Rotor Noise from CAMRAD II

This note describes the calculation of rotor noise from CAMRAD II results for blade motion and aerodynamic loading. The loading, thickness, and broadband noise sources are considered. The loading and thickness noise calculations are based on Ffowcs Williams and Hawkings (1969) and Farassat (1975, 1981). Not considered is high-speed impulsive noise, which requires the quadrupole source terms for the FW-H equation. The broadband noise calculations are based on Pegg (1979) or Brooks (Brooks, Pope, and Marcolini (1989) and Weir, Jumper, Burley, and Golub (1995)). The model of Brooks covers airfoil self noise sources: turbulent boundary-layer trailing-edge noise, laminar boundary-layer vortex-shedding noise, trailing-edge bluntness noise, and tip vortex formation noise. Not considered are blade-wake interaction noise, which requires the random loading on the blade caused by random wake features; and turbulence ingestion noise, which requires the spectrum of the atmospheric turbulence.

Ffowcs Williams-Hawkings Equation

Ffowcs Williams and Hawkings (1969) extended Lighthill's acoustic analogy to include moving surfaces. Consider a surface S defined by the function $f(x) = 0$. The normal to the surface is $n_i = \frac{1}{|\nabla f|}\partial f/\partial x_i$, and the normal velocity of the surface is $v_n = -\frac{1}{|\nabla f|}\partial f/\partial t$. The surface divides the volume into interior ($f < 0$) and exterior ($f > 0$) domains. The normal and normal velocity are outward, directed into the exterior domain. It is possible to define f such that $|\nabla f| = 1$ on the surface.

Manipulating the Navier-Stokes equations in the volume V, bounded in the far field by the surface A and including the moving surface S (Johnson (2013)), gives the wave equation for density ρ

$$\frac{\partial^2\rho}{\partial t^2} - c_s^2\nabla^2\rho = \frac{\partial^2}{\partial x_i x_j}\big(T_{ij}H(f)\big) - \frac{\partial}{\partial x_i}\big((p-p_0)\delta_{ij} - \tau_{ij}\big)n_j|\nabla f|\delta(f) + \frac{\partial}{\partial t}\big(\rho_0 v_n|\nabla f|\delta(f)\big)$$

This is the Ffowcs Williams-Hawkings (FW-H) equation, which is the foundation of current rotor noise calculations. It is a rearrangement of the Navier-Stokes equations (without approximation) to an inhomogeneous wave equation for the density, with two surface source terms and a volume source term. The equation was derived by adding an artificial interior flow to the exterior flow, so the equations of motion are for the flow in unbounded space, with discontinuities at the moving surface. So the free-space Green's function can be used to solve the wave equation.

For small disturbances, the wave equation for the acoustic pressure $\tilde{p} = c_s^2\tilde{\rho}$ is:

$$\Box^2\tilde{p} = \frac{\partial^2}{\partial x_i x_j}\big(T_{ij}H(f)\big) - \frac{\partial}{\partial x_i}\big(\ell_i|\nabla f|\delta(f)\big) + \frac{\partial}{\partial t}\big(\rho_0 v_n|\nabla f|\delta(f)\big)$$

where $\ell_i = P_{ij}n_j = \big((p-p_0)\delta_{ij} - \tau_{ij}\big)n_j$, and v_n is the normal velocity at the surface. The three terms on the right-hand side are quadrupole, dipole, and monopole sources of noise, respectively. The quadrupole term is Lighthill's turbulent stress tensor, in the volume exterior to the surface S. The dipole term is from the loading, the surface forces on S; the stress τ_{ij} is usually not a significant contribution to ℓ_i. The monopole term is from the thickness, the volume displacement of the surface.

Integral Formulations

The free-space Green's function can be used to obtain the solution of the inhomogeneous wave equation as integrals of source terms over the surface and volume. For the acoustic analogy, the result is not an integral equation, rather the source flow field is considered to be an input to the acoustic solution. This approach allows separation of the aerodynamic problem (for the sources) and the acoustic problem (for the sound). The acoustic analogy is particularly useful if the volume sources can be ignored, requiring only integrals over surfaces. The integral formulation is not unique. The choice among the numerous manipulations possible is based on implementation considerations, as well as facilitating exposition.

Considering only the thickness and loading terms (surface sources), the Ffowcs Williams-Hawkings equation is

$$\Box^2 \tilde{p} = \frac{\partial}{\partial t}\big(\rho_0 v_n \delta(f)\big) - \frac{\partial}{\partial x_i}\big(\ell_i \delta(f)\big)$$

(assuming $|\nabla f| = 1$). The solution (Johnson (2013)) is developed from the free-space Green's function for the wave equation, which is a function of $g = \tau - t + r/c_s$ for an observer at $(\mathbf{x}, t)$ and a source at $(\mathbf{y}, \tau)$. The position of the observer relative to the source is $\mathbf{r} = \mathbf{x} - \mathbf{y}$, and $r = |\mathbf{x} - \mathbf{y}|$, so $\hat{r} = \mathbf{r}/r$ is the unit vector in the radiation direction. The velocity of the source relative to the undisturbed fluid is $v_i = \partial y_i/\partial \tau$, and $M_r = \hat{r}_i v_i/c_s$ is the Mach number of the source (point $\mathbf{y}$) in the radiation direction. The derivatives of r are: $\frac{\partial r}{\partial x_i} = \hat{r}_i$, $\frac{\partial r}{\partial y_i} = -\hat{r}_i$, and $\frac{\partial r}{\partial \tau} = \frac{\partial r}{\partial y_i}\frac{\partial y_i}{\partial \tau} = -\hat{r}_i v_i = -v_r$. The Doppler factor is

$$\frac{\partial g}{\partial \tau} = 1 + \frac{1}{c_s}\frac{\partial r}{\partial y_i}\frac{\partial y_i}{\partial \tau} = 1 - \frac{\hat{r}_i v_i}{c_s} = 1 - M_r$$

The solution of $g = \tau - t + |\mathbf{x} - \mathbf{y}(\tau)|/c_s = 0$ is the retarded time, or emission time τ_e. At τ_e, the emission position is $\mathbf{y}_e$, and the emission distance is $r_e = |\mathbf{x} - \mathbf{y}_e| = c_s(t - \tau_e)$. For subsonic sources, $g = 0$ has only one solution, so there is only one emission point and time. The solution is

$$\begin{aligned}
4\pi\tilde{p}(\mathbf{x}, t) &= \frac{\partial}{\partial t}\int_{-\infty}^{t}\int \rho_0 v_n \delta(f)\frac{\delta(g)}{r} d\mathbf{y}\, d\tau - \frac{\partial}{\partial x_i}\int_{-\infty}^{t}\int \ell_i \delta(f)\frac{\delta(g)}{r} d\mathbf{y}\, d\tau \\
&= \frac{\partial}{\partial t}\int_{-\infty}^{t}\int \big(\rho_0 v_n + \ell_i \hat{r}_i/c_s\big)\frac{\delta(f)\delta(g)}{r} d\mathbf{y}\, d\tau + \int_{-\infty}^{t}\int \ell_i \hat{r}_i \frac{\delta(f)\delta(g)}{r^2} d\mathbf{y}\, d\tau \\
&= \frac{\partial}{\partial t}\int_{f=0}\left[\frac{\rho_0 v_n + \ell_r/c_s}{r(1 - M_r)}\right]_{\text{ret}} dS + \int_{f=0}\left[\frac{\ell_r}{r^2(1 - M_r)}\right]_{\text{ret}} dS
\end{aligned}$$

where $\ell_r = \ell_i \hat{r}_i$ is the loading in the radiation direction. The subscript "ret" means the integrand is evaluated at the retarded time, $\tau = t - r/c_s$, the time when the sound was emitted. This is Farassat's Formulation 1; see Farassat (1975, 1981). The loading is

$$\ell_i = P_{ij} n_j = \big((p - p_0)\delta_{ij} - \tau_{ij}\big) n_j \cong (p - p_0) n_i$$

so

$$\ell_r = P_{ij} n_j \hat{r}_i \cong (p - p_0) n_i \hat{r}_i = (p - p_0)\cos\theta$$

where θ is the angle between the surface normal and the radiation direction. While the spatial derivatives have been eliminated, the time derivative remains, requiring numerical differential in implementations.

In order to move the time derivative inside the integral, note:

$$\frac{\partial}{\partial t}\Big[Q(\mathbf{y}, \tau)\Big]_{\text{ret}} = \left[\frac{\partial Q/\partial \tau}{|\partial t/\partial \tau|}\right]_{\text{ret}} = \left[\frac{1}{(1 - M_r)}\frac{\partial Q}{\partial \tau}\right]_{\text{ret}}$$

Then

$$4\pi\tilde{p}(\mathbf{x}, t) = \int_{f=0}\left[\frac{1}{(1 - M_r)}\frac{\partial}{\partial \tau}\frac{\rho_0 v_n + \ell_r/c_s}{r(1 - M_r)}\right]_{\text{ret}} dS + \int_{f=0}\left[\frac{\ell_r}{r^2(1 - M_r)}\right]_{\text{ret}} dS$$

Now define $\dot{v}_n = \partial v_n/\partial\tau = \partial(v_i n_i)/\partial\tau$, and $\dot{\ell}_r = \dot{\ell}_i \hat{r}_i = (\partial\ell_i/\partial\tau)\hat{r}_i$. From

$$
\begin{aligned}
\frac{\partial}{\partial\tau}\frac{1}{r(1-M_r)} &= -\frac{1}{r^2(1-M_r)^2}\frac{\partial}{\partial\tau}(r - rM_r) \\
&= -\frac{1}{r^2(1-M_r)^2}\frac{\partial}{\partial\tau}(r - r_i v_i/c_s) \\
&= -\frac{1}{r^2(1-M_r)^2}(-\hat{r}_i v_i + v_i^2/c_s - r_i \dot{v}_i/c_s) \\
&= \frac{1}{r^2(1-M_r)^2}(c_s M_r - c_s M^2 + r\dot{M}_r) \\
\frac{\partial\ell_r}{\partial\tau} = \frac{\partial(\ell_i\hat{r}_i)}{\partial\tau} &= \frac{\partial\ell_i}{\partial\tau}\hat{r}_i + \ell_i\frac{\hat{r}_i v_r - v_i}{r} \\
&= \dot{\ell}_i\hat{r}_i + \frac{c_s}{r}\big(\ell_r M_r - \ell_i M_i\big)
\end{aligned}
$$

there follows

$$
4\pi\tilde{p}(\mathbf{x},t) = \int_{f=0}\left[\frac{\rho_0\dot{v}_n + \dot{\ell}_r/c_s}{r(1-M_r)^2} + \frac{\ell_r - \ell_i M_i}{r^2(1-M_r)^2} + \frac{\rho_0 v_n + \ell_r/c_s}{r^2(1-M_r)^3}\big(r\dot{M}_r + c_s M_r - c_s M^2\big)\right]_{\text{ret}} dS
$$

This is Farassat's Formulation 1A, which is well suited for numerical evaluation of the noise from helicopter rotors with subsonic tip speeds. This equation is the retarded time formulation. The numerical implementation of this solution is robust and efficient. One approach is to fix the observer time t, and solve for the retarded time τ as integrate over S. An alternative is to fix the source time τ, and find t to which the retarded surface element contributes (simple for a fixed observer), then interpolate to get the required observer time.

Compact Sources

The aerodynamic model of CAMRAD II does not produce the loading on the surface of the blade, instead the interface between the wing solution and wake solution (and with the beam elements in the structural model) is in terms of section loading: lift, drag, and moment at radial stations along the high aspect-ratio blade. For higher fidelity, computational fluid dynamics (CFD, perhaps coupled with CAMRAD II for the blade motion and trim) calculations of the near field provide input for the acoustic integral formulations to calculate the radiated noise. To calculate the rotor noise from the CAMRAD II loading, a compact source form of the noise analysis is required.

Assuming that the chord is small, so r, M, and the retarded time do not vary significantly over the section, the loading noise is

$$
4\pi\tilde{p}_L(\mathbf{x},t) = \int\left[\frac{\dot{L}_r/c_s}{r(1-M_r)^2} + \frac{L_r - L_i M_i}{r^2(1-M_r)^2} + \frac{L_r/c_s}{r^2(1-M_r)^3}\big(r\dot{M}_r + c_s M_r - c_s M^2\big)\right]_{\text{ret}} ds
$$

where $L = \int \ell dx$ is the section loading vector, and s is the spanwise integration variable (Bres, Brentner, Perez, and Jones (2004)).

For the thickness noise, Lopes (2017) writes the Ffowcs Williams-Hawkings equation as

$$
\Box^2\tilde{p}_T = \frac{\partial}{\partial t}\big(\rho_0 v_n|\nabla f|\delta(f)\big) = \frac{\partial^2}{\partial t^2}\big(\rho_0[1 - H(f)]\big)
$$

Solving this using the free space Green's function, and then introducing the compact source assumption, gives

$$
4\pi\tilde{p}_T(\mathbf{x},t) = \frac{\partial^2}{\partial t^2}\int_{f<0}\left[\frac{\rho_0}{r(1-M_r)}\right]_{\text{ret}} dV \cong \frac{\partial^2}{\partial t^2}\int\left[\frac{\rho_0 A_{xs}}{r(1-M_r)}\right]_{\text{ret}} ds
$$

where A_{xs} is the cross section area of the blade. Then moving the time derivative inside the integral gives

$$\begin{aligned} 4\pi\tilde{p}_T(\mathbf{x},t) &= \int \left[\frac{\rho_0 A_{xs}}{1-M_r} \frac{\partial}{\partial\tau} \left\{ \frac{1}{1-M_r} \frac{\partial}{\partial\tau} \left(\frac{1}{r(1-M_r)} \right) \right\} \right]_{\text{ret}} ds \\ &= \int \left[\frac{\rho_0 A_{xs}}{1-M_r} \frac{\partial}{\partial\tau} \left\{ \frac{1}{r^2(1-M_r)^3} (r\dot{M}_r + c_s M_r - c_s M^2) \right\} \right]_{\text{ret}} ds \end{aligned}$$

(Yang, Brentner, and Walsh (2018)). The cross section area is approximately $A_{xs} = 0.685ct$.

Doppler Shift

The rotational noise of the rotor in forward or vertical flight has been derived for an observer moving with the helicopter. For a fixed observer, these solutions can still be used to evaluate the time history of the sound pressure, by using the instantaneous observer position relative to the rotor. The relative motion between the observer and the rotor produces a shift of the frequencies of the perceived sound, which is the Doppler effect. An acoustic source at frequency ω produces a sound pressure proportional to $e^{i\omega\tau}$, where $\tau = t - s(\tau)/c_s$ is the retarded time. The frequency of this sound at the observer then is

$$\omega_{\text{obs}} = \frac{\partial}{\partial t}\omega\tau = \omega\frac{\partial\tau}{\partial t} = \frac{\omega}{1-M_r}$$

since $dt/d\tau = d(\tau + s/c_s)/d\tau = 1 - M_r$, with M_r the Mach number of the helicopter in the direction of the observer. A fixed observer hears a frequency increase as the helicopter approaches and a frequency decrease as the helicopter recedes.

Broadband Noise: Pegg

Broadband noise of the helicopter rotor can be calculated using the model of Pegg (1979). The peak broadband noise frequency (Hz) is

$$f_P = -240 \log T + K_V V_{\text{tip}} + K_0$$

where $T = |\mathbf{F}|$ is the rotor thrust, V_{tip} is the tip speed; and $K_V = 2.448$, $K_0 = 942$ for SI units or $K_V = 0.746$, $K_0 = 786$ for English units. The peak frequency f_P is then identified with a standard one-third-octave frequency band. The one-third-octave sound pressure level spectrum in dB is then

$$SPL_{1/3} = 60 \log M_{\text{tip}} + 10 \log \left[\frac{A_b}{r^2}(\cos^2\theta + 0.1) \right] + S_{1/3} + f(\bar{c}_\ell) + 130$$

where M_{tip} is the tip Mach number, A_b is the rotor blade area, r is the distance from source to observer, and θ is the angle between the thrust axis and the line from the hub to the observer ($\cos\theta = F_i r_i / Tr$). $S_{1/3}$ is a normalized spectrum shape (table), and $f(\bar{c}_\ell)$ is a function of the average blade lift coefficient $\bar{c}_\ell = 6C_T/\sigma$. For most helicopters with $\bar{c}_\ell \leq 0.48$, $f = 10\log(\bar{c}_\ell/0.4)$; and $f = 0.7918 + 80\log(\bar{c}_\ell/0.48)$ for $0.48 \leq \bar{c}_\ell \leq 0.6$.

From Pegg (1979), the sound pressure level increments $S_{1/3}$ in this table are based on an octave-band spectra. A continuous third-octave band spectrum is obtained preserving the total mean-square pressure. Distributing the mean-square pressure equally over the three third-octave bands around the octave-band would give an adjustment $S_{1/3} - 10\log 3$. For a smoother spectrum, the mean-square pressure is distributed as 1/3 to the center octave-band, 2/9 to the two adjacent third-octave bands, and 1/9 to the next two third-octave bands. The result is the third-octave spectrum in the table.

relative band	$S_{1/3}$ (dB)	third-octave	relative band	$S_{1/3}$ (dB)	third-octave
-17		-38.54	4		-16.46
-16		-35.53	5		-16.66
-15	-29.0	-33.77	6	-12.1	-16.87
-14		-31.71	7		-17.91
-13		-30.32	8		-19.27
-12	-24.5	-29.27	9	-16.5	-21.27
-11		-26.91	10		-21.43
-10		-25.39	11		-21.60
-9	-19.5	-24.27	12	-17.0	-21.77
-8		-22.39	13		-22.87
-7		-21.08	14		-24.34
-6	-15.3	-20.07	15	-21.8	-26.57
-5		-18.52	16		-27.64
-4		-17.37	17		-29.05
-3	-11.7	-16.47	18	-26.4	-31.17
-2		-14.59	19		-32.07
-1		-13.28	20		-33.22
0	-7.5	-12.27	21	-30.0	-34.77
1		-13.24	22		-36.53
2		-14.50	23		-39.54
3	-11.5	-16.27			

Broadband Noise: Brooks

Broadband noise of the helicopter rotor can be calculated using the model of Brooks (Brooks, Pope, and Marcolini (1989) and Weir, Jumper, Burley, and Golub (1995)). Rotor broadband noise is produced by turbulent flow interactions with the blade surface. The sources include turbulence ingestion noise, blade-wake interaction noise, and blade self-noise. Airfoil self-noise is produced by the interaction between an airfoil and the turbulence produced in its own boundary layer and wake. The following four self-noise sources are considered.
1) Turbulent boundary-layer trailing-edge noise: At high Reynolds numbers, turbulent boundary layers develop over most of the airfoil, and noise is produced as this turbulence passes over the trailing edge. At high angle-of-attack, the flow separates near the trailing edge to produce noise due to the shed turbulent vorticity. At very high angle-of-attack, large scale separation radiates low frequency noise similar to that of a bluff body.
2) Laminar boundary-layer vortex-shedding noise: At low Reynolds numbers, a laminar boundary layer develops, with instabilities that result in vortex shedding, and associated noise from the trailing edge.
3) Bluntness vortex-shedding noise: Noise is produced by vortex shedding in the small separated flow regions aft of a blunt trailing edge.
4) Tip vortex formation noise: Noise is produced by the vortex shedding and highly turbulent flow occurring at the blade tips.

The Brooks model gives the mean square acoustic pressure from these four sources, for an airfoil segment of chord c, length L, section angle-of-attack $\alpha_\star$, and section velocity U. The noise depends on the section Mach number $M = U/c_s$ and Reynolds number $R_c = cU/\mu$. The model gives the mean square pressure at standard atmospheric conditions, relative the reference pressure $p_{\rm ref} = 20$ μPa. Therefore the pressure results are scaled to the actual density and speed of sound ($\rho_0 c_s^2$).

The bluntness noise calculation requires the trailing edge height h_{TE} and angle ψ_{TE}. The tip noise calculation (only for airfoil segments at the blade tip) depends on whether the tip is rounded on not, and the tip section angle-of-attack can be corrected for the spanwise gradient of lift relative the reference wing (high aspect ratio, untwisted). The laminar boundary-layer noise depends on the boundary layer thickness, and the turbulent boundary-layer and bluntness noise depends on the displacement thickness. These boundary layer characteristics are estimated from the angle-of-attack and Reynolds number, based on measured results for an NACA 0012 airfoil. Boundary layer states untripped (natural transition), tripped, and lightly tripped are considered.

The empirical models used to calculate the broadband noise were developed from measured data for an NACA 0012 airfoil. Hence the angle-of-attack $\alpha_\star$ must be positive, and measured from the zero lift angle: $\alpha_\star = |\alpha - \alpha_{zl}|$, where α is the section angle-of-attack, and α_{zl} is the zero-lift angle for the airfoil profile used. The model's stall criteria for $\alpha_\star$ are still based on the thick, symmetric NACA 0012 profile. Alternatively, the required angle-of-attack can be obtained from the section lift coefficient: $\alpha_\star = |c_\ell|/c_{\ell\alpha}$, where $c_{\ell\alpha}$ is the NACA 0012 lift-curve slope at Mach number M. Then α_{zl} is not needed, but $\alpha_\star$ does not reflect angles beyond the maximum lift. Negative lift is accounted for in the direction of the section normal.

The vector $\mathbf{r}$ from source to observer is resolved in the local coordinates of the airfoil section: x chordwise (towards the trailing edge), y spanwise, and z normal (in the direction of the lift). The orientation of the observer is given by the polar angle $\cos\theta = r_z/r$, and the azimuth angle $\tan\phi = r_z/r_y$. The directivity of the noise is determined by θ and ϕ. Two directivity model are implemented, from Brooks (Brooks, Pope, and Marcolini (1989)) and from ROTONET (Weir, Jumper, Burley, and Golub (1995)).

The Brooks model gives the noise spectrum for source frequencies. The observer frequency is $f_{\text{obs}} = df_{\text{src}}$, where $d = 1/(1 - M_r)$ is the Doppler shift. The mean square pressure spectrum is linearly interpolated (in the logarithm of the frequency) from source to observer frequencies, preserving the total mean square pressure (overall sound pressure).

The broadband spectrum is evaluated for each aerodynamic panel, for the source time τ and source frequencies, at the observer time $t = \tau + r/c_s$ shifted to observer frequencies. The noise is averaged over a period T in observer time:

$$p^2_{\text{ave}} = \frac{1}{T}\int_0^T p^2(t)\,dt = \frac{1}{T}\int_0^T p^2(\tau = t - r/c_s)\,\frac{dt}{d\tau}\,d\tau = \frac{1}{T}\int_0^T p^2(\tau)\,(1 - M_r)\,d\tau$$

and summed over all panels.

Noise Metrics

Sound is measured in units of decibels (dB), defined as

$$10\log_{10}\frac{\text{sound power}}{\text{reference power}}$$

A logarithmic scale is used because of the need to handle orders-of-magnitude differences in the sound levels encountered, and because the human ear has basically a logarithmic response to sound. The energy flux at a given point in the sound field is given by the acoustic intensity $I = E(pu)$. Here p is the perturbation pressure and u is the velocity due to the sound waves, so the instantaneous intensity pu is the power radiated per unit area. In the far field, the velocity and pressure disturbances of a sound wave are related by $u = p/\rho_0 c_s$, so the intensity is

$$I = E(pu) = \frac{E(p^2)}{\rho_0 c_s} = \frac{p^2_{\text{rms}}}{\rho_0 c_s}$$

where ρ_0 is the mean air density and c_s is the speed of sound. Thus the rms pressure is a measure of the acoustic intensity. Moreover, the human ear and the aircraft structure respond to the pressure deviations from the mean atmospheric value.

Hence noise is measured in terms of the sound pressure level, defined as

$$\mathrm{SPL} = 10 \log \frac{p_{\mathrm{rms}}^2}{p_{\mathrm{ref}}^2} = 20 \log \frac{p_{\mathrm{rms}}}{p_{\mathrm{ref}}}$$

in units of dB (re p_{ref}). For the reference pressure, $p_{\mathrm{ref}} = 20$ μPa is normally used. The spectrum of the rms pressure can then be regarded as the distribution of the sound energy over frequency.

The overall sound pressure level (OASPL) is the total rms pressure. Common practice is to measure and present the noise data in terms of its spectrum, either by octave band, third-octave band, or narrow band measurements. Since the subjective perception of sound depends on the frequency content, a number of frequency-weighted measures of the sound pressure level have been developed, notably the A-weighted sound level (dBA) and the perceived noise level (PNdB). The A-weighted sound level is intended to account for the sensitivity of the human ear. The perceived noise level (PNL) is a metric developed for aircraft, using a weighting that depends on both the magnitude and the frequency of the spectrum, based on the sound annoyance level. Further corrections have been developed for aircraft noise. The sound exposure level (SEL) accounts for the sound duration, using the A-weighted sound level. The effective perceived noise level (EPNL) accounts for the sound duration and the presence of discrete frequency tones. SEL, PNL, and EPNL are defined in FAR Part 36.

CAMRAD II Data

In order to calculate the rotor noise, the following information is obtained from the CAMRAD II output file. The job can have more than one case. The rotorcraft model has N_{rotor} rotors, each with N_{blade} blades. The aircraft in the trim task is operating in steady flight (including climb or descent, and turns), so the trim solution is periodic relative airframe axes. The blade motion and loading can be the same for all blades (and results given only for the last blade), or different (results given for all blades).

a) Atmosphere: density ρ_0, speed of sound c_s, kinematic viscosity μ.
b) Flight condition: aircraft speed V (or wind speed W for constrained operation), orientation of airframe relative inertial axes (roll ϕ_F, pitch θ_F, yaw ψ_F), orientation of velocity relative inertial axes (climb θ_V, side ψ_V).
c) Rotor geometry: direction of rotation, radius R, total blade area $A_b = \sigma \pi R^2$ from solidity, reference azimuth ψ_{ref}, aerodynamic panel radial stations r_A (midpoint) and width Δr_A, chord c.
d) Rotor operating condition: rotational speed Ω, tip speed $V_{\mathrm{tip}} = \Omega R$, blade loading C_T/σ.
e) Blade sound sensor: loading and motion at each aerodynamic collocation point, as function of azimuth ψ.
  e1) Discrete force $L_i \Delta s$ (in airframe axes F).
  e2) Source position y_i (in airframe axes F, relative origin F).
  e3) Source velocity v_i (in airframe axes F, relative inertial axes).
  e4) Three-quarter chord position y_{3i} (in airframe axes F, relative origin F).
f) Blade aerodynamic sensors: section velocity U, section angle of attack α, and lift coefficient c_ℓ at aerodynamic collocation points.

In additional to these quantities obtained from the CAMRAD II calculations, the following information is required:

x) Blade cross section area $A_{xs} = 0.684 \tau c^2$, as a function of radial station r/R, in terms of the equivalent thickness ratio τ.
y) For broadband noise: zero lift angle-of-attack α_{zl}, trailing edge height h_{TE}/c, trailing edge angle ψ_{TE}; as a function of radial station r/R.
z) Observer location x_i (in airframe axes F, relative origin F).
  z1) Trajectory of aircraft (origin airframe axes F) relative inertial frame.
  z2) Observer position relative inertial frame, or relative airframe axes.

The source velocity v_i is relative the inertial frame, which is taken as the undisturbed fluid for the free flight case. For a constrained model (wind tunnel), the wind velocity must be added:

$$v = u + V^{FI/F} = u + C^{FV} \begin{pmatrix} W \\ 0 \\ 0 \end{pmatrix}$$

where $C^{FV} = C^{FI} = X_{\phi_F} Y_{\theta_F} Z_{\psi_F}$ for wind tunnel operation. The transformation C^{FI} from inertial axes to airframe axes may also be required for the observer location.

For broadband noise, the rotor loading and motion (mean at hub) are required. The total aerodynamic hub force F_i (in airframe axes F) is calculated from panel force $L_i \Delta s$ (summed radially and over blades, averaged over azimuth). The hub position h_i (in airframe axes F, relative origin F) is calculated from the root panel position y_i (averaged over blades and azimuth). Then $T = |\mathbf{F}|$, $r = |\mathbf{x} - \mathbf{h}|$, $\cos\theta = F_i r_i / Tr$.

When the rotor blades have identical motion (relative each blade's azimuth), the CAMRAD II solution is only for the reference (last) blade. The loading as a function of azimuth and time for the other blades is constructed from that of the reference blade, with the appropriate azimuthal phase shift.

Acoustic Source Data

The acoustic source data are derived from the CAMRAD II data by first, if necessary, converting to SI units (N and m). Then the structures are derived for time derivatives and interpolation of the loading and motion.

The following quantities are required for each rotor/blade/panel, as a function of time: $r_i = x_i - y_i$, $r = |\mathbf{r}|$, $\hat{r}_i = r_i/r$, $M_i = v_i/c_s$, $M_r = \hat{r}_i M_i$, $\dot{M}_r = \hat{r}_i \dot{v}_i / c_s$. This motion is periodic relative the airframe axes, so differentiation and interpolation are implemented by harmonic analysis: y_i, v_i, $\dot{v}_i$ are evaluated at time t from the harmonics of y_i and v_i.

The loading noise requires for each rotor/blade/panel, as a function of time: L_i, $\dot{L}_i$, $L_r = \hat{r}_i L_i$, $\dot{L}_r = \hat{r}_i \dot{L}_i$. The loading is periodic, but often with significant higher-harmonic content. There are three options for interpolating and differentiating the loading. The loading time derivatives can be calculated using finite differences, and then L_i and $\dot{L}_i$ are evaluated at time t by linear interpolation. Alternatively, differentiation and interpolation can be implemented by harmonic analysis: L_i, $\dot{L}_i$ are evaluated at time t from the harmonics of L_i. A third option is linear-harmonic interpolation, which uses a harmonic analysis of the linearly interpolated time series, implemented by multiplying the n-th harmonic by $K_n = (\sin(\pi n/J)/(\pi n/J))^2$ (J is the length of the time series). Linear-harmonic interpolation rounds the corners of the linear interpolation, and suppresses the overshoot of the derivative from harmonic interpolation.

The retarded time is $\tau = t - r/c_s$, so for integration over source time, the observer time is $t = \tau + |\mathbf{x} - \mathbf{y}(\tau)|/c_s$. The source azimuth is $\psi = \Omega\tau + \psi_{\mathrm{ref}}$.

Broadband noise calculation requires local axes of the airfoil section: x chordwise (towards the trailing edge), y spanwise, and z normal (in the direction of the lift). These axes are constructed from the positions of the quarter-chord points y_i and three-quarter-chord points y_{3i} on the blade, as for the CAMRAD II lifting line wing. The chordwise vector at section k is $c_k = y_{3QCk} - y_{QCk}$. From $a_k = y_{QC(k+1)} - y_{QC(k-1)}$, the normal vector is $n_k = \widetilde{c}_k a_k$. Then the radial vector is $t_k = \widetilde{n}_k c_k$. For a clockwise-rotating rotor, the aerodynamic panel index goes from right tip to left tip, so the sign of a_k must be changed. Unit vectors are constructed from c_k, t_k, n_k. Here the local section x-axis is towards the aerodynamic trailing edge, so in reverse flow ($|\alpha| > 90$), the signs of c_k and n_k are changed. The local section z-axis is in the direction of the lift, so for negative lift coefficient the signs of t_k and n_k are changed.

Observer Position

The noise certification requirements are specified in FAR Part 36, Appendix H, "Noise Requirements for Helicopters." The aircraft is flown over three microphones, at the centerline and 150 m to starboard and port.

H36.3(a), Operating conditions: sea level pressure, temperature 25 degC.
H36.3(c), Takeoff profile: start 500 m from the center microphone, 20 m above ground level; fly at best rate of climb, V_y (best rate of climb speed), 100% maximum rated power.
H36.3(d), Flyover profile: 150 m above ground level; fly at minimum of speeds $V = 0.9V_H$, $0.45V_H + 65$, $0.9V_{NE}$, $0.45V_{NE} + 65$) (knots).
H36.3(e), Speeds: V_H is speed at 100% maximum continuous power, sea level and 25 degC, maximum certificated weight; V_{NE} is the never-exceed speed.
H36.3(f): Approach profile: 6 degree descent angle, V_y (best rate of climb speed), rate of descent $V_y \tan(6)$; 120 m above ground level at center microphone.

The aircraft trajectory $\mathbf{x}^I_{ac}$ is in general specified as the position of the center-of-gravity (origin of airframe frame F), in inertial frame axes (x forward, y to the right, z down). The aircraft flight speed is V, and the climb angle θ_V (positive for climb, negative for descent). Thus $x = x_{\text{begin}}$ to x_{end} (negative for the microphone ahead of the aircraft), $t = x/V \cos\theta_V$, $y = 0$, and $z = -z_{\text{ref}} - \tan\theta_V(x - x_{\text{ref}})$ (z_{ref} is the altitude at x_{ref}).

The microphone position is $\mathbf{x}^I_{\text{mic}}$, relative inertial axes. Thus the observer position (microphone relative aircraft center-of-gravity, in airframe axes) is $\mathbf{x} = C^{FI}(\mathbf{x}^I_{\text{mic}} - \mathbf{x}^I_{ac})$. Then the certification profiles are:

a) Takeoff profile: $x_{\text{ref}} = -500$, $z_{\text{ref}} = 0$.
b) Flyover profile: x_{ref} arbitrary (since $\theta_V = 0$), $z_{\text{ref}} = 150$.
c) Approach profile: $x_{\text{ref}} = 0$, $z_{\text{ref}} = 120$.
d) Microphone: $x = z = 0$ and $y = 0$ (centerline), or $y = +150$ (starboard), or $y = -150$ (port).

The speed V, climb angle θ_V, and flight conditions are obtained from the CAMRAD II output.

Alternatively, the observer position $\mathbf{x}$ can be specified directly, either in airframe axes, or inertial axes so $\mathbf{x}^F = C^{FI}\mathbf{x}^I$.

Sound Calculation

The rotor noise is calculated for each CAMRAD II case, and each observer position. The basic sound value for rotational (loading and thickness) noise is the pressure as a function of time, over one period. From this time series, a fast-Fourier-transform gives the narrow band spectrum (harmonics). The BVI (blade-vortex interaction) sound pressure level is calculated from the narrow band spectrum over a specified frequency range, typically 10 go 50 blade-passage harmonics. These harmonics are collected based on frequency to produce the third-octave spectra. The following table gives the third-octave center frequencies, and the upper and lower frequencies of each band. These spectra are calculated in terms of the mean-square pressure, from which the sound pressure level is calculated: $\text{SPL} = 10\log(p^2_{\text{rms}}/p^2_{\text{ref}})$ in dB. The sum over the spectrum gives the overall mean-square pressure, from which the overall sound pressure level OASPL (dB) is calculated. Applying a correction (given in the following table) to the third-octave spectrum gives the A-weighted overall sound pressure level (dBA). For rotors operating at different rotational speeds, the noise can be added in the frequency domain but not in the time domain (since the present analysis deals only with periodic rotor loading and motion).

The broadband noise is calculated in the frequency domain, so the sound values consist of the third-octave spectrum (dB), from which the mean-square-pressure spectrum can be calculated, and the overall sound pressure level

OASPL (dB and dBA). The total rotor noise, which is the sum of the rotational and broadband noise from all rotors, is thus also only available in the frequency domain.

For each case and observer position, the loading noise is evaluated for each rotor and each blade by summing (integrating) the contributions from the loading acting on all aerodynamic panels of the blade. For each panel, the sound is calculated for the source time over one period, $\tau = 0$ to T, at N_{time} time steps. The sound pressure contribution Δp is saved with the observer time $t = \tau + r/c_s$ (monotonic with source time for subsonic M_r). When the complete period has been calculated, the sound pressure $\Delta p(t)$ is linearly interpolated to N_{time} uniformly-spaced time steps over the period $t = 0$ to T. The sum of the pressure for all aerodynamic panels gives the sound produced by a blade over one period of the observer time.

The thickness noise is evaluated for each rotor and each blade by summing (integrating) the contribution from the cross-section area A_{xs} of N_{span} radial stations along the blade, uniformly spaced from root cutout to the tip. The time derivative in the integrand of the thickness noise for compact sources is evaluated by numerical differentiation. The sum of the pressure for all span stations gives the sound produced by a blade over one period of the observer time.

The sound pressure from all blades is summed to obtain the loading and thickness sound for each rotor. The loading and thickness sound pressure are summed to obtain the rotational sound of each rotor. For each blade and each rotor, from the pressure time histories of the loading, thickness, and rotational sound, the other sound values are calculated, including narrow band and third-octave spectra (mean-square pressure and dB), total mean-square pressure, and OASPL and A-weighted OASPL.

For each case and observer position, the broadband noise is calculated for each rotor, in terms of the third-octave spectrum. Then the other sound values are calculated (mean-square pressure spectrum, OASPL, and OASPLA). The total rotor sound is the sum of the third-octave pressure spectra of the rotational sound and broadband sound, from which the spectrum (dB), OASPL, and OASPLA are calculated.

For rotors that have the same rotational speed and period (probably because they are connected by a drive train, with gear ratios equal 1.0), the time histories of loading, thickness, and rotational sound pressures can be added. The third-octave, mean-square pressure spectra of the broadband sound can be summed over all rotors, to obtain the aircraft broadband sound. The third-octave, mean-square pressure spectra of the total sound can be summed over all rotors, to obtain the aircraft total sound.

The components of calculated sound are summarized in the table below. Loading and thickness noise (and the sum, rotational noise) are calculated in the time domain. Broadband noise is calculated in the frequency domain, and summing noise from rotors with different periods must be done in the frequency domain, so in general the total noise is also in the frequency domain. From the total sound calculated for an aircraft trajectory (as third-octave spectrum), the sound exposure level (SEL) and the effective perceived noise level (EPNL) are obtained, as defined in FAR Part 36.

each blade	loading	thickness			
sum blades	$\Downarrow$	$\Downarrow$			
each rotor	loading	+ thickness	= rotational	+ broadband (F)	= total (F)
sum rotors with same period	$\Downarrow$	$\Downarrow$			
each rotor	loading	+ thickness	= rotational		
sum all rotors	$\Downarrow$	$\Downarrow$			
total aircraft (F)	loading	+ thickness	= rotational	+ broadband	= total

F = frequency domain only

band	1/3-octave band frequencies (Hz) center	lower	upper	dBA correction
0	1	0.9	1.12	-148.58
1	1.25	1.12	1.4	-140.83
2	1.6	1.4	1.8	-132.28
3	2	1.8	2.24	-124.55
4	2.5	2.24	2.8	-116.85
5	3.15	2.8	3.55	-108.89
6	4	3.55	4.5	-100.72
7	5	4.5	5.6	-93.14
8	6.3	5.6	7.1	-85.40
9	8	7.1	9	-77.55
10	10	9	11.2	-70.43
11	12.5	11.2	14	-63.58
12	16	14	18	-56.42
13	20	18	22.4	-50.39
14	25	22.4	28	-44.82
15	31.5	28	35.5	-39.52
16	40	35.5	45	-34.54
17	50	45	56	-30.27
18	63	56	71	-26.22
19	80	71	90	-22.40
20	100	90	112	-19.14
21	125	112	140	-16.19
22	160	140	180	-13.24
23	200	180	224	-10.85
24	250	224	280	-8.67
25	315	280	355	-6.64
26	400	355	450	-4.77
27	500	450	560	-3.25
28	630	560	710	-1.91
29	800	710	900	-0.79
30	1000	900	1120	0.00
31	1250	1120	1400	0.58
32	1600	1400	1800	0.99
33	2000	1800	2240	1.20
34	2500	2240	2800	1.27
35	3150	2800	3550	1.20
36	4000	3550	4500	0.96
37	5000	4500	5600	0.55
38	6300	5600	7100	-0.12
39	8000	7100	9000	-1.15
40	10000	9000	11200	-2.49

Program CIISound

The calculation of rotor noise from CAMRAD II results is implemented in the Fortran program CIISound. This is a Fortran program, which should be compiled with double precision. Input is read through the namelist CIISound_input; the input variables are defined in the table below. Output consists of text files with tab-delimited data:

a) Performance file (logical name fileperf): rotor and rotorcraft perfromance data from the CAMRAD II output file.
b) Sensor file (logical name filesen): sound sensor data extracted from the CAMRAD II output file.
c) Source file (logical name filesrc): sound source data, calculated from the sensor data.
d) Sound file (logical name filesnd): noise results, consisting of spectra and time histories for
  d1) Aircraft: total, broadband, rotational, loading, thickness.
  d2) Summed rotors: rotational, loading, thickness (rotors with same period).
  d3) Each rotor: total, broadband, rotational, loading, thickness.
  d4) Each rotor and each blade: loading, thickness.

Rotational noise is the sum of loading and thickness noise, which are calculated in the time domain for each blade of each rotor, summed for all blades, and then summed for all rotors with the same period. Spectra and metrics (OASPL, OASPLA, PNL, PNLT, BVISOL) are calculated from the pressure time histories. Broadband noise is calculated in the frequency domain (SPL or mean-square pressure). The aircraft noise is the sum of the rotational noise and broadband noise spectra of all rotors.

The blade loading, motion, and velocity are obtained using the CAMRAD II sound sensor. For best results a high azimuthal resolution is needed, using the post-trim analysis. The noise can also be calculated using the standard resolution, without the post-trim analysis. For BVI airloads using lifting-line theory and a small azimuthal step size, a large vortex core radius is recommended (Boyd, Greenwood, Watts, and Lopes (2017)). The required CAMRAD II input is:

NLJOB input: OUTRES=0 (standard) or OUTRES=1 (enhanced)
TRIM input: MPSI=24,OPPOST=1,MPSIH=240 (post-trim analysis)
TRIM ROTOR input: MPSEN=1,MPTIME=240 (sound sensor)
ROTOR STRUCTURE input: OPSND=1 (sound sensor)
ROTOR WAKE input: CORE=0.8,COREWG=0.8 (vortex core size), OPVCG=5,5,RVCG=2.,2. (core growth)

The Brooks broadband noise model requires the CAMRAD II aerodynamic sensors: section velocity, angle-of-attack, and lift coefficient (all three, in that order). The required CAMRAD II input is:

ROTOR AERODYNAMICS input: NSEN=3,QUANT=24,25,61,IDENT=0,0,1,OPSCL=0,1,0 (aero sensors)
TRIM ROTOR input: MASEN=1,MATIME=240 (aero sensors)

Namelist CIISound_input

Variable Name	Type	Definition	Default
KIND_obs(ncase)	int	observer definition (0 trajectory, 1 input (airframe), 2 input (inertial))	1
		trajectory (KIND_obs = 0)	
KIND_traj(ncase)	int	trajectory defn (0 general (xref, zref), 1 takeoff, 2 flyover, 3 approach)	0
KIND_mic(ncase)	int	microphone definition (0 general (xmic), 1 center, 2 starboard, 3 port)	0
Ntraj	int	number of trajectory steps	51
xbegin(ncase)	real	beginning aircraft position, negative for microphone ahead (m)	-500.
xend(ncase)	real	ending aircraft position (m)	500.
xref(ncase)	real	trajectory reference position (m)	0.
zref(ncase)	real	trajectory altitude above ground level, at xref (m)	0.
xmic(3,ncase)	real	microphone position, inertial axes (m)	0.
		input observers (KIND_obs = 1 or 2)	
nObserver(ncase)	int	number of observers	1
position_obs(3,nobs,ncase)			
	real	observer position, relative airframe origin, airframe or inertial axes (m)	0
		analysis	
Ntime	int	number of time steps	1024
Nspan	int	number of radial stations, thickness noise (root cutout to tip)	20
interp	int	loading interpolation (1 linear, 2 harmonic, 3 linear-harmonic)	3
NfreqBVI(2)	int	blade passage frequency range for BVI SPL	10,50
		model	
nprop	int	number radial stations	2
rprop(nprop,nrtr)	real	radial stations (r/R)	0.,1.
thick(nprop,nrtr)	real	thickness ratio ($A_{xs} = 0.685$*thick*chord^2)	0.12
MODEL_broadband	int	broadband noise model (1 Pegg, 2 Brooks)	1
OMIT_broadband	int	broadband sound (1 to omit from total sound)	0
		Brooks broadband noise model	
READ_aero	int	read aerodynamic sensors (0 not); required for Brooks model	0
OMIT_TBLTE	int	turbulent boundary-layer, trailing edge noise (1 to omit)	0
OMIT_LBLVS	int	laminar boundary-layer, vortex shedding noise (1 to omit)	0
OMIT_BLUNT	int	trailing-edge bluntness noise (1 to omit)	0
OMIT_TIP	int	tip vortex formation noise (1 to omit)	0
MODEL_direct	int	directivity (0 Brooks, 1 ROTONET)	1
MODEL_alpha	int	$\alpha_\star$ (1 from angle-of-attack, 2 from lift coefficient)	1
MODEL_trip	int	boundary layer (0 untripped (natural trans.), 1 tripped, 2 lightly tripped)	1
Nbbave	int	number of time steps in average	24
alpha_zl(nprop,nrtr)	real	zero-lift angle-of-attack (deg) (for MODEL_alpha = 1)	0
hTE(nprop,nrtr)	real	trailing edge thickness ratio h/c (BLUNT noise)	0.00252
psiTE(nprop,nrtr)	real	trailing edge angle (deg) (BLUNT noise)	15.974
isRound(nrtr)	int	blade tip cap shape (1 rounded) (TIP noise)	0
ftipalpha(nrtr)	real	blade tip angle-of-attack correction factor (TIP noise)	1.
		output	
OUT_perf	int	write performance file (logical name fileperf); 0 no write	0
OUT_sensor	int	write sound sensor file (logical name filesen); 0 no write, 1 English, 2 SI	0
OUT_source	int	write sound source file (logical name filesrc); 0 no write	0
OUT_sound	int	write sound file (logical name filesnd); 0 no write, 1 basic, 2 third-octave spectra, 3 narrow band and time history, 4 blades	1
OUT_soundfiles	int	write sound file (0 single file, 1 separate files)	0

ncase is the number of cases in the CAMRADII output file, and nrtr is the number of rotors

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