 & 11.9 & 11.1 & 25.9 & 10.6 & 10.1 & 9.5 & 9.4 & 9.5 & 9.1 & 8.9 & 7.5 \\
\hline PSI \(=300.0\) & 11.5 & 12.2 & 11.7 & 12.2 & 12.1 & 12.1 & 12.6 & 11.0 & 9.6 & 8.8 & 8.9 & 9.2 & 8.9 & 9.0 & 7.7 \\
\hline PSI \(=315.0\) & 10.4 & 10.7 & 11.1 & 11.0 & 11.0 & 10.9 & 10.5 & 9.8 & 8.6 & 7.9 & 8.3 & 8.6 & 8.4 & 8.6 & 7.4 \\
\hline PSI \(=330.0\) & 8.1 & 9.3 & 9.7 & 10.0 & 10.0 & 10.0 & 9.8 & 9.0 & 7.7 & 7.1 & 7.6 & 8.1 & 7.9 & 8.2 & 7.2 \\
\hline PSI \(=345.0\) & 8.7 & 8.6 & 8.7 & 8.8 & 8.8 & 8.7 & 8.5 & 7.6 & 6.5 & 6.0 & 6.4 & 7.0 & 6.9 & 7.1 & 6.3 \\
\hline PSI \(=360.0\) & 7.2 & 7.5 & 7.5 & 7.4 & 7.2 & 7.0 & 6.7 & 6.0 & 5.1 & 4.6 & 4.9 & 5.3 & 5.3 & 5.6 & 4.9 \\
\hline
\end{tabular}



MEAN INFLOW RATIO: VELOCITY MAGNITUDE AND COMPONENTS IN SHAFT AXES (AVERAGE OVER ROTOR DISK) FIGURE OF MERIT IS MEASURE OF ROTOR EFFICIENCY APPROPRIATE FOR AXIAL FLOW

BOTH DEFINITIONS INCLUDE CONVENTIONAL HOVER FIGURE OF MERIT, M = T*VIDEAL/P = T*SQRT(T/2*RHO*A)/P
\(\mathrm{M}=(\mathrm{PM}+\mathrm{PINT}) /(\mathrm{PO}+\mathrm{PI})=\mathrm{T} *(\mathrm{VIDEAL}+\mathrm{VINT}) /(\mathrm{PO}+\mathrm{PI})\)
\(M=P I D E A L / P=1-P N / P=T *(V+V I D E A L) / P\) FOR AXIAL FLOW (NOT VALID FOR ZERO POWER)
SECOND DEFINITION NEARLY PROPULSIVE EFFICIENCY ETA = TV/P FOR HIGH INFLOW
PROPULSIVE EFFICIENCY IS MEASURE APPROPRIATE FOR HIGH SPEED PROPULSION
ETA \(=-\mathrm{X} * \mathrm{~V} / \mathrm{P}=(\mathrm{PC}+\mathrm{PP}) / \mathrm{P}\) (NOT VALID FOR ZERO VELOCITY OR ZERO POWER)
ROTOR LIFT-TO-DRAG RATIO IS MEASURE OF ROTOR LIFTING EFFICIENCY, APPROPRIATE FOR EDGEWISE FLOW
\(\mathrm{L} / \mathrm{D}=\mathrm{L} * \mathrm{~V} /(\mathrm{PI}+\mathrm{PO})\) (NOT USEFUL FOR ZERO VELOCITY); ROTOR EQUIVALENT DRAG \(\mathrm{D}=\mathrm{P} / \mathrm{V}+\mathrm{X}=(\mathrm{PI}+\mathrm{PO}) / \mathrm{V}\) WING LIFT-TO-DRAG RATIO AND INDUCED EFFICIENCY E ARE MEASURES APPROPRIATE FOR FIXED WING
\(D I / Q=(L / Q) * * 2 /(E * P I * B * * 2)\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{16}{|l|}{BLADE SECTION ANGLE OF ATTACK (DEG)} \\
\hline RADIAL STATION & 0.560 & 0.615 & 0.665 & 0.710 & 0.750 & 0.785 & 0.815 & 0.845 & 0.870 & 0.890 & 0.910 & 0.930 & 0.950 & 0.970 & 0.990 \\
\hline AZIMUTH (DEG) & ALPHA & & & & & & & & & & & & & & \\
\hline PSI \(=0.0\) & 8.0 & 8.2 & 8.1 & 7.9 & 7.6 & 7.3 & 7.0 & 6.5 & 3.9 & 8.2 & 1.9 & 8.2 & 4.1 & 6.0 & 4.9 \\
\hline PSI \(=15.0\) & 6.9 & 6.8 & 6.7 & 6.5 & 6.3 & 6.1 & 5.8 & 5.3 & 3.6 & 5.5 & 2.5 & 5.5 & 3.9 & 4.7 & 4.0 \\
\hline PSI \(=30.0\) & 5.4 & 5.2 & 5.0 & 4.6 & 4.3 & 3.9 & 3.6 & 2.3 & 6.1 & -1.6 & 8.8 & -2.8 & 5.5 & 0.9 & 2.1 \\
\hline PSI \(=45.0\) & 2.8 & 2.4 & 2.1 & 1.9 & 1.7 & 1.5 & 1.7 & -0.3 & 7.6 & -5.1 & 10.7 & -6.8 & 7.7 & -2.2 & 1.4 \\
\hline PSI \(=60.0\) & 2.1 & 1.4 & 0.9 & 0.6 & 0.3 & 0.0 & 0.8 & -2.0 & 7.1 & -7.5 & 9.1 & -8.6 & 7.7 & -3.8 & 1.0 \\
\hline PSI \(=75.0\) & 2.3 & 2.0 & 1.6 & 1.2 & 0.9 & 0.2 & 1.2 & -2.2 & 7.0 & -7.9 & 7.9 & -8.8 & 6.7 & -4.5 & 0.8 \\
\hline PSI \(=90.0\) & 1.6 & 1.0 & 0.5 & 0.1 & -0.1 & -0.7 & 0.6 & -2.9 & 5.7 & -9.2 & 6.4 & -9.3 & 6.0 & -4.9 & 1.2 \\
\hline PSI \(=105.0\) & 3.1 & 2.7 & 2.3 & 1.8 & 1.4 & 0.8 & 0.9 & 0.1 & -2.0 & 3.6 & -3.9 & -0.8 & 4.0 & -4.1 & 1.0 \\
\hline PSI \(=120.0\) & 3.7 & 3.2 & 2.6 & 2.0 & 1.5 & 1.1 & 0.4 & 2.1 & -5.9 & 8.9 & -7.2 & 6.7 & -4.8 & 2.7 & -0.9 \\
\hline PSI \(=135.0\) & 4.6 & 3.9 & 3.2 & 2.7 & 2.3 & 2.1 & 1.0 & 3.4 & -5.4 & 11.7 & -5.9 & 9.9 & -5.8 & 5.9 & -1.8 \\
\hline PSI \(=150.0\) & 5.6 & 4.9 & 4.4 & 3.9 & 3.5 & 3.4 & 2.3 & 4.5 & -3.8 & 13.5 & -4.7 & 11.4 & -4.4 & 7.0 & -1.6 \\
\hline PSI \(=165.0\) & 7.0 & 6.4 & 6.0 & 5.5 & 5.1 & 4.9 & 3.9 & 5.9 & -2.1 & 14.1 & -3.0 & 11.9 & -3.1 & 7.5 & -0.9 \\
\hline PSI \(=180.0\) & 8.9 & 8.3 & 7.8 & 7.3 & 6.8 & 6.5 & 5.7 & 7.5 & -0.3 & 14.4 & -1.2 & 11.7 & -1.6 & 7.6 & 0.1 \\
\hline PSI \(=195.0\) & 10.6 & 10.1 & 9.7 & 9.2 & 8.7 & 8.4 & 7.6 & 9.3 & 1.6 & 15.1 & 0.5 & 11.4 & 0.0 & 7.7 & 1.1 \\
\hline PSI \(=210.0\) & 12.6 & 12.0 & 11.4 & 10.9 & 10.4 & 10.0 & 9.3 & 10.9 & 3.5 & 15.7 & 2.1 & 11.5 & 1.4 & 7.9 & 2.3 \\
\hline PSI \(=225.0\) & 14.4 & 13.6 & 12.9 & 12.3 & 11.8 & 11.5 & 10.9 & 12.1 & 5.8 & 15.6 & 3.8 & 11.6 & 3.0 & 8.1 & 3.5 \\
\hline PSI \(=240.0\) & 15.8 & 14.8 & 14.2 & 13.6 & 13.1 & 12.7 & 12.2 & 12.8 & 8.3 & 14.6 & 5.8 & 11.7 & 4.6 & 8.5 & 4.8 \\
\hline PSI \(=255.0\) & 15.6 & 15.0 & 14.7 & 14.3 & 14.1 & 13.7 & 13.5 & 13.1 & 10.1 & 13.0 & 7.4 & 11.4 & 6.2 & 8.8 & 5.9 \\
\hline PSI \(=270.0\) & 21.6 & 21.1 & 12.6 & 23.2 & 12.2 & 11.8 & 24.9 & 11.5 & 10.2 & 11.6 & 8.4 & 11.2 & 7.7 & 9.3 & 6.9 \\
\hline PSI \(=285.0\) & 13.2 & 14.0 & 10.7 & 23.9 & 11.0 & 10.9 & 26.4 & 11.0 & 10.0 & 10.7 & 8.7 & 11.1 & 8.6 & 9.7 & 7.7 \\
\hline PSI \(=300.0\) & 11.4 & 11.2 & 12.4 & 12.9 & 11.8 & 11.8 & 12.4 & 10.9 & 9.2 & 9.6 & 8.0 & 10.2 & 8.4 & 9.3 & 7.7 \\
\hline PSI \(=315.0\) & 10.0 & 10.5 & 10.4 & 10.4 & 10.7 & 10.6 & 10.2 & 9.6 & 7.9 & 8.6 & 7.1 & 9.2 & 7.8 & 8.5 & 7.2 \\
\hline PSI \(=330.0\) & 7.6 & 8.7 & 9.3 & 9.5 & 9.5 & 9.6 & 9.4 & 8.7 & 6.9 & 8.0 & 6.3 & 8.8 & 7.3 & 8.2 & 7.0 \\
\hline PSI \(=345.0\) & 8.6 & 8.4 & 8.4 & 8.5 & 8.5 & 8.4 & 8.1 & 7.4 & 5.5 & 7.8 & 4.3 & 8.2 & 6.0 & 7.1 & 6.1 \\
\hline PSI \(=360.0\) & 8.0 & 8.2 & 8.1 & 7.9 & 7.6 & 7.3 & 7.0 & 6.5 & 3.9 & 8.2 & 1.9 & 8.2 & 4.1 & 6.0 & 4.9 \\
\hline
\end{tabular}


MEAN INFLOW RATIO: VELOCITY MAGNITUDE AND COMPONENTS IN SHAFT AXES (AVERAGE OVER ROTOR DISK FIGURE OF MERIT IS MEASURE OF ROTOR EFFICIENCY APPROPRIATE FOR AXIAL FLOW

BOTH DEFINITIONS INCLUDE CONVENTIONAL HOVER FIGURE OF MERIT, M = T*VIDEAL/P = T*SQRT(T/2*RHO*A)/P
\(\mathrm{M}=(\mathrm{PM}+\mathrm{PINT}) /(\mathrm{PO}+\mathrm{PI})=\mathrm{T} *(\mathrm{VIDEAL}+\mathrm{VINT}) /(\mathrm{PO}+\mathrm{PI})\)
\(\mathrm{M}=\mathrm{PIDEAL} / \mathrm{P}=1-\mathrm{PN} / \mathrm{P}=\mathrm{T} *(\mathrm{~V}+\mathrm{VIDEAL}) / \mathrm{P}\) FOR AXIAL FLOW (NOT VALID FOR ZERO POWER)
SECOND DEFINITION NEARLY PROPULSIVE EFFICIENCY ETA = TV/P FOR HIGH INFLOW
PROPULSIVE EFFICIENCY IS MEASURE APPROPRIATE FOR HIGH SPEED PROPULSION
ETA \(=-X * V / P=(P C+P P) / P \quad\) (NOT VALID FOR ZERO VELOCITY OR ZERO POWER)
ROTOR LIFT-TO-DRAG RATIO IS MEASURE OF ROTOR LIFTING EFFICIENCY, APPROPRIATE FOR EDGEWISE FLOW
\(L / D=L * V /(P I+P O)\) (NOT USEFUL FOR ZERO VELOCITY); ROTOR EQUIVALENT DRAG D \(=P / V+X=(P I+P O) / V\)
WING LIFT-TO-DRAG RATIO AND INDUCED EFFICIENCY E ARE MEASURES APPROPRIATE FOR FIXED WING
\(\mathrm{DI} / \mathrm{Q}=(\mathrm{L} / \mathrm{Q}) * * 2 /(\mathrm{E} * \mathrm{PI} * \mathrm{~B} * * 2)\)

INFLOW RATIO, INFLOW = VELOCITY / VTIP
REFERENCE ROTOR RADIUS \(R=24.7240 \mathrm{FT}\), ROTATIONAL SPEED OMEGA \(=260.000 \mathrm{RPM}\); TIP SPEED VTIP \(=\) OMEGA*R COEFFICIENT CF = F/QA, WING COEFFICIENT CFW \(=\mathrm{F} / \mathrm{QS}\)

ROTOR AREA A
RADIUS \(R=24.7240 \mathrm{FT}, \mathrm{SOLIDITY}\) RATIO \(S I G M A=0.09100\), AIR DENSITY RHO \(=0.001918\) SLUG/FT**3
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{16}{|l|}{BLADE SECTION ANGLE OF ATTACK (DEG)} \\
\hline RADIAL STATION & \(=0.560\) & 0.615 & 0.665 & 0.710 & 0.750 & 0.785 & 0.815 & 0.845 & 0.870 & 0.890 & 0.910 & 0.930 & 0.950 & 0.970 & 0.990 \\
\hline AZIMUTH (DEG) & ALPHA & & & & & & & & & & & & & & \\
\hline PSI \(=0.0\) & 7.1 & 7.5 & 7.4 & 7.3 & 7.1 & 6.8 & 6.5 & 5.8 & 4.8 & 4.3 & 4.5 & 4.7 & 4.3 & 3.8 & 24.7 \\
\hline PSI \(=15.0\) & 5.4 & 5.5 & 5.5 & 5.4 & 5.2 & 5.0 & 4.7 & 4.1 & 3.5 & 3.1 & 3.2 & 3.3 & 3.0 & 2.7 & 17.8 \\
\hline PSI \(=30.0\) & 4.1 & 4.0 & 3.8 & 3.6 & 3.4 & 3.2 & 3.0 & 2.6 & 2.1 & 1.8 & 1.9 & 2.0 & 1.8 & 1.6 & 11.1 \\
\hline PSI \(=45.0\) & 2.6 & 2.4 & 2.2 & 2.0 & 1.8 & 1.6 & 1.4 & 1.2 & 0.9 & 0.8 & 0.8 & 0.8 & 0.8 & 0.7 & 5.4 \\
\hline PSI \(=60.0\) & 1.7 & 1.5 & 1.3 & 1.1 & 1.0 & 0.8 & 0.6 & 0.5 & 0.3 & 0.2 & 0.2 & 0.2 & 0.2 & 0.1 & 1.4 \\
\hline PSI \(=75.0\) & 1.2 & 1.0 & 0.8 & 0.6 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & -0.1 & -0.1 & -0.1 & -0.1 & -0.1 & -0.5 \\
\hline PSI \(=90.0\) & 1.0 & 0.9 & 0.7 & 0.6 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & -0.1 & -0.1 & -0.1 \\
\hline PSI \(=105.0\) & 1.4 & 1.2 & 1.0 & 0.9 & 0.7 & 0.7 & 0.6 & 0.5 & 0.5 & 0.3 & 0.3 & 0.2 & 0.1 & 0.1 & 0.9 \\
\hline PSI \(=120.0\) & 2.0 & 1.8 & 1.7 & 1.6 & 1.5 & 1.4 & 1.3 & 1.1 & 1.0 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 2.5 \\
\hline PSI \(=135.0\) & 2.9 & 2.8 & 2.7 & 2.6 & 2.4 & 2.2 & 2.0 & 1.9 & 1.7 & 1.3 & 1.0 & 0.8 & 0.7 & 0.5 & 4.9 \\
\hline PSI \(=150.0\) & 4.2 & 4.2 & 4.0 & 3.7 & 3.5 & 3.2 & 3.0 & 2.9 & 2.6 & 2.0 & 1.6 & 1.3 & 1.1 & 0.8 & 7.7 \\
\hline PSI \(=165.0\) & 5.9 & 5.6 & 5.3 & 5.0 & 4.7 & 4.4 & 4.1 & 4.0 & 3.8 & 3.0 & 2.3 & 2.0 & 1.6 & 1.2 & 11.0 \\
\hline PSI \(=180.0\) & 7.5 & 7.2 & 6.8 & 6.4 & 6.0 & 5.7 & 5.4 & 5.3 & 5.0 & 4.0 & 3.2 & 2.7 & 2.3 & 1.7 & 14.5 \\
\hline PSI \(=195.0\) & 9.1 & 8.7 & 8.3 & 7.9 & 7.5 & 7.1 & 6.8 & 6.7 & 6.4 & 5.2 & 4.1 & 3.5 & 3.0 & 2.2 & 18.3 \\
\hline PSI \(=210.0\) & 10.6 & 10.1 & 9.7 & 9.3 & 8.9 & 8.5 & 8.2 & 8.1 & 7.7 & 6.3 & 5.1 & 4.4 & 3.7 & 2.8 & 22.0 \\
\hline PSI \(=225.0\) & 12.0 & 11.5 & 11.0 & 10.6 & 10.2 & 9.8 & 9.5 & 9.3 & 8.8 & 7.3 & 6.1 & 5.4 & 4.6 & 3.5 & 25.6 \\
\hline PSI \(=240.0\) & 13.0 & 12.5 & 12.1 & 11.7 & 11.3 & 10.9 & 10.6 & 10.3 & 9.7 & 8.2 & 7.1 & 6.4 & 5.5 & 4.2 & 28.8 \\
\hline PSI \(=255.0\) & 13.6 & 13.2 & 12.9 & 12.6 & 12.3 & 11.9 & 11.6 & 11.1 & 10.2 & 8.8 & 7.9 & 7.3 & 6.3 & 5.0 & 31.7 \\
\hline PSI \(=270.0\) & 18.1 & 16.5 & 13.4 & 13.0 & 13.0 & 12.8 & 12.5 & 11.6 & 10.3 & 9.1 & 8.5 & 8.0 & 7.1 & 5.6 & 34.2 \\
\hline PSI \(=285.0\) & 12.8 & 13.3 & 12.3 & 12.6 & 12.7 & 12.6 & 12.4 & 11.5 & 10.1 & 9.1 & 8.8 & 8.5 & 7.5 & 6.1 & 36.5 \\
\hline PSI \(=300.0\) & 11.5 & 11.6 & 12.0 & 11.8 & 11.7 & 11.5 & 11.2 & 10.3 & 9.0 & 8.2 & 8.1 & 8.1 & 7.3 & 6.0 & 37.1 \\
\hline PSI \(=315.0\) & 10.2 & 10.6 & 10.7 & 10.7 & 10.6 & 10.5 & 10.2 & 9.3 & 8.0 & 7.3 & 7.4 & 7.5 & 6.7 & 5.7 & 35.2 \\
\hline PSI \(=330.0\) & 8.1 & 9.2 & 9.6 & 9.8 & 9.8 & 9.7 & 9.4 & 8.5 & 7.2 & 6.6 & 6.9 & 7.1 & 6.4 & 5.4 & 34.1 \\
\hline PSI \(=345.0\) & 8.7 & 8.5 & 8.6 & 8.6 & 8.6 & 8.5 & 8.2 & 7.2 & 6.1 & 5.6 & 5.8 & 6.1 & 5.5 & 4.7 & 30.7 \\
\hline PSI \(=360.0\) & 7.1 & 7.5 & 7.4 & 7.3 & 7.1 & 6.8 & 6.5 & 5.8 & 4.8 & 4.3 & 4.5 & 4.7 & 4.3 & 3.8 & 24.7 \\
\hline
\end{tabular}
SUCCESSIVE SUBSTITUTION ITERATION, NO VARIABLES, NUMBER OF LEVELS \(=12, ~ L O O P=\) WAKE
ITERATION, COUNT \(=1\) FOR LEVEL \(1=\) UNIFORM INFLOW
\(; \quad\) LOOP \(=\) WAK
ITERATION, COUNT \(=1\), FOR LEVEL \(1=\) UNIFORM INFLOW ; LOOP = WAKE

BOUND CIRCULATION PEAKS (ENTIRE BLADE)
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline & BOUN & IRCUL & ATON & \multicolumn{3}{|c|}{RADIAL STATION} & WAKE GEOMEIRY \\
\hline AZIMUTH (DEG) & GMAX & GO & GI & RMAX & RO & RI & RGI \\
\hline PSI \(=0.0\) & 0.03448 & 0.03448 & -0.00058 & 0.89000 & 0.89000 & 0.25000 & -0.60000 \\
\hline PSI \(=15.0\) & 0.02837 & 0.02837 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & -0.60000 \\
\hline PSI \(=30.0\) & 0.01968 & 0.01968 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & -0.60000 \\
\hline PSI \(=45.0\) & 0.01032 & 0.01032 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & -0.60000 \\
\hline PSI \(=60.0\) & 0.00715 & -0.00005 & 0.00715 & 0.56000 & 0.97000 & 0.56000 & -0.60000 \\
\hline \(\mathrm{PSI}=75.0\) & 0.00638 & -0.00315 & 0.00638 & 0.49500 & 0.95000 & 0.49500 & -0.60000 \\
\hline PSI \(=90.0\) & 0.00694 & -0.00328 & 0.00694 & 0.49500 & 0.97000 & 0.49500 & -0.60000 \\
\hline PSI \(=105.0\) & 0.00845 & -0.00103 & 0.00845 & 0.56000 & 0.97000 & 0.56000 & -0.60000 \\
\hline PSI \(=120.0\) & 0.01064 & 0.01064 & 0.00000 & 0.66500 & 0.66500 & -0.60000 & -0.60000 \\
\hline PSI \(=135.0\) & 0.01564 & 0.01564 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=150.0\) & 0.02050 & 0.02050 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=165.0\) & 0.02478 & 0.02478 & 0.00000 & 0.93000 & 0.93000 & -0.60000 & -0.60000 \\
\hline PSI \(=180.0\) & 0.02877 & 0.02877 & 0.00000 & 0.93000 & 0.93000 & -0.60000 & -0.60000 \\
\hline PSI \(=195.0\) & 0.03117 & 0.03117 & 0.00000 & 0.95000 & 0.95000 & -0.60000 & -0.60000 \\
\hline PSI \(=210.0\) & 0.03228 & 0.03228 & -0.00475 & 0.95000 & 0.95000 & 0.25000 & -0.60000 \\
\hline PSI \(=225.0\) & 0.03178 & 0.03178 & -0.00438 & 0.95000 & 0.95000 & 0.34000 & 0.25000 \\
\hline PSI \(=240.0\) & 0.03040 & 0.03040 & -0.01064 & 0.95000 & 0.95000 & 0.34000 & 0.25000 \\
\hline PSI \(=255.0\) & 0.02904 & 0.02904 & -0.00728 & 0.95000 & 0.95000 & 0.42000 & 0.25000 \\
\hline PSI \(=270.0\) & 0.02802 & 0.02802 & -0.00334 & 0.95000 & 0.95000 & 0.49500 & 0.25000 \\
\hline PSI \(=285.0\) & 0.02842 & 0.02842 & -0.00813 & 0.91000 & 0.91000 & 0.42000 & 0.25000 \\
\hline PSI \(=300.0\) & 0.03042 & 0.03042 & -0.01571 & 0.89000 & 0.89000 & 0.34000 & 0.25000 \\
\hline PSI \(=315.0\) & 0.03322 & 0.03322 & -0.00968 & 0.89000 & 0.89000 & 0.34000 & 0.25000 \\
\hline PSI \(=330.0\) & 0.03514 & 0.03514 & -0.01169 & 0.89000 & 0.89000 & 0.25000 & -0.60000 \\
\hline PSI \(=345.0\) & 0.03591 & 0.03591 & -0.00541 & 0.89000 & 0.89000 & 0.25000 & -0.60000 \\
\hline PSI \(=360.0\) & 0.03448 & 0.03448 & -0.00058 & 0.89000 & 0.89000 & 0.25000 & -0.60000 \\
\hline
\end{tabular}

\footnotetext{
GO = OUTBOARD PEAK, GI = INBOARD PEAK, GMAX = PEAK WITH MAXIMUM MAGNITUDE CORRESPONDING RADIAL STATIONS OF PEAKS ARE RO,RI,RMAX; VALUE OUTSIDE BLADE IF PEAK NOT FOUND RGI = SPAN STATION OF INBOARD CIRCULATION PEAK
}

FOR DUAL PEAK WAKE MODEL: CONVERGENCE OF WAKE GEOMETRY ITERATION REQUIRES RGI = RI

GAMMA \(=\) BOUND CIRCULATION / OMEGA*R**2
REFERENCE ROTOR RADIUS \(R=24.7240\) FT, ROTATIONAL SPEED OMEGA \(=260.000\) RPM; TIP SPEED VTIP \(=\) OMEGA*R
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline BOUND CIRCULATION & \multicolumn{3}{|l|}{(ENTIRE BLADE)} & \multicolumn{3}{|c|}{RADIAL STATION} & WAKE GEOMETRY \\
\hline AZIMUTH (DEG) & GMAX & GO & GI & RMAX & RO & RI & RGI \\
\hline PSI \(=0.0\) & 0.02641 & 0.02641 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & 0.25000 \\
\hline PSI \(=15.0\) & 0.02287 & 0.02287 & 0.00000 & 0.87000 & 0.87000 & -0.60000 & -0.60000 \\
\hline PSI \(=30.0\) & 0.01707 & 0.01707 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & -0.60000 \\
\hline PSI \(=45.0\) & 0.01002 & 0.01002 & 0.00000 & 0.71000 & 0.71000 & -0.60000 & -0.60000 \\
\hline PSI \(=60.0\) & 0.00758 & 0.00758 & 0.00000 & 0.61500 & 0.61500 & -0.60000 & 0.56000 \\
\hline PSI \(=75.0\) & 0.00631 & -0.00073 & 0.00631 & 0.42000 & 0.99000 & 0.42000 & 0.49500 \\
\hline PSI \(=90.0\) & 0.00588 & 0.00588 & 0.00000 & 0.49500 & 0.49500 & -0.60000 & 0.49500 \\
\hline PSI \(=105.0\) & 0.00756 & 0.00756 & 0.00000 & 0.49500 & 0.49500 & -0.60000 & 0.56000 \\
\hline PSI \(=120.0\) & 0.00967 & 0.00967 & 0.00000 & 0.75000 & 0.75000 & -0.60000 & -0.60000 \\
\hline PSI \(=135.0\) & 0.01329 & 0.01329 & 0.00000 & 0.71000 & 0.71000 & -0.60000 & -0.60000 \\
\hline PSI \(=150.0\) & 0.01639 & 0.01639 & 0.00000 & 0.75000 & 0.75000 & -0.60000 & -0.60000 \\
\hline PSI \(=165.0\) & 0.01872 & 0.01872 & 0.00000 & 0.75000 & 0.75000 & -0.60000 & -0.60000 \\
\hline PSI \(=180.0\) & 0.02038 & 0.02038 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=195.0\) & 0.02195 & 0.02195 & 0.00000 & 0.93000 & 0.93000 & -0.60000 & -0.60000 \\
\hline PSI \(=210.0\) & 0.02276 & 0.02276 & 0.00000 & 0.93000 & 0.93000 & -0.60000 & 0.25000 \\
\hline PSI \(=225.0\) & 0.02302 & 0.02302 & 0.00000 & 0.93000 & 0.93000 & -0.60000 & 0.34000 \\
\hline PSI \(=240.0\) & 0.02266 & 0.02266 & 0.00000 & 0.93000 & 0.93000 & -0.60000 & 0.34000 \\
\hline PSI \(=255.0\) & 0.02252 & 0.02252 & 0.00000 & 0.93000 & 0.93000 & -0.60000 & 0.42000 \\
\hline PSI \(=270.0\) & 0.02346 & 0.02346 & -0.00151 & 0.87000 & 0.87000 & 0.34000 & 0.49500 \\
\hline PSI \(=285.0\) & 0.02414 & 0.02414 & -0.00347 & 0.89000 & 0.89000 & 0.34000 & 0.42000 \\
\hline PSI \(=300.0\) & 0.02338 & 0.02338 & -0.00442 & 0.89000 & 0.89000 & 0.34000 & 0.34000 \\
\hline PSI \(=315.0\) & 0.02486 & 0.02486 & -0.00218 & 0.87000 & 0.87000 & 0.42000 & 0.34000 \\
\hline PSI \(=330.0\) & 0.02662 & 0.02662 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & 0.25000 \\
\hline PSI \(=345.0\) & 0.02893 & 0.02893 & -0.00433 & 0.87000 & 0.87000 & 0.34000 & 0.25000 \\
\hline PSI \(=360.0\) & 0.02641 & 0.02641 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & 0.25000 \\
\hline
\end{tabular}

\footnotetext{
GO = OUTBOARD PEAK, GI = INBOARD PEAK, GMAX = PEAK WITH MAXIMUM MAGNITUDE CORRESPONDING RADIAL STATIONS OF PEAKS ARE RO,RI,RMAX; VALUE OUTSIDE BLADE IF PEAK NOT FOUND RGI = SPAN STATION OF INBOARD CIRCULATION PEAK
}

FOR DUAL PEAK WAKE MODEL: CONVERGENCE OF WAKE GEOMETRY ITERATION REQUIRES RGI = RI

REFERENCE ROTOR RADIUS \(R=24.7240\) FT, ROTATIONAL SPEED OMEGA \(=260.000\) RPM; TIP SPEED VTIP \(=\) OMEGA*R
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline BOUND CIRCULATION & \multicolumn{3}{|l|}{PEAKS (ENTIRE BLADE) BOUND CIRCULATION} & \multicolumn{3}{|c|}{RADIAL STATION} & WAKE GEOMETRY \\
\hline AZIMUTH (DEG) & GMAX & GO & GI & RMAX & RO & RI & RGI \\
\hline PSI \(=0.0\) & 0.02538 & 0.02538 & 0.00000 & 0.87000 & 0.87000 & -0.60000 & -0.60000 \\
\hline PSI \(=15.0\) & 0.02162 & 0.02162 & 0.00000 & 0.84500 & 0.84500 & -0.60000 & -0.60000 \\
\hline PSI \(=30.0\) & 0.01593 & 0.01593 & 0.00000 & 0.78500 & 0.78500 & -0.60000 & -0.60000 \\
\hline PSI \(=45.0\) & 0.01017 & 0.01017 & 0.00000 & 0.71000 & 0.71000 & -0.60000 & -0.60000 \\
\hline PSI \(=60.0\) & 0.00761 & 0.00761 & 0.00000 & 0.56000 & 0.56000 & -0.60000 & -0.60000 \\
\hline PSI \(=75.0\) & 0.00617 & -0.00019 & 0.00617 & 0.42000 & 0.99000 & 0.42000 & 0.42000 \\
\hline PSI \(=90.0\) & 0.00569 & 0.00569 & 0.00000 & 0.49500 & 0.49500 & -0.60000 & -0.60000 \\
\hline PSI \(=105.0\) & 0.00750 & 0.00750 & 0.00000 & 0.49500 & 0.49500 & -0.60000 & -0.60000 \\
\hline PSI \(=120.0\) & 0.00962 & 0.00962 & 0.00000 & 0.75000 & 0.75000 & -0.60000 & -0.60000 \\
\hline PSI \(=135.0\) & 0.01329 & 0.01329 & 0.00000 & 0.71000 & 0.71000 & -0.60000 & -0.60000 \\
\hline PSI \(=150.0\) & 0.01630 & 0.01630 & 0.00000 & 0.71000 & 0.71000 & -0.60000 & -0.60000 \\
\hline PSI \(=165.0\) & 0.01863 & 0.01863 & 0.00000 & 0.78500 & 0.78500 & -0.60000 & -0.60000 \\
\hline PSI \(=180.0\) & 0.02040 & 0.02040 & 0.00000 & 0.89000 & 0.89000 & -0.60000 & -0.60000 \\
\hline PSI \(=195.0\) & 0.02203 & 0.02203 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=210.0\) & 0.02294 & 0.02294 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=225.0\) & 0.02334 & 0.02334 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=240.0\) & 0.02338 & 0.02338 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=255.0\) & 0.02353 & 0.02353 & 0.00000 & 0.91000 & 0.91000 & -0.60000 & -0.60000 \\
\hline PSI \(=270.0\) & 0.02380 & 0.02380 & -0.00235 & 0.91000 & 0.91000 & 0.34000 & 0.34000 \\
\hline PSI \(=285.0\) & 0.02444 & 0.02444 & -0.00358 & 0.89000 & 0.89000 & 0.34000 & 0.34000 \\
\hline PSI \(=300.0\) & 0.02391 & 0.02391 & -0.00451 & 0.89000 & 0.89000 & 0.34000 & 0.34000 \\
\hline PSI \(=315.0\) & 0.02433 & 0.02433 & -0.00200 & 0.89000 & 0.89000 & 0.42000 & 0.42000 \\
\hline PSI \(=330.0\) & 0.02622 & 0.02622 & 0.00000 & 0.87000 & 0.87000 & -0.60000 & -0.60000 \\
\hline PSI \(=345.0\) & 0.02680 & 0.02680 & -0.00317 & 0.87000 & 0.87000 & 0.25000 & 0.34000 \\
\hline PSI \(=360.0\) & 0.02538 & 0.02538 & 0.00000 & 0.87000 & 0.87000 & -0.60000 & -0.60000 \\
\hline
\end{tabular}

\footnotetext{
GO = OUTBOARD PEAK, GI = INBOARD PEAK, GMAX = PEAK WITH MAXIMUM MAGNITUDE CORRESPONDING RADIAL STATIONS OF PEAKS ARE RO,RI,RMAX; VALUE OUTSIDE BLADE IF PEAK NOT FOUND RGI = SPAN STATION OF INBOARD CIRCULATION PEAK
FOR DUAL PEAK WAKE MODEL: CONVERGENCE OF WAKE GEOMETRY ITERATION REQUIRES RGI = RI
}

GAMMA \(=\) BOUND CIRCULATION / OMEGA*R**2
REFERENCE ROTOR RADIUS \(R=24.7240 \mathrm{FT}\), ROTATIONAL SPEED OMEGA \(=260.000 \mathrm{RPM}\); TIP SPEED VTIP \(=\) OMEGA*R

Chapter 8

\section*{ROTORCRAFT INPUT}

GENERAL REFERENCE FOR CHAPTER: CAMRAD II Documentation, Volume VI, Rotorcraft Input

\section*{ROTORCRAFT SHELL INPUT PARAMETERS}

COMPLETE DESCRIPTION IN ROTORCRAFT INPUT MANUAL

INPUT PARAMETERS ORGANIZED BY SHELL CLASS AND TYPE
ALL SHELL INPUT BLOCKS NEEDED BY JOB MUST BE DEFINED BY USER
even if only default parameter values are required usually include all blocks that might be needed in shell input file, at least with default values

USER SHOULD KNOW WHAT OPTIONS ARE AVAILABLE THROUGH INPUT PARAMETERS
rely on input manual for details

JAVA PROGRAM AVAILABLE TO CREATE AND EDIT ROTORCRAFT SHELL INPUT

\section*{UNITS OF INPUT PARAMETERS}

CONVENTION:
english or metric (SI) units, using consistent length-masstime system
foot-slug-second or meter-kilogram-second
angles are input in degrees

\section*{EXCEPTIONS TO THIS CONVENTION IN ROTORCRAFT} SHELL:
aircraft gross weight is in lb or kg
wind speed and aircraft speed can be in knots

\section*{8-1 CAMRAD II Analysis}

\section*{NAMELIST NLJob PARAMETERS:}

NCASES = number of cases
PLFILE \(=1\) to write plot file for job
OPSHLL = shell input file read
defaults: one case, no plot file written, shell input file read for first case only (OPSHLL = 1)

OPINIT = initialization from previous case
trim solution can be initialized from previous case using the trim loop variables (OPINIT \(=1\) ); or using the trim part total solution (OPINIT = 6); or both (OPINIT = 7); default is no initialization
default values generally used for other parameters
8-2 Class = CASE

\section*{CASE DESCRIPTION}

OPUNIT = identify units (English or metric)
TMTASK = select trim task
TNTASK = select transient task
FLTASK = select flutter task
skip, execute, or initialize
CONTROL PRINT OF INPUT DATA
usually do not print all core input or derived data (too much information)

\section*{PLUGINS}

PLUGIN = run installed shell plugins

\section*{TIMER AND DEBUG PARAMETERS}

ENVIRONMENT PARAMETERS
AERODYNAMIC
EARTH (GENERALLY USE DEFAULTS)

\section*{8-3 Class = TRIM}

\section*{OPERATING CONDITION}
initial values required for variables of trim loop; remaining parameters specify operating condition

FLIGHT SPEED AND WIND SPEED
four input formats; other three input values ignored wind speed usually zero for free flight operation, but can be used
only wind speed used for wind tunnel operation, flight speed ignored

ROTOR ROTATIONAL SPEED
four input formats; other input values ignored
CONTROL SETTINGS

\section*{SOLUTION PROCEDURE}

WAKE LOOP
LEVEL = wake analysis level
uniform inflow
nonuniform inflow, prescribed wake geometry nonuniform inflow, free wake geometry
any level except last executed can be skipped
typically skip levels when trim solution initialized from previous job (OPITJR >0) or from previous case

\section*{TRIM LOOP}

OPTRIM \(=0\) for no trim, 1 for trim iteration
MTRIM, MNAME, VNAME define trim problem
trim variables VNAME: convenient to identify using name of corresponding initial value parameter
can also use descriptive name, or name of symbol
trim quantities MNAME: can identify using descriptive name, name of symbol, or name of corresponding target value parameter
typical free flight trim:
\[
\begin{aligned}
& \text { MTRIM }=6, \\
& \text { VNAME }=\text { 'COLL','LATCYC','LNGCYC', } \\
& \text { 'PEDAL','PITCH','ROLL', } \\
& \text { MNAME = 'FORCE X','FORCE Y','FORCE Z', } \\
& \text { 'MOMENT X','MOMENT Y','MOMENT Z', }
\end{aligned}
\]
typical wind tunnel trim:
\[
\begin{aligned}
& \text { MTRIM }=3 \\
& \text { VNAME }=\text { 'COLL','LATCYC','LNGCYC', } \\
& \text { MNAME }=\text { 'CT } / \text { S' ','BETAS',''BETAC', }^{\text {MNA }}
\end{aligned}
\]
can select flight speed, wind speed, or rotor rotational speed as trim variable
with drive train model, should trim torque with throttle (rotor speed perturbation is additional system rigid degree of freedom)

\section*{PART SOLUTIONS}

MPSI = number of azimuth steps per revolution
shell generally uses same \(\Delta \psi\) throughout solution motion solution can use MPSIR < MPSI output solution can use MxTIME

OPPER \(=\) common period and full interaction default is to suppress vibratory interaction at inconsistant period

OPPART = rotor part definition solve only reference blade ( \(N\)-th blade): assuming identical motion
solve each blade separately: needed if system changed (using core input) so motion of blades no longer identical
solve all blades together: needed if blades are structurally coupled
analysis requires this option for rotor with gimbal or teeter joint perhaps needed with flexible swashplate consistently set filter of response, using OPFLTx

OPCHLD \(=\) identical motion of all blades option if solve all blades (OPPART \(=3\) ) harmonic solution for blade degree of freedom replaced by parent, with phase shift; parent is average (with phase shifts) of solution for all blades; for gimbal and teeter degree of freedom, also only \(p N \pm 1\) harmonics nonzero

METHOD = rotor part solution method harmonic or time finite element

\section*{EXTERNAL AEROACOUSTIC ANALYSIS}

OPPOST \(=\) post-trim solution
for high resolution, or to obtain partial angle-ofattack

OPEXT of rotor aerodynamics for prescribed coefficient increments

OPEXT of rotor wake for partial angle-of-attack MPSIH \(=\) resolution in time and wake age

\section*{DEGREES OF FREEDOM}

DOFx = zero, dynamic, or quasistatic

\section*{OUTPUT}

TRACEL \(=2\) and TRACEP \(=3\) to follow convergence and use TIMERP of class \(=\) CASE to see execution of all parts always get trace of trim loop

NAPRNT, NDPRNT, NRPRNT: control print of part solution (degree of freedom and constraint variables)
generally not used (blade position sensor gives physically meaningful motion)

NWPRNT \(=1\) for output from each wake iteration

\section*{SENSORS}
response calculated:
airframe sensors (MSSEN)
frame motion
output selection: N×PRNT, N×FILE, N×PLOT
number of azimuth steps: MxTIME
number of harmonics: MxHARM
harmonics calculated from time history at MxTIME azimuth steps

\section*{ELEMENTARY SCENARIO}
using initial values
azimuth step \(\Delta \psi=15 \mathrm{deg}\) solve reference blade only
no airframe or drive train degrees of freedom

\section*{8-4 Class = TRIM ROTOR}

\section*{CONTROL SETTINGS}

COMPONENT CONTROLS
HIGHER HARMONIC CONTROL HIGHER HARMONIC AUXILIARY FORCES

\section*{SOLUTION PROCEDURE}

\section*{DEGREES OF FREEDOM}

DOFx = zero, dynamic, or quasistatic
use DOFB and/or DOFM, depending on OPMODE
should not use
DOFS \(=0\) with swashplate not locked; or DOFD \(=0\) with trailing-edge flap not locked; or \(\operatorname{DOFB}(3)=0\) with pitch bearing not locked:
get zero motion, so zero control
instead use quasistatic motion (DOFx = 2)
or locked structure (LOCKPx \(=0\) )
similar issue when use blade modes:
should not use DOFM \(=0\) for blade pitch mode
option to use multiblade coordinates
use of transformed degree of freedom determined by product of appropriate DOFx and DOFMBC

\section*{OUTPUT}

\section*{HUB AND BLADE SENSORS}
response calculated:
hub load sensors (MHSEN)
control load sensors (MCSEN)
blade load sensors (MBSEN)
blade position sensors (MPSEN)
blade aerodynamics (MASEN)
wake velocity off rotor (MVSEN)
wake geometry (MWSEN)
output selection: N×PRNT, NxFILE, NxPLOT
number of azimuth steps: MxTIME
number of harmonics: MxHARM
harmonics calculated from time history at MxTIME azimuth steps

\section*{ELEMENTARY SCENARIO}
using initial values
only flap degree of freedom

\section*{8-5 Class = TRANSIENT}

\section*{TRANSIENT CONDITION}

TIMEB to TIMEE \(=\) time range of transient solution
SEPARATE PRESCRIBED CONTROL COMPONENT CONSTRUCTED FOR EACH ELEMENT OF INPUT
constructed with same time history kind, as specified by shell input
amplitudes specified by control settings in shell input
use core input for different time history kind for particular element

CONTROL SETTINGS

\section*{SOLUTION PROCEDURE}
parameters controlling convergence and accuracy of transient solution have already been discussed

\section*{PART SOLUTIONS}

METHOD and OPINIT define integration method
TRESP \(=\) response time step, \(\Delta t\)
to solve implicit parts, and save integration solution
TSTEPS \(=\) numerical integration time increment, \(\delta t\)
determines accuracy of integration
DEGREES OF FREEDOM
DOFx \(=\) zero, dynamic, or quasistatic

\section*{OUTPUT}

TRACEP \(=2\) to follow progress of transient integration and assess convergence

NPPRNT, NAPRNT, NRPRNT: control print of part solution (degree of freedom and constraint variables)
generally not used (blade position sensor gives physically meaningful motion)

\section*{SENSORS}
response calculated:
airframe sensors (MSSEN)
output selection: N×PRNT, NxFILE, N×PLOT time step: TxOUT
should calculate output for trim also

\section*{ELEMENTARY SCENARIO}
using initial values
implicit filters
no airframe or drive train degrees of freedom

\section*{8-6 Class = TRANSIENT ROTOR}

\section*{CONTROL SETTINGS}

\section*{AERODYNAMIC MODEL}

OPWAKE = rotor wake model
OPGEOM = wake geometry model

\section*{SOLUTION PROCEDURE}

DEGREES OF FREEDOM
DOFx = zero, dynamic, or quasistatic
use DOFB and/or DOFM, depending on OPMODE
option to use multiblade coordinates
use of transformed degree of freedom determined by product of appropriate DOFx and DOFMBC

\section*{OUTPUT}

\section*{HUB AND BLADE SENSORS}
response calculated:
hub load sensors (MHSEN)
control load sensors (MCSEN)
blade load sensors (MBSEN)
blade position sensors (MPSEN)
blade aerodynamics (MASEN)
wake velocity off rotor (MVSEN)
wake geometry (MWSEN)
output selection: N×PRNT, NxFILE, NxPLOT time step: TxOUT
should calculate output for trim also

\section*{ELEMENTARY SCENARIO}
using initial values
dynamic inflow, with aerodynamic interference
only flap degree of freedom only collective and cyclic multiblade coordinates

\section*{8-7 Class = FLUTTER}

\section*{TASK DEFINITION}

OPFLUT \(=\) time invariant or periodic
OPMEAN = constant coefficient approximation (averaged)
generally analyze time invariant equations, averaged in forward flight

MPSIAV = number of steps per rev in average
OPBLD \(=\) independent blade analysis
typically used for free vibration calculations
OPSTAB = flight dynamics analysis (quasistatic reduction of all states except aircraft rigid body motion)
should print stability derivatives as well (by printing loop equations)

\section*{AERODYNAMIC MODEL}

OPAERO = airframe aerodynamics
can trim in air, but obtain flutter equations without aerodynamics (free vibration)

\section*{SOLUTION PROCEDURE}
parameters controlling accuracy of flutter solution have already been discussed

MPSIAV = number of steps per rev in average

\section*{FLUTTER VARIABLES}

DOFx \(=\) degrees of freedom (zero, dynamic, or quasistatic)
CONx \(=\) controls (zero, used)
GUST = gust (zero, used)
M×SEN = output (zero, used)

DOFORD can be used to override internal determination of order of degree of freedom (based on columns of mass and spring matrices)
option to use symmetric/antisymmetric transform (for tiltrotor)
use of transformed degree of freedom determined by product of appropriate DOFx and DOFSYM, etc

\section*{OUTPUT}

N×PRNT: control print of loop solution for linearized equations allows examination of subsystem equations flutter analysis can also print final system equations

\section*{ANALYSIS OF FLUTTER EQUATIONS}

TASK = analysis tasks
OPEQN = equation sets analyzed
eigenanalysis, time history response, frequency response, rms gust response
for nine subsets of flutter equations

\section*{OUTPUT SELECTION}
can print flutter equations, or send to plot file for use outside CAMRAD II

\section*{VARIABLES AS OUTPUT}
can select degrees of freedom and controls as output variables in flutter analysis
must identify by name: run analysis to get list of names in flutter task "DESCRIPTION OF VARIABLES"

\section*{PERIODIC EQUATIONS}
evaluate and interpolate equations: MPSI, OPNTRP, MHARM integrate equations: MSTEP, TSTEP

\section*{EIGENANALYSIS}
eigenvalues are most common product of flutter analysis with experience, simplest to identify modes from eigenvalues (frequency and damping)
eigenvectors of states are influenced by normalization and units of degrees of freedom eigenvectors of output are more useful

TIME HISTORY RESPONSE
more efficient than transient task
FREQUENCY RESPONSE
RMS GUST RESPONSE
rms response not properly defined for system with unstable modes

\section*{ELEMENTARY SCENARIO}
using initial values
six rigid body degrees of freedom (free flight) quasistatic filters analysis of equations: eigenvalues only
no drive train degrees of freedom print rotorcraft equations and matrices of flutter equations

\section*{8-8 Class = FLUTTER ROTOR}

\section*{AERODYNAMIC MODEL}

OPAERO = rotor aerodynamics
can trim in air, but obtain flutter equations without aerodynamics (free vibration)

OPWAKE = rotor wake model
trim inflow: inflow not perturbed at all
dynamic inflow: DOFL controls inflow states
uniform inflow: perturbed trim model, similar to quasistatic dynamic inflow
lagged inflow: perurbed trim model with time lag, DOFL controls inflow state

\section*{SOLUTION PROCEDURE}

\section*{FLUTTER VARIABLES}

DOFx \(=\) degrees of freedom (zero, dynamic, or quasistatic)
CONx \(=\) controls (zero, used)
MxSEN = output (zero, used)
use DOFB and/or DOFM, depending on OPMODE
option to use multiblade coordinates
use of transformed degree of freedom determined by product of appropriate DOFx and DOFMBC, etc rotor in forward flight has periodic equations from aerodynamics of edgewise flow; with multiblade
coordinates, constant coefficient approximation (giving time invariant equations) is adequate up to advance ratio of about \(\mu=0.5\)

\section*{ELEMENTARY SCENARIO}
using initial values
dynamic inflow (but DOFL = 0), with aerodynamic interference
only flap degree of freedom

\section*{8-9 Class = AIRFRAME, Type \(=\) STRUCTURE}

\section*{ROTORCRAFT DESCRIPTION}

CONFIG \(=\) configuration
OPFREE \(=\) wind tunnel (constrained) or free flight
OPTRAN = drive train model
OPAERO = airframe aerodynamics
OPFIX = fixed wing

\section*{ROTORCRAFT INERTIA}

\section*{EQUIVALENT HUB INERTIA}
gross weight and center of gravity
input for entire system, including rotors
moments of inertia and elastic mode generalized masses
input for rotorcraft system with:
all mass defined by rotor input parameters (blade and flexbeam mass, point masses) removed
"equivalent hub inertia" added
often MASSR = total rotor inertia (blade and flexbeam mass and point masses, for all blades)
analysis prints calculated blade mass \(M_{b}\)
(can use to define MASSR \(=N M_{b}\) )

\section*{LOCATION OF ROTORCRAFT COMPONENTS} GENERAL LOCATIONS
first NROTOR locations: rotor hubs next NROTOR locations: rotor swashplates
only need OPAXES for these locations
swashplate locations created even if rotor model does not have swashplate
location number \(>2\) NROTOR: define as required for swashplate nonrotating actuators, auxiliary forces, tail boom, sensors, slung loads, airframe wings and bodies, and rotor-induced velocity calculation

\section*{ROTOR PYLON}

\section*{ELASTIC AIRFRAME}

SOURCE \(=\) mode properties from namelist arrays or table file mode shapes, LSHAPE and ASHAPE namelist input: use one line ( \(x, y\), and \(z\) components) for each location and each mode; group by location; see sample jobs for example with more than 40 locations or more than 40 modes, must use table file
aerodynamic control coefficient QAEROC

\section*{AIRFRAME AUXILIARY FORCES}

\section*{TAIL BOOM}

OPTAIL \(=\) configuration (circulation-control boom, reaction jet)
definition of tail boom parameters completed using core input

SLUNG LOAD
OPSLNG \(=\) model (rigid load, elastic load)
core input may be required to complete slung load model

\section*{AIRFRAME SENSORS}

OUTPUT SELECTED SEPARATELY FOR TRIM, TRANSIENT, AND FLUTTER TASKS

\section*{ELEMENTARY SCENARIO}
using initial values
constant rotational speed
no airframe elastic modes
no auxiliary forces
no sensors
wing-body center of action at rotorcraft center of gravity

\section*{8-10 Class \(=\) AIRFRAME, Type \(=\) AERODYNAMICS}

\section*{RIGID AIRFRAME AERODYNAMICS}

TNMODL and FLMODL = model for transient and flutter tasks
NONLINEAR AERODYNAMIC MODEL
NLSCR = from namelist variables (equations) or table file table kind (2D or 3D or 4D) identified by NLTABL in class \(=\) TABLES input
some derivatives from namelist variables are used with tables

STABILITY DERIVATIVE AERODYNAMIC MODEL
SDSRC = from namelist variables or table file

\section*{AIRFRAME FLOW FIELD}

VISRC = interference from calculation or table file (0 for no interference)

\section*{ELEMENTARY SCENARIO}
using initial values
nonlinear aerodynamic model, from namelist variables no airframe flow field
minimum input:
DRGOW \(=\) wing-body drag, \(f=D_{0} / q\left(\mathrm{ft}^{2}\right.\) or \(\left.\mathrm{m}^{2}\right)\)
DRGVW \(=\) vertical drag, \(g=D_{V} / q\left(\mathrm{ft}^{2}\right.\) or \(\left.\mathrm{m}^{2}\right)\)
required for correct rotor propulsive force
LFTAH, or MOMAW including tail effect; required for correct angle-of-attack stability in flight dynamics

\section*{8-11 Class \(=\) AIRFRAME, Type \(=\) CONTROL}

\section*{CONTROL MATRIX INPUT FORMAT}

TCxIN \(=\) calculated or input \(T_{C}\) matrix typically use calculated matrix, unless not adequate to represent connection of redundant controls to pilot's controls

CONTROL MATRIX

CONTROL MATRIX CALCULATION
SWASHPLATE GAINS AND PHASE ANGLES matrices defined in theory manual

\section*{ELEMENTARY SCENARIO}
using initial values calculated control matrix, from unit swashplate gains

\section*{8-12 Class \(=\) AIRFRAME, Type \(=\) DRIVE TRAIN}

\section*{TRANSMISSION AND ENGINE}

CONFIG \(=\) configuration
OPGOV = rotor speed governor
existence of drive train model determined by OPTRAN airframe structure input

GOVERNOR
FEEDBACK TO ROTOR COLLECTIVE OR ENGINE THROTTLE careful with units of throttle

\section*{ELEMENTARY SCENARIO}
using initial values airframe structure scenario selects constant rotational speed (no drive train model)

\section*{8-13 Class \(=\) ROTOR, Type \(=\) STRUCTURE}

\section*{ROTOR DESCRIPTION}

\section*{GEOMETRY AND STRUCTURE}

CONFIG = blade root configuration
OPAERO = blade aerodynamics
OPAZ = blade spacing
SHAFT = rotor shaft spring or free rotation
GIMBAL = gimbal/teeter joint
CONTRL = control system configuration
HINGE = flap and lag hinge configuration
PITCH = pitch bearing configuration
OPTEF = trailing-edge flaps

\section*{TIP PATH PLANE SENSOR}
normal displacement at ETPP (should be inboard of swept tip), or flap hinge rotation, or gimbal/teeter hinge rotation
can use core input to replace with blade sensor

\section*{REFERENCE LINE}
two straight segments, divided at droop-sweep node (which may be absent)
defined by torque offset, undersling, precone, droop, sweep, and span station of droop-sweep node
figure 1 shows geometry of blade and control system
all joints on first segment of reference line, inboard of droop-sweep node

PITCH BEARING ALIGNED WITH PRECONED REFERENCE LINE


Figure 8-1a Geometry of blade and control system.
counter-clockwise rotation shown;
positive input X 's for clockwise rotation, input X's still and Z's shown positive toward trailing edge


Figure 8-1b Geometry of blade and control system.
counter-clockwise rotation shown;
positive input X 's
for clockwise rotation, input X's still positive toward trailing edge and \(Z\) 's shown

\(\begin{array}{ll}\text { ITHETA } I_{\theta}=\int\left(y^{2}+z^{2}\right) d m & \text { about } \\ \text { IPOLAR } I_{P}=\int\left(y^{2}-z^{2}\right) d m & \text { beam } \\ \text { axis }\end{array}\)


KP \(\quad E A k_{P}^{2}=\int E\left(y^{2}+z^{2}\right) d A \quad\) about beam axis
EIFLAP \(E I_{z z}=\int E\left(z-z_{C}\right)^{2} d A\) about principal axes,
EILAG \(E I_{y y}=\int E\left(y-y_{C}\right)^{2} d A \quad\) relative tension center

Figure 8-1c Geometry of blade and control system.

\section*{BLADE SECTION PROPERTIES}

INPUT AS PIECEWISE LINEAR FUNCTIONS OF SPAN STATION
\begin{tabular}{llll}
\hline \hline & \begin{tabular}{l} 
chordwise \\
offset
\end{tabular} & \begin{tabular}{l} 
normal \\
offset
\end{tabular} & \begin{tabular}{l} 
twist or \\
pitch
\end{tabular} \\
\hline beam axis & XEA & ZEA & \\
quarter chord & XQC & ZQC & TWISTA \\
tension center & XC & ZC & THETAC \\
center of gravity & XI & ZI & THETAI \\
\hline \hline structure & KP & & \\
isotropic stiffness & EIFLAP, EILAG, EA, GJ, KT & \\
anisotropic stiffness & SUU, STU, SWU, SVU, STT \\
anisotropic stiffness & SWT, SVT, SWW, SVW, SVV & \\
inertia & MASS, ITHETA, IPOLAR & \\
\hline \hline
\end{tabular}

\section*{INPUT GEOMETRY}

RADIUS in ft or m , all other lengths input as fraction radius
blade radial stations Exxx measured along \(y\)-axis of rotating blade frame (not along reference line)
input in axes of rotating blade frame: B axes
\(X\) chordwise (+ aft, towards trailing edge)
Y spanwise
Z normal (+ up)
frame origin is hub node (center of rotation)
\(y\)-axis is span station variable (and wing reference line)
quantities measured in blade \(B\) axes, from hub node:
pitch link attachment to swashplate \(\times \mathrm{SP}\) pitch link attachment to pitch horn \(\times \mathrm{PH}\) torque offset XTO and undersling ZUS
quantities measured in blade \(B\) axes, from reference line:
locus of beam axis, XEA and ZEA
locus of quarter chord, XQC and ZQC damper attachments, XDAMP and ZDAMP locations of point masses, XPM and ZPM
aerodynamic twist TWISTA measured about wing reference line, from \(x-y\) plane of B frame
quarter chord locus and aerodynamic twist only required if blade aerodynamics used chord measured in plane perpendicular to wing reference line (in \(x-z\) plane of B axes)
quantities measured in section principal axes, from (straight) beam axis of element:
tension center, XC and ZC
center of gravity, XI and ZI
pitch angle of principle axes (THETAC and THETAI) measured about beam axis, from element \(x-y\) plane
quantities measured in twisted aerodynamic axes, from quarter chord:
trailing-edge flap hinge (XHTEF and ZHTEF)
and center of gravity (XCGTEF and ZCGTEF)
quantities measured in hub \(S\) axes:
actuator attachment to swashplate (xACTSP) actuator attachment to airframe (xACTAF)

\section*{CONVENTIONS}
beam axis must be straight within each element (between nodes)
such a beam axis is constructed using input beam axis positions at \(10 \%\) and \(90 \%\) element length, so jumps at nodes are possible
most important that relative positions of center of gravity, quarter chord, and feathering axis are correct
section mass specified per unit length along span axis (not along element axis)
structural and inertial properties within elastic beam element only used at Gaussian integration points (and at sensor or interface location)
inertial properties of rigid element also calculated using Gaussian integration

\section*{POINT MASSES}

\section*{REACTION JETS}

\section*{BLADE ANALYSIS}

OPBEAM = rigid body, rigid beam, or elastic beam elements
short elements usually rigid (DRELST sets criterion)
OPMAT = isotropic with elastic axis, or anisotropic/composite material

OPGEOM \(=\) second order, almost exact, or exact geometry NDOFx \(=\) number of elastic degrees of freedom that exist order reduction can be specified separately for trim, transient, and flutter tasks

KNODE and RNODE \(=\) specification of elements in blade structural model

OPWING = separate structural and wing components, or rigid wing component for last element (not recommended)

RIGID BLADE IS SIMPLEST MODEL:
OPBEAM \(=0, \mathrm{KNODE}=0\)

\author{
BLADE POSITION SENSORS \\ BLADE CFD POSITION SENSORS \\ BLADE MODE SENSORS \\ BLADE LOADS SENSORS
}

OUTPUT SELECTED SEPARATELY FOR TRIM, TRANSIENT, AND FLUTTER TASKS

\section*{ELEMENTARY SCENARIO}

RIGID, FLAPPING BLADE CONSTRUCTED
using initial values
single load path configuration
no swashplate mechanism, locked pitch bearing
flap hinge, no lag hinge
uniform properties, no point masses, linear twist
rigid body elements, no additional nodes
rigid wing model
structural dynamic properties estimated from section mass, blade radius, number of blades, normal tip speed, and solidity
required parameters:
TITLE, RADIUS, NBLADE, ROTATE, SIGMA, VTIPN
OPAERO (default 1)
EFLAP, KFLAP, DFLAP, AFLAP (defaults all 0.)
TWISTL
MASS (only first value used)

\section*{8-14 Class \(=\) ROTOR, Type \(=\) FLEXBEAM}

\section*{BLADE GEOMETRY AND STRUCTURE}

SNUB \(=\) snubber configuration
blade connected to hub or to flexbeam
snubber axes fixed to hub/flexbeam, or rotate with blade

\section*{CONFIGURATION}
blade extends from EBLADE to tip (torque tube is inboard part of blade)
connected to flexbeam at EFB (inboard of droopsweep node)
blade root (inboard end of torque tube) may be attached to hub or flexbeam through snubber, at radial station EROOT
flexbeam extends from hub to blade
swashplate required for pitch link (connected through pitch horn to torque tube) and control

\section*{FLEXBEAM SECTION PROPERTIES INPUT GEOMETRY}
quantities measured in blade \(B\) axes, from hub node:
snubber attachment to hub xSNUB
quantities measured in blade \(B\) axes, from reference line:
snubber attachment to flexbeam, xSNUB
snubber attachment to blade, xROOT

\section*{POINT MASSES}

\section*{FLEXBEAM ANALYSIS}

BLADE POSITION SENSORS
BLADE MODE SENSORS
BLADE LOADS SENSORS

\section*{OUTPUT SELECTED SEPARATELY FOR TRIM, TRANSIENT, AND FLUTTER TASKS}

\section*{ELEMENTARY SCENARIO}
using initial values
blade structure scenario selects single load patch configuration

\section*{8-15 Class \(=\) ROTOR, Type \(=\) AERODYNAMICS}

\section*{AERODYNAMIC PANELS}

DEFINED BY EDGES OF PANELS
at least two panels needed, in order to generate wing surface
aerodynamic collocation points are at midpoints of panels
\(\operatorname{REDGE}(1)=\) blade root, \(\operatorname{REDGE}(\) NPANEL +1\()=\) blade tip

\section*{GEOMETRY}

WING REFERENCE LINE
\(y\)-axis of rotating blade frame is wing reference line not structural reference line (which is defined by precone, droop, sweep)
\(r=y / R\) is span station variable

\section*{SPAN SCALE FACTOR}

RSCALE \(=\) blade radius \(R\) (constant)
with large droop, use core input to define correct RSCALE, such that (RSCALE \(\times \Delta r \times\) CHORD) is panel area

\section*{AERODYNAMIC PROPERTIES}

INPUT AS PIECEWISE LINEAR FUNCTIONS OF SPAN STATION
used at collocation points, wing tips, and other points as required

CHORD = wing chord (ft or m)
chord measured in plane perpendicular to wing reference line (in \(x-z\) plane of B axes)

ASWEEP = sweep angle of quarter chord angle between tangent to quarter chord locus and tangent to wing reference line required for swept/yawed flow model (OPYAW); in particular, controls use of Mach number normal to swept quarter chord

TAF, XAF, XAFF = airfoil table references positions airfoil section relative quarter chord and aerodynamic twist, consistent with definition of zero angle of attack and moment axis in airfoil table

\section*{AERODYNAMIC MODEL}

RTVTX \(=\) tip vortex rollup location
unless blade has highly tapered tip, use RTVTX \(=\) ETIP (default is 1.0) to avoid problems

NTRAIL = number of panels for far wake trailed vorticity specify panel boundaries TEDGE
defines structure and rollup of inboard vorticity in wake model

USMODL, OPUS = unsteady aerodynamic loads (attached flow)
DSMODL, OPDS = dynamic stall model

\section*{EXTERNAL AEROACOUSTIC ANALYSIS}

OPEXT \(=\) prescribed coefficient increment from table coefficient \(\Delta c_{\ell}, \Delta c_{d}, \Delta c_{n}, \Delta c_{x}, \Delta c_{m}, \Delta c_{r}, \Delta c_{g}\) \(\Delta c_{\ell f}, \Delta c_{d f}, \Delta c_{n f}, \Delta c_{x f}, \Delta c_{m f}, \Delta c_{r f}\) or \(\Delta\left(M^{2} c\right)\) form or load \(\Delta F_{Q C}, \Delta F_{3 Q C}, \Delta M_{Q C}, \Delta F_{f}, \Delta M_{f}\)

\section*{COMPUTATIONAL FLUID DYNAMICS ANALYSIS}

\section*{AERODYNAMIC SENSORS}

OUTPUT SELECTED SEPARATELY FOR TRIM, TRANSIENT, AND FLUTTER TASKS

\section*{ELEMENTARY SCENARIO}
using initial values
uniform properties, rectangular planform
static stall
minimum input: CHORD (only first value)

\section*{8-16 Class = ROTOR, Type = INFLOW}

\section*{ROTOR INFLOW}

KINTFx = forward flight interference factor KINTHx \(=\) hover interference factor

\section*{INFLOW PARAMETERS}

KHLMDA and KFLMDA \(=\) momentum theory correction factors
OPFFLI = linear inflow variation in forward flight FMLMDA \(=\) linear inflow variation from hub moments OPHZT = wake model for hover near zero thrust OPDMT \(=\) momentum theory model (global or differential) OPDUCT = ducted fan model

OPDW = dynamic wake model

\section*{ELEMENTARY SCENARIO}
using initial values
minimum input:
KHLMDA (default \(=1.1\) )
KFLMDA \((\) default \(=1.8)\)

\section*{8-17 Class = ROTOR, Type = WAKE}

\section*{ROTOR WAKE}

NONUNIFORM INFLOW CALCULATION OFF ROTORS
at locations defined in airframe axes

\section*{NONUNIFORM INFLOW PARAMETERS}

OPFW = far wake rollup
OPNW \(=\) lifting line theory order
generally use single-peak model, unless observe dualpeak character of calculated loading
second-order lifting-line theory most accurate
increase vortex core radius by \(15 \%\) chord if use first-order (quarter chord collocation point)
far wake extent should be at least two revolutions, more at low speed

OPRTV \(=\) tip vortex initial span station prescribed RTVTX (separate specification for wake geometry and loading) or Betz rollup

\section*{EXTERNAL AEROACOUSTIC ANALYSIS}

OPEXT = exclude vortex elements inside computational domain to calculate partial angle-of-attack in post-trim solution; must use fixed circulation strength

\section*{WAKE GEOMETRY}

OPRWG = rigid or prescribed wake geometry model
prescribed geometry calculated using input TWIST value (specified separate from aerodynamic twist input)

OPFWG = free wake geometry model tip vortex core size COREWG usually same as value CORE(1) used to calculate induced velocity
extent of calculated distorted geometry MFWG can be less than far wake extent RFW or RDW (calculated distortion extrapolated as necessary)

\section*{SUMMARY OF WAKE MODELS, OPTIONS, AND SCENARIOS}

Rotor Wake Scenarios
Rolled-Up Wake Model
Multiple-Trailer Wake Model
Multiple-Trailer Wake Model, With Consolidation
Tip Vortex Formation
Vortex Core Radius
Display Geometry
empirical models introduce many input parameters, most of which are used with default values or ignored

\section*{SCENARIOS}
using initial values
trim input:
LEVEL specifies rigid/prescribed or free wake geometry default azimuth step \(=15 \mathrm{deg}\)

\section*{ROTOR IN FORWARD FLIGHT}
single peak model
two revolutions of far wake second-order lifting-line theory, swept lifting line near wake extent = 60 deg tip vortex core radius = 20\% chord core type: Scully model, constant core radius wake vorticity elements: line segment with linear strength suppress inboard blade-vortex interaction rigid wake geometry model (not prescribed) free wake geometry model (Johnson method) minimum input:

OPFW = far wake rollup model (for dual peak)
RFW = far wake extent

\section*{ROTOR IN HOVER}
single peak model
five revolutions of far wake
three-quarter chord collocation point, swept lifting line
near wake extent \(=60\) deg
tip vortex core radius \(=20 \%\) chord
core type: Scully model, constant core radius wake vorticity elements: line segment with linear strength no shed wake vorticity prescribed wake geometry model (Kocurek and Tangler) free wake geometry model (Johnson method) minimum input:

TWIST = blade twist
RICWG \(=\) initial convection

\section*{ROTOR IN LOW SPEED FLIGHT}
single peak model
four revolutions of far wake second-order lifting-line theory, swept lifting line near wake extent \(=60\) deg tip vortex core radius \(=20 \%\) chord core type: Scully model, constant core radius wake vorticity elements: line segment with linear strength suppress inboard blade-vortex interaction rigid wake geometry model (not prescribed) free wake geometry model (general method)
minimum input:
OPFW = far wake rollup model (for dual peak)
RFW = far wake extent

\section*{8-18 Class = TABLES}

NAME, FILE UNIT NUMBER, AND FILE NAME OR LOGICAL NAME FOR FOR ALL TABLES REQUIRED BY ANALYSIS

\section*{ROTOR AERODYNAMICS}

AIRFOIL TABLE
AFTABL = same table for all rotors, or separate tables default file name or logical name: BLADEAIRFOILn

AIRFRAME STRUCTURE
MODE PROPERTIES INPUT
TAIL BOOM COEFFICIENT INPUT

\section*{AIRFRAME AERODYNAMICS \\ NONLINEAR AERODYNAMIC MODEL INPUT \\ \[
\text { NLTABL }=2 \mathrm{D} \text { or } 3 \mathrm{D} \text { or } 4 \mathrm{D} \text { table }
\] \\ STABILITY DERIVATIVE AERODYNAMIC MODEL INPUT INTERFERENCE VELOCITY}

\section*{INPUT AND COMPONENTS INPUT}

GENERAL REFERENCE FOR CHAPTER: CAMRAD II Documentation, Volume IV, Input; and Volume V, Components Input

\section*{FOLLOWING SECTIONS ARE EXAMPLES OF USING CORE INPUT TO MODIFY ROTORCRAFT CONSTRUCTED BY SHELL \\ input parameters of core system pieces and components}

\section*{SEE ALSO DOCUMENTS DEMONSTRATING USE OF CAMRAD II CORE INPUT:}

TILTROTOR ANALYSIS construction of elastic cantilever wing, replacing normal modes representation of rotor support

BEARINGLESS ROTOR ANALYSIS construction of snubber-damper with axes following blade pitch, replacing snubber with axes fixed to hub

CLOSED-LOOP HIGHER-HARMONIC CONTROL ANALYSIS completion of regulator loop, including construction of filters to obtain harmonic loads for feedback

PENDULUM ABSORBER ANALYSIS construction of flapwise absorber

\section*{SEVERAL EXAMPLES OF CORE INPUT ARE INCLUDED IN CAMRAD II EXTRAS FOLDER}
frozen wake geometry for second case or job (solution procedure change)
individual blade control
interblade damper (extensive core input)
elastic beams for trailing-edge flap structure tip-path-plane sensors for hingeless and bearingless rotors autopilot for transient task prescribed airframe motion for transient task revised trim variables or input trim matrix

\section*{9-1 Lag Damper}

ROTORCRAFT SHELL DEFINES LINEAR SPRING AND DAMPING VALUE FOR EACH JOINT CONSTRUCTED

CORE INPUT NEEDED FOR NONLINEAR SPRING OR DAMPING
example: hydraulic lag damper

\section*{IDENTIFY NAME OF COMPONENT WITH LAG DAMPER}

ELEMENTS CONSTRUCTED ARE LISTED AT BEGINNING OF JOB
name: 'ROTOR n BLADE m ELEMENT k'

IDENTIFY SPRING/DAMPER (j) FOR LAG JOINT PRINT CORE INPUT FOR COMPONENT
using class = CASE parameters:
\[
\begin{aligned}
& \text { TMTASK }=2 \\
& \text { NPRNTC }=1 \\
& \text { PCLASS }=\text { 'COMPONENT' } \\
& \text { PTYPE }=\text { 'RIGID BODY' or 'BEAM' } \\
& \text { PNAME }=\text { name }
\end{aligned}
\]

\section*{OBTAIN DAMPER CHARACTERISTICS}

HYDRAULIC DAMPER MOMENT \(M\), AS FUNCTION OF LAG RATE \(\dot{\zeta}\) :
\[
M=\min \left(M_{\max }, M_{\max }\left(\dot{\zeta} / \dot{\zeta}_{\mathrm{ref}}\right)^{2}\right)=\min \left(x, y \dot{\zeta}^{2}\right)
\]

\section*{PREPARE CORE INPUT}

IDENTIFY INPUT BY CLASS, TYPE, AND NAME usually for all blades

LINEAR + HYDRAULIC DAMPER: CTYPE \(=3\)
NO LINEAR DAMPING: CLIN \(=0\). keep CEQUIV, for flutter and solution procedures

HYDRAULIC DAMPING: polynomial order and coefficients
figure 1 illustrates input required
```

....
....
....
\&NLDEF action='end of shell',\&END
! hydraulic lag damper (core input)
!================================================================================
\&NLDEF class='COMPONENT',type='BEAM',name='ROTOR n BLADE m ELEMENT k',\&END
\&NLVAL
CTYPE(j)=3,CLIN(j)=0.,
NCHYDA(j)=0,CHYDA (1, j)=x,
NCHYDB(j)=2,CHYDB(1,j)=0.,0.,Y,
\&END
!===============================================================================
\&NLDEF action='end of core',\&END

```

Figure 9-1 Lag damper.

\section*{9-2 Additional Output}

\section*{ROTORCRAFT SHELL CONSTRUCTS OUTPUT FOR VARIOUS STRUCTURAL AND AERODYNAMIC SENSORS}

\section*{CORE INPUT NEEDED FOR OUTPUT OF SYSTEM VARIABLES}
example: output for input/output interface variable

\section*{IDENTIFY NAME OF INPUT/OUTPUT INTERFACE}

USE INPUT PROGRAM TO EXAMINE INTERFACES CONSTRUCTED BY SHELL
these variables are already being solved, so only need to add output to system
can also examine trim loops, to determine where to put write module

\author{
name: 'interfacename'
}

\section*{EXAMPLES OF NAMES}
'ROTOR n PITCH CONTROL BLADE m'
'AIRFRAME AERODYNAMIC FORCE WB'
'ROTOR \(n\) FORCE (RTR AXES) '
'ROTOR n WAKE V, PNL kkk W mm'
see CAMRAD II Documentation, Volume III, Rotorcraft Theory; Chapter "Rotorcraft System"

PREPARE CORE INPUT
IDENTIFY INPUT BY CLASS, TYPE, AND NAME
CREATE OUTPUT PIECE FOR INTERFACE VARIABLE
specify number of time steps and number of harmonics desired
name of output piece must be unique

\section*{ADD WRITE MODULE TO WAKE LOOP}
here " 20 " is intended to be larger than current number of write modules in loop
if add write module to some other loop, should consider where the analysis solves for the variable
figure 2 illustrates input required
```

....
••••
\&NLDEF action='end of shell',\&END

```

```

\&NLDEF class='OUTPUT', name='NEWOUTPUT', \&END
\&NLVAL KINDY=2,YNAMEV='interfacename' ,MTIME=n,MHARM=m,\&END
\&NLDEF class='TRIM LOOP',type='SUCCESSIVE SUBSTITUTION', name='WAKE' , \&END
\&NLVAL NWRT=20,KINDW(20)=7,WRT(20)='NEWOUTPUT', \&END

```

```

\&NLDEF action='end of core',\&END

```

Figure 9-2 Additional output.

\section*{9-3 General Approach}

\section*{ROTORCRAFT SHELL CONSTRUCTS TYPICAL SYSTEM CORE INPUT NEEDED TO MODIFY MODEL \\ often just modify structural model, perhaps just change geometry \\ ASSISTANCE AVAILABLE FROM MAINTENANCE/SUPPORT CONTRACTOR}

\section*{STEPS IN DEVELOPING CORE INPUT TO MODIFY ROTORCRAFT SHELL MODEL:}
1) PREPARE SHELL MODEL

SHELL INPUT AND JOBS
get as close as possible to desired system usually shell creates all of aerodynamic model
2) EXAMINE SYSTEM CONSTRUCTED BY SHELL

CAN USE Input PROGRAM TO IDENTIFY SYSTEM PIECES
or analysis with TMTASK \(=2\), NPRNTD \(=1\)

PRINT CORE INPUT FOR SPECIFIC SYSTEM PIECES OR ENTIRE MODEL
using class \(=\) CASE parameters:
TMTASK=2 (perhaps TNTASK=2,FLTASK=2) and NPRNTC=n,PCLASS= ...,PTYPE= . . .,PNAME= ...
to print core input for entire system, use
NPRNTC=1,PCLASS='

\section*{USE CORE INPUT CREATED BY SHELL AS MODEL AND GUIDE, AND AS INPUT MANUAL \\ follow pattern for joints, degrees of freedom, interfaces, sensors}

IDENTIFY (BY NAME) PIECES TO BE MODIFIED

TYPICALLY INTERESTED IN STRUCTURE
structural dynamic components
structural dynamic interfaces
modes
response for degrees of freedom
response for constraint forces
rigid degrees of freedom require rigid response piece
other quantities (elastic degrees of freedom, joint degrees of freedom, constraint forces) require variable response pieces

MAY NOT NEED TO CHANGE SOLUTION PROCEDURE ASSIGN NEW COMPONENTS AND INTERFACES TO EXISTING PARTS AND MODES
following pattern constructed by shell
3) SKETCH SYSTEM CONSTRUCTED BY SHELL GEOMETRY AND LOGICAL RELATION, FOR COMPONENTS AND INTERFACES
see CAMRAD II Documentation, Volume III, Rotorcraft Theory; Volume IV, Input; Volume V, Components Input STRUCTURAL MODEL REQUIRES GEOMETRY
summary of geometry in rotorcraft shell: Table 7.6 of Volume III
identify body axes of all elements
airframe: \(z\)-axis down, \(x\)-axis forward
hub: \(z\)-axis in \(+T\) direction, \(x\)-axis downstream beams: \(x\)-axis axial
and axes of structural dynamic interfaces
locations, joints, connections
4) DEFINE MODIFIED SYSTEM

GEOMETRY AND LOGICAL RELATION, FOR COMPONENTS AND INTERFACES
sketch geometry of each body and beam in isolation, showing body axes, location axes, joint axes, connection axes (ignore frames)
connection axes define position and orientation of interfaces
typically interface axes are rotated from body axes back to frame; then rotation matrix of location or connection is transpose of rotation matrix of nominal response
convenient approach, but in general not necessary to involve frame in description of component geometry
conventions for describing rotations:
Euler angles, \(C=X Y Z\) (most useful)
arbitrary rotations, \(C=U V W\)
Unit, \(C=I\); or input matrix
system assembled by gluing connection axes together
create sensors (with output) for all new joints and degrees of freedom
perhaps create sensors for loads as well
use blade position sensor solution procedures (MPSEN =1)
identify frames for description of motion (new frames probably not needed)
frames and nominal (rest position or rotating) are required to describe motion of structural dynamic components motion of body axes is measured relative frame of component
nominal position and orientation defined in rigid response piece
sketch each input/output interface, and connection to components

\section*{5) PREPARE CORE INPUT}

\section*{DELETE EXISTING SYSTEM PIECES AS REQUIRED}

IDENTIFY INPUT FOR NEW OR MODIFIED SYSTEM PIECES BY CLASS, TYPE, AND NAME
set up all NLDEF namelists
define parameters in NLVAL namelists (guided by input manuals, and values constructed by shell)

SET UP INPUT FOR ONE BLADE, THEN DUPLICATE FOR OTHER BLADES
reference (last) and other blades are different
reference blade has sensors and aerodynamics interfaces, other blades defined as child response
follow pattern created by shell
RUN ANALYSIS TO FIND ERRORS

\section*{6) CHECK MODIFICATIONS}

\section*{USE Input PROGRAM TO DRAW GEOMETRY OF SYSTEM WITH NEW COMPONENTS AND INTERFACES}
draws rest position, as specified by nominal in response pieces
rest position should be close to where component will be when the system is assembled
can draw axes at structural dynamic interfaces (change drawing option to get orientation, in separate file)

CHECK STRUCTURAL DYNAMICS AND GEOMETRY (WITHOUT AERODYNAMICS)
trim solution (NRPRNT \(=7\) )
if geometry of connections and rest positions is correct, trim solution for rigid motion of structural dynamic components should be small
units of rigid motion: ft or m for linear, radians for angular
trim position sensors (MPSEN), and sensors for new joints and new degrees of freedom
free vibration calculation of frequencies (flutter task)
solution for new input/output interfaces

\section*{7) RUN TEST CASES}

\section*{CORE INPUT CONSTRUCTED BY SHELL CAN BE CHANGED BY SHELL PARAMETERS}

IN EXISTING STRUCTURAL DYNAMIC COMPONENTS, NUMBER OF LOCATIONS, JOINTS, CONNECTIONS, INTERFACES, AND SENSORS CAN BE CHANGED
for example, different models are constructed with and without aerodynamics, or modes, or sensors

CARE REQUIRED SO CORE CHANGES REMAIN CONSISTENT WITH MODEL CREATED BY SHELL

LOCATIONS, JOINTS, CONNECTIONS, INTERFACES, SENSORS ON A STRUCTURAL DYNAMIC COMPONENT CAN BE IGNORED
default is to ignore location, joint, connection, interface, or sensor
so core input can add items without knowing exactly the current number
add locations, joints, connections, interfaces, sensors beyond the maximum possible created by the shell

GEOMETRY DEFINITION FOR STRUCTURAL DYNAMIC COMPONENTS

CAN CREATE NEW COMPONENTS AND INTERFACES USING CORE INPUT

MAY ENCOUNTER ERROR MESSAGE:
matrix singular
part differential equations
inverting PHIXOB
OR FIND VERY LARGE ANGLES (1 or 2 radians) IN TRIM SOLUTION (printed using NRPRNT = 7)

\author{
MAY BE CAUSED BY 180 DEGREE MISMATCH OF CONNECTION AXES (REST POSITION) ON TWO SIDES OF STRUCTURAL DYNAMIC INTERFACE \\ produces singular constraint equations \\ CAN USE input PROGRAM TO DRAW AXES AT STRUCTURAL DYNAMIC INTERFACES
}

Chapter 10
CONCLUSION

\section*{it is DIFFICULT to analyze rotorcraft}

COMPLEX, MULTIDISCIPLINARY SYSTEM
STILL A LOT WE DO NOT KNOW ABOUT AERODYNAMICS, DYNAMICS, AND STRUCTURES OF ROTORCRAFT

STILL NEED ENGINEERING JUDGEMENT, EXPERIENCE, AND MUCH TESTING OF THE ACTUAL SYSTEM

\section*{IT IS IMPOSSIBLE TO ANALYZE ROTORCRAFT EFFECTIVELY WITHOUT THE PROPER TOOLS}


CAMRAD II IS A SOPHISTICATED AND MATURE AEROMECHANICAL ANALYSIS OF HELICOPTERS AND ROTORCRAFT
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[^0]:    for each time value constraint equations for each solution pass for each vector evaluate component output: total $f_{l}=x$ translate total to difference output equations for each vector evaluate component output: total $y_{q}=x$ translate total to difference implement order reduction: $f_{l}, y_{q}$

[^1]:    ORIGIN = FILE (IDENT = HH:MM:SS DD-MMM-YY), MODIFICATION BY NAMELIST
    ....
    . . .

